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ANALYSES OF COMPOSITE STRUCTURES

by Stephen W. Tsai, Donald F. Adams, and Douglas R. Doner

Prepared by
PHILCO CORPORATION
Newport Beach, Calif.
for Western Operations Office

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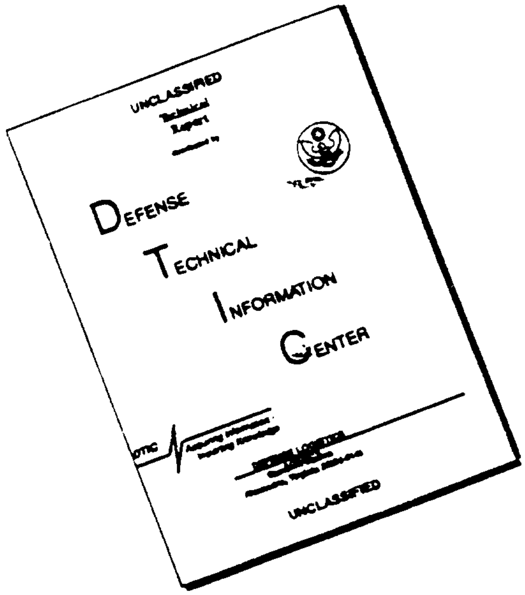

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FOREWORD

This is an annual report of the work done under National Aeronautics and Space Administration Contract NAS 7-215, entitled "Structural Behavior of Composite Materials," for the period January 1965 to January 1966. The program is monitored by Mr. Norman J. Mayer, Chief, Advanced Structures and Materials Application, Office of Advanced Research and Technology.

The authors wish to acknowledge the contributions of their consultants Dr. G. S. Springer of the Massachusetts Institute of Technology, Dr. A. B. Schultz of the University of Illinois, and Dr. H. B. Wilson, Jr. of the University of Alabama. The assistance of Mr. R. L. Thomas and Mrs. V. A. Tischler of Aeronutronic is also gratefully acknowledged.

Particular recognition is given to Dr. Wilson for his work in establishing the fundamental concepts upon which the periodic inclusion problems of Sections 3 and 4 are based.

ABSTRACT

The stiffness and strength analyses of composite materials previously presented have been reviewed and extended to cross-ply and helical-wound cylinders, as well as flat laminates. Consideration has been given to the composite behavior after initial yielding, including the influence of filament crossovers in helical-wound cylinders. In doing so, a modified "netting analysis" has been used in conjunction with the continuum analysis to predict both initial yielding and post-yielding behavior.

Cylinders were assumed to be subjected to various loading conditions, including axial tension and compression, torsion, and internal pressure. Theoretical results were then compared with experimental data obtained using glass-epoxy composites.

Investigations have also been made of the relative contributions of the constituent material properties to the gross behavior of a unidirectional fiber-reinforced composite when subjected to various loading conditions. Theoretical values obtained for the prediction of the stiffness and strength of the composite as a function of constituent properties have been compared with experimental data obtained using both glass-epoxy and boron-epoxy systems.

Complete digital computer programs, developed in conjunction with the strength analyses of flat laminates and laminated composite cylinders, and the investigation of stress distributions in the fibers and matrix of a composite subjected to either longitudinal shear or transverse normal loading, are presented in Appendices A, B, and C.

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CONTENTS (Continued)

ILLUSTRATIONS

ILLUSTRATIONS (Continued)

ILLUSTRATIONS (Continued)

NOMENCLATURE

A_{ij}	=	A	=	In-plane stiffness matrix, lb/in.
A^*_{ij}	=	A^*	=	Intermediate in-plane matrix, in./lb
A'_{ij}	=	A'	=	In-plane compliance matrix, in./lb
a	=			Length of the upper and lower boundaries of the first quadrant of the fundamental region surrounding one inclusion, in.
B_{ij}	=	B	=	Stiffness coupling matrix, lb
B^*_{ij}	=	B^*	=	Intermediate coupling matrix, in.
B'_{ij}	=	B'	=	Compliance coupling matrix, 1/lb
b	=			Length of the left and right boundaries of the first quadrant of the fundamental region surrounding one inclusion, in.
C_{ij}	=			Anisotropic stiffness matrix, psi
D_{ij}	=	D	=	Flexural stiffness matrix, lb-in.
D^*_{ij}	=	D^*	=	Intermediate flexural matrix, lb-in.
D'_{ij}	=	D'	=	Flexural compliance matrix, 1/lb-in.
E	=			Modulus of elasticity, psi
E_{11}	=			Composite axial stiffness, psi
E_{22}	=			Composite transverse stiffness, psi

G	=			Shear modulus, psi
H^*_{ij}	=	H^*	=	Intermediate coupling, matrix, in.
h	=			Total thickness, in.
M_i	=	M	=	Distributed bending (and twisting) moments, lb
M_i^T	=	M^T	=	Thermal moments, lb
$\overline{M}_i$	=	$\overline{M}$	=	Effective moment $= M_i + M_i^T$
m	=			$\cos\theta$ or cross-ply ratio (total thickness of odd layers over that of even layers)
N_i	=	N	=	Stress resultant, lb/in.
N_i^T	=	N^T	=	Thermal stress resultant, lb/in.
$\overline{N}_i$	=	$\overline{N}$	=	Effective stress resultant $= N_i + N_i^T$
N_f	=			Stress in the direction of the fibers per inch of thickness, lb/in.
n	=			$\sin\theta$, or total number of layers
P	=			Internal pressure, psi
R	=			Radius, in.
r	=			Ratio of normal strengths $= X/Y$
S	=			Shear strength of unidirectional composite, psi
s	=			Shear strength ratio $= X/S$, or standard deviation of fiber strength
SCF	=			Stress concentration factor
T	=			Temperature, degree F
u, v, w	=			Displacement components, in.
v_f	=			Percent fiber content by volume
X	=			Axial tensile strength of unidirectional composite, psi

X' = Axial compressive strength of unidirectional composite, psi

Y = Transverse tensile strength of unidirectional composite, psi

Y' = Transverse compressive strength of unidirectional composite, psi

z = Distance as measured from the middle surface, in.

α_i = Thermal expansion coefficient, in./in./degree F

β = Matrix effectiveness in "shear transfer"

ϵ_i = Strain component, in./in.

ϵ_i^o = In-plane strain component, in./in.

θ = Fiber orientation or lamination angle, degree

κ_i = Curvature, 1/in.

ν = Poisson's ratio

σ_i = Stress component, psi

σ_B = Fiber bundle strength, psi

$\overline{\sigma}$ = Average deviation of the fiber strength

τ_{ij} = Shear stress, psi

SUBSCRIPTS

f = fiber

m = matrix

i, j, k = 1, 2, ... 6 or x, y, z in 3-dimensional space, or 1, 2, 6 or x, y, s in 2-dimensional space

SUPERSCRIPTS

k	=	kth layer of a laminated composite
-1	=	Inverse matrix
H	=	Hoop layers (odd layers) of a cross-ply cylinder or pressure vessel
L	=	Longitudinal layers (even layers) of a cross-ply cylinder or pressure vessel

SECTION 1

INTRODUCTION

This is a continuing attempt to develop a rational approach to the design and utilization of composite materials in structural applications. Previous efforts[1,2*] were concerned with the establishment of the independent elastic moduli and strength parameters from the macroscopic viewpoint.

The current effort is concerned with the development of guidelines for the design of composite structures. The determination of the deformation and load-carrying capacity of filamentary structures is outlined. Helical-wound tubes subjected to various loading conditions are examined in detail. The behavior of this structural element is expressed in terms of various lamination parameters including the helical wrap angle, number of layers, etc., and material parameters such as the properties of the constituent materials, the cross-sectional shape of the filaments, etc. The present theory of design of composite materials can be applied to the analysis and design of filamentary structures.

The weak link in a fiber-reinforced composite, as exhibited by the initial yielding, is closely associated with the low strength levels attainable in a direction transverse to the fibers and in shear. For this reason, the transverse and shear properties of a unidirectional composite are analyzed, the results providing information needed in improving composite materials.

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[*] References are listed at the end of this report.

The present theory of design of composite materials is only preliminary. A number of refinements and appropriate experimental verification remain to be explored. In particular, inelastic behavior both on the macroscopic and microscopic levels and the effect of filament crossovers are two problems that deserve immediate attention. It is hoped that as the theory is improved, the extent of empiricism can be substantially reduced in the design and utilization of composite materials.

SECTION 2

STRENGTH ANALYSIS

Anisotropic Yield Condition

The anisotropic yield condition, as reported in Reference 2, is derived from a generalization of the von Mises yield condition for isotropic materials.[3] It is assumed that the yield condition is a quadratic function of the stress components

$$
\begin{aligned}
2f(\sigma_{ij}) &= F(\sigma_y - \sigma_z)^2 + G(\sigma_z - \sigma_x)^2 + H(\sigma_x - \sigma_y)^2 \\
&\quad + 2L\,\tau_{yz}^2 + 2M\,\tau_{zx}^2 + 2N\,\tau_{xy}^2 = 1
\end{aligned}
\tag{1}
$$

where F, G, H, L, M, N are material coefficients characteristic of the state of anisotropy, and x, y, z, are the axes of the assumed orthotropic material symmetry. Equation (1) reduces to the von Mises condition if

$$F = G = H = 1/6k^2$$

$$L = M = N = 1/2k^2$$

where k is a material parameter governing the yielding of isotropic materials.

Since the composite material of present interest is in a form of relatively thin plates, a state of plane stress is assumed. Equation (1) can be reduced to:

$$\left(\frac{\sigma_x}{X}\right)^2 - \frac{1}{r}\frac{\sigma_x}{X}\frac{\sigma_y}{Y} + \left(\frac{\sigma_y}{Y}\right)^2 + \left(\frac{\sigma_s}{S}\right)^2 = 1 \qquad (2)$$

The validity of this yield condition has been demonstrated in Reference 2, using unidirectional glass-epoxy composites subjected to tensile loads.

For the strength analysis of a filamentary structure subjected to combined loading, compressive properties must be known. Analogous to the tensile strengths X and Y, the compressive strengths X' and Y' are determined from 0- and 90-degree specimens subjected to uniaxial compressive loads, respectively. Shear has no directional property, hence, S = S'.

It is assumed that the anisotropic yield condition remains applicable for materials with properties different in tension and compression. It is only necessary to use the principal strengths compatible with the prevailing stress components, i.e., tensile strength for positive normal stress and compressive strength for negative normal stress. This method of taking into account different tensile and compressive properties follows those used previously by other investigators.[4,5] Equation (2) can now be written in four forms corresponding to the four quadrants of the σ_x - σ_y stress space. The quadrant descriptions are as follows:

Quadrant	σ_x	σ_y	Axial Strength	Transverse Strength	Strength Ratio
1	positive	positive	X	Y	r_1 = X/Y
2	negative	positive	X'	Y	r_2 = X'/Y
3	negative	negative	X'	Y'	r_3 = X'/Y'
4	positive	negative	X	Y'	r_4 = X/Y'

In terms of these definitions, the yield condition given by Equation (2) becomes, in the order of the corresponding quadrant:

$$\left(\frac{\sigma_x}{X}\right)^2 - \frac{1}{r_1}\frac{\sigma_x}{X}\frac{\sigma_y}{Y} + \left(\frac{\sigma_y}{Y}\right)^2 + \left(\frac{\sigma_s}{S}\right)^2 = 1 \tag{3}$$

$$\left(\frac{\sigma_x}{X'}\right)^2 - \frac{1}{r_2}\frac{\sigma_x}{X'}\frac{\sigma_y}{Y} + \left(\frac{\sigma_y}{Y}\right)^2 + \left(\frac{\sigma_s}{S}\right)^2 = 1 \tag{4}$$

$$\left(\frac{\sigma_x}{X'}\right)^2 - \frac{1}{r_3}\frac{\sigma_x}{X'}\frac{\sigma_y}{Y'} + \left(\frac{\sigma_y}{Y'}\right)^2 + \left(\frac{\sigma_s}{S}\right)^2 = 1 \tag{5}$$

$$\left(\frac{\sigma_x}{X}\right)^2 - \frac{1}{r_4}\frac{\sigma_x}{X}\frac{\sigma_y}{Y'} + \left(\frac{\sigma_y}{Y'}\right)^2 + \left(\frac{\sigma_s}{S}\right)^2 = 1 \tag{6}$$

The signs for the principal strengths are always positive; those for the stress components are positive or negative, corresponding to the appropriate quadrant in the stress space. Diagrammatically, the yield surface can be represented in dimensionless form as shown in Figure 1.

For unidirectional glass-epoxy composites (v_f = 70%),

$r_1 = X/Y = 150/4 = 37.5$

$r_2 = X'/Y = 150/4 = 37.5$

$r_3 = X'/Y' = 150/20 = 7.5$

$r_4 = X/Y' = 150/20 = 7.5$

This is represented by the solid curves in Figure 2.

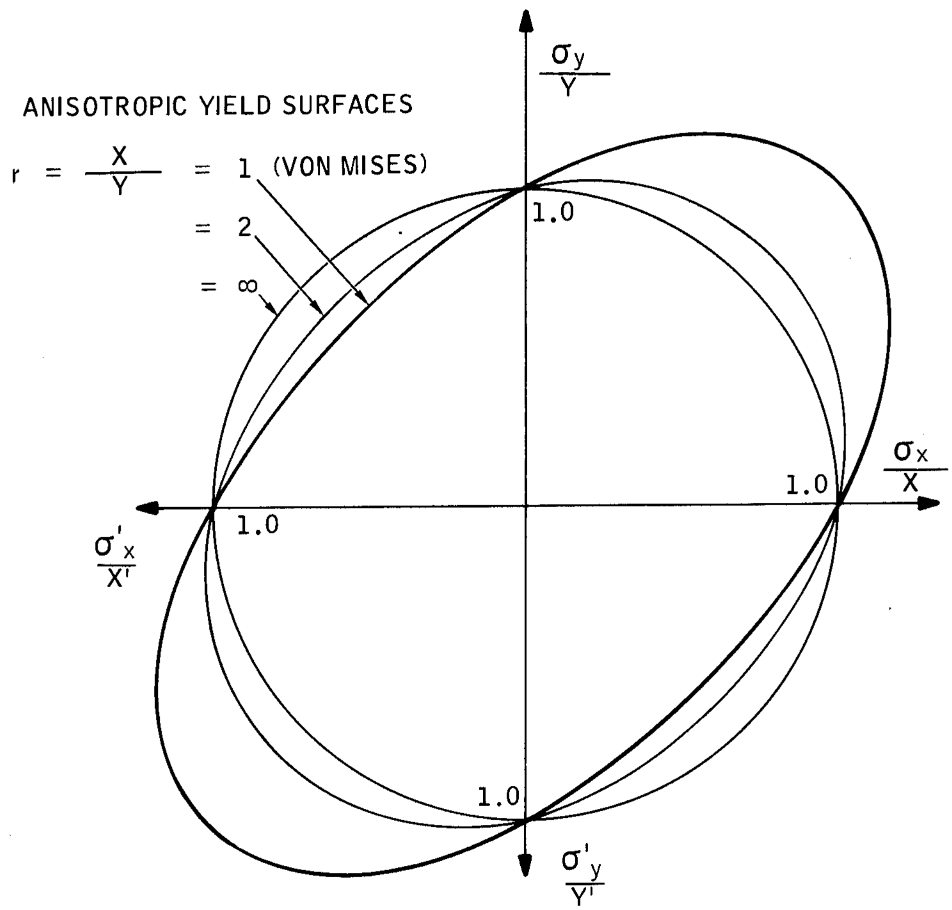


Figure 1. Comparative Yield Surfaces

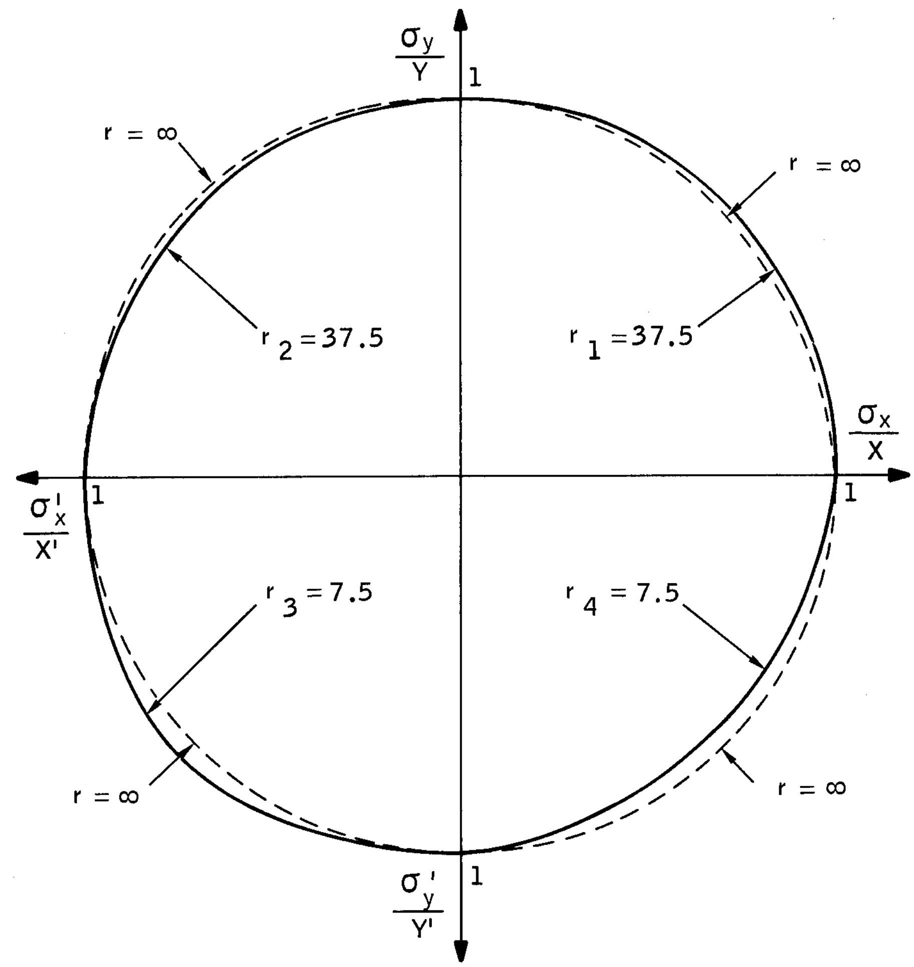


Figure 2. Yield Surfaces for Glass-Epoxy Composites

The yield conditions of Equations (2) through (6) apply to an orthotropic material in the directions of its material symmetry axes. For unidirectional composites, the symmetry axes are parallel and perpendicular to the fibers. If the fibers are oriented other than 0- or 90-degrees with respect to the externally applied load, the applied stress components σ_i, $i = 1, 2, 6$, must be transformed to the symmetry axes, $i = x, y, s$, before the yield condition can be applied.[2] The usual transformation equation for stress components, in matrix form, is

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_s \end{bmatrix} = \begin{bmatrix} m^2 & n^2 & 2mn \\ n^2 & m^2 & -2mn \\ -mn & mn & m^2-n^2 \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_6 \end{bmatrix} \tag{7}$$

For uniaxial tension,

$$\sigma_1 = \text{positive}, \quad \sigma_2 = \sigma_6 = 0 \tag{8}$$

From Equation (7),

$$\sigma_x = m^2\sigma_1, \quad \sigma_y = n^2\sigma_1, \quad \sigma_s = -mn\sigma_1 \tag{9}$$

Substituting these values into the appropriate yield condition, Equation (3), one obtains:

$$m^4 + \left(s_1^2 - 1\right) m^2n^2 + r_1^2 n^4 = (X/\sigma_1)^2 \tag{10}$$

which is identical with Equation (9) of Reference 2, where

$$s_1 = s = X/S, \quad r_1 = r = X/Y$$

In the same manner, for uniaxial compression, the appropriate yield condition equation is

$$m^4 + \left(s_3^2 - 1\right) m^2 n + r_3^2 n^4 = (X'/\sigma_1)^2 \tag{11}$$

where $s_3 = s = X'/S, \quad r_3 = r = X'/Y'$

For pure shear, the yield condition corresponding to the second or fourth quadrant will be needed. This can easily be derived by taking σ_6 as the only nonzero stress component. If r_2 and r_4 are different, which is usually the case, the shear strength of a unidirectional composite will have different values depending on the direction of the applied shear, i.e., positive or negative shear.

In summary, the initial yielding of a unidirectional composite, when subjected to a complex state of stress, is governed by one of four possible yield conditions. The appropriate condition to be used is determined by the signs of the normal stress components. If the tensile and compressive strengths are equal, the four conditions reduce to one equation; such is the case in Equation (4) of Reference 3.

Compressive Properties

In a previous study,[2] the principal strengths were limited to tensile loading only. However, in the strength analysis of a structure subjected to combined loading, the compressive properties of unidirectional composites must also be known.

Compressive elastic moduli have been found to be approximately the same as tensile moduli for glass-epoxy composites[1] and boron-epoxy composites.[6] Compressive axial and transverse strengths, X' and Y',

respectively, can be determined by the compressive loading of 0- and 90-degree specimens. Compression tests are known to be difficult to perform. Test results often are affected by the geometric configuration of the specimen. Competing modes of failure, i.e., buckling and strength, are operative.

As an indication of the difficulty of direct measurement of the compressive axial strength, X', the numerical value of X' for glass-epoxy composites has been reported as anywhere within a range of from 100 to 250 ksi, depending upon the test method used. In flexural tests of 0-degree specimens, which include a hoop-wound ring pin-loaded at diametrically opposite points, most failures are of the tensile type. It appears reasonable to assume that the compressive strength is at least equal to, if not higher than, the tensile strength. In the present work, a value of 150 ksi is assumed for both the tensile and compressive strengths of the glass-epoxy composite. This value is undoubtedly conservative.

The compressive transverse strength Y' is comparatively simple to determine because of its low numerical value. For glass-epoxy composites, with v_f = 70 percent, the value of Y' is between 16 and 24 ksi. The lower values were obtained using specimens having rectangular cross sections; the higher values, circumferentially wound tubes with over-wound (reinforced) ends. No gross buckling of the specimens was observed. Using the experimentally determined principal strengths,

$$X' = 150 \text{ ksi}$$

$$Y' = 20 \text{ ksi}$$

$$S = 6 \text{ ksi}$$

from which,

$$r_3 = X'/Y' = 150/20 = 7.5$$

$$s_3 = X'/S = 150/6 = 25$$

one can determine, using Equation (11), the uniaxial compressive strength σ_1 as a function of fiber orientation. The resulting curve, together with experimental data, is shown in Figure 3. The corresponding uniaxial stiffness and tensile strength are also shown. The tensile and compressive stiffnesses are practically identical when the strain is small, i.e., in the order of 0.1 percent.

Strength of Laminated Composites

For the sake of completeness, the strength analysis of laminated composites described in Reference 2 is summarized here. Essentially, the strength of materials approach is used, whereby the normals to the middle surface remain undeformed during the stretching and bending of the composite plate. The total strain at any point in the plate is defined as

$$\epsilon_i = \epsilon_i^o + z\kappa_i \tag{12}$$

It is further assumed that each constituent layer of the laminated composite is mechanically and thermally anisotropic, i.e.,

$$\sigma_i = C_{ij}(\epsilon_j - \alpha_j T) \tag{13}$$

where i, j = 1, 2, and 6.

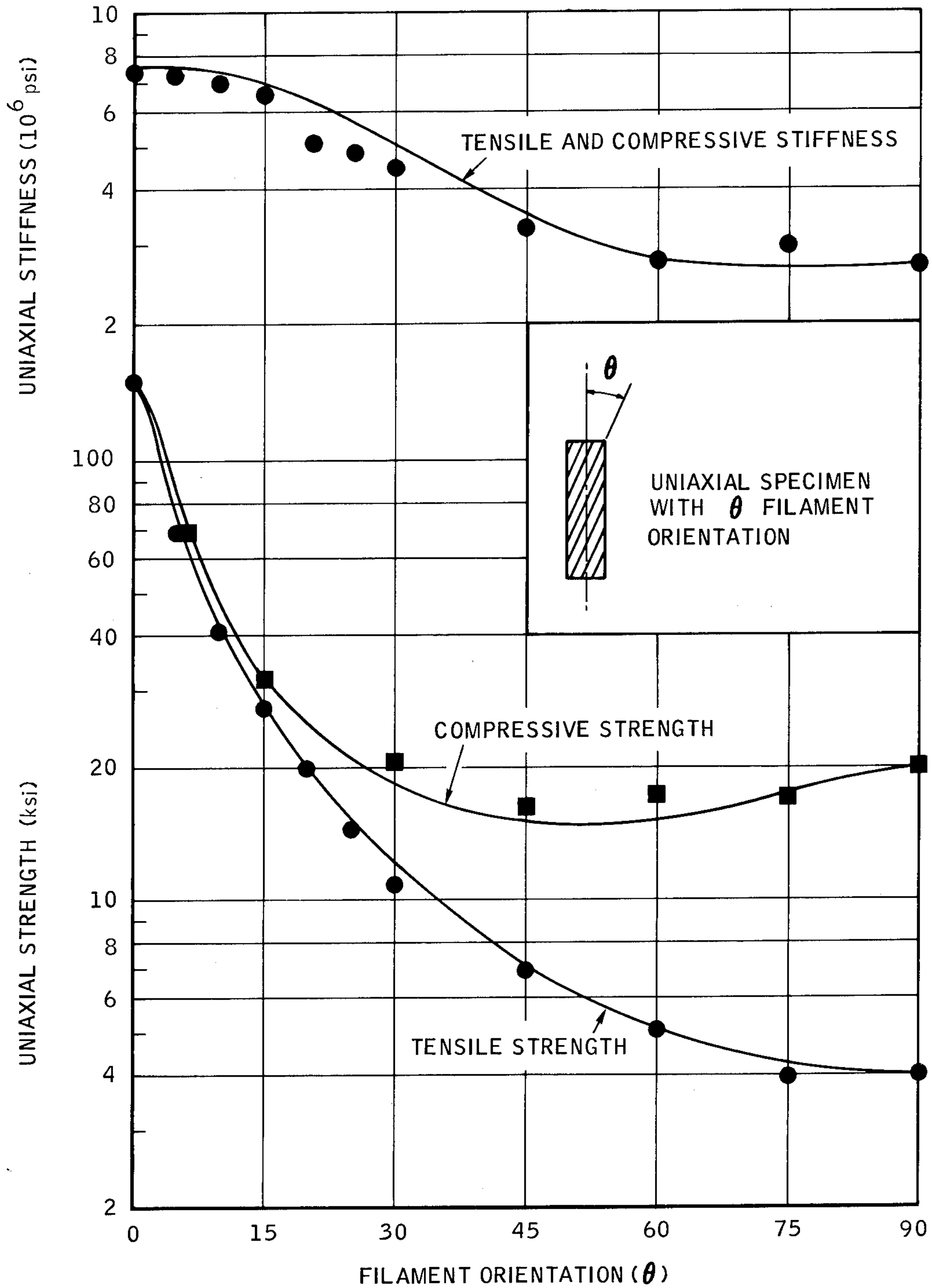


Figure 3. Uniaxial Properties of Glass-Epoxy Composites

Equation (13), when integrated across the thickness of the laminated composite, becomes:

$$\overline{N}_i = N_i + N_i^T = A_{ij}\,\epsilon_j^o + B_{ij}\,\varkappa_j \tag{14}$$

$$\overline{M}_i = M_i + M_i^T = B_{ij}\,\epsilon_j^o + D_{ij}\,\varkappa_j \tag{15}$$

where

$$(N_i, M_i) = \int_{-h/2}^{h/2} \sigma_i \,(1, z)\, dz \tag{16}$$

$$(N_i^T, M_i^T) = \int_{-h/2}^{h/2} C_{ij}\,\alpha_j T\,(1, z)\, dz \tag{17}$$

$$(A_{ij}, B_{ij}, D_{ij}) = \int_{-h/2}^{h/2} C_{ij}\,(1, z, z^2)\, dz \tag{18}$$

Equations (14) and (15) are the basic constitutive equations for a laminated anisotropic composite, taking into account equivalent thermal loadings.

The stress at any location across the thickness of the composite can be expressed in the following manner.[2] Having established that

$$\begin{bmatrix} \overline{N} \\ \\ \overline{M} \end{bmatrix} = \left[\begin{array}{c|c} A & B \\ \hline B & D \end{array}\right] \begin{bmatrix} \epsilon^o \\ \\ \varkappa \end{bmatrix} \tag{19}$$

then, by matrix inversion,

$$\begin{bmatrix} \epsilon^{o} \\ \\ \overline{M} \end{bmatrix} = \left[\begin{array}{c|c} A^* & B^* \\ \hline H^* & D^* \end{array}\right] \begin{bmatrix} \overline{N} \\ \\ \varkappa \end{bmatrix} \tag{20}$$

$$\begin{bmatrix} \epsilon^{o} \\ \\ \varkappa \end{bmatrix} = \left[\begin{array}{c|c} A' & B' \\ \hline H' & D' \end{array}\right] \begin{bmatrix} \overline{N} \\ \\ \overline{M} \end{bmatrix} \tag{21}$$

where

$$\begin{aligned} A^* &= A^{-1} \\ B^* &= -A^{-1}B \\ H^* &= BA^{-1} \\ D^* &= D - BA^{-1}B \\ A' &= A^* - B^* D^{*-1} H^* \\ B' &= H' = B^* D^{*-1} \\ D' &= D^{*-1} \end{aligned} \tag{22}$$

Substituting Equation (21) into (12)

$$\epsilon_i = (A'_{ij} + zB'_{ij})\,\overline{N}_j + (B'_{ij} + zD'_{ij})\,\overline{M}_j \tag{23}$$

From Equation (13), the stress components for the kth layer are:

$$\sigma_i^{(k)} = C_{ij}^{(k)} (\epsilon_j - \alpha_j^{(k)} T) \tag{24}$$

$$= C_{ij}^{(k)} \left[(A'_{jk} + zB'_{jk}) \overline{N}_k + (B'_{jk} + zD'_{jk}) \overline{M}_k - \alpha_j^{(k)} T \right]$$

This is the most general expression for stresses as functions of stress resultants, bending moments, and temperature. The same material coefficients A', B', and D', as reported in Reference 2, can be used for the thermal stress analysis. This simple link between the isothermal and nonisothermal analyses is achieved by treating thermal effects as equivalent mechanical loads, e.g., N_i^T and M_i^T in Equation (17). Determining the level of external load N_i and/or bending moment M_i that will initiate failure in one or several of the constituent layers is not a straightforward calculation. This is due to the fact that the stress components σ_i (i = 1, 2, 6) computed from Equation (24) must be transformed into the x-y coordinates (i = x, y, s), which represent the material symmetry axes, before the signs of the stresses σ_x and σ_y, whether positive or negative, can be determined. Only after the signs of σ_x and σ_y are known, can the proper yield condition be selected. The actual numerical method by which the maximum allowable loadings (N_i and/or M_i) are determined is outlined in detail in Appendix A.

A cylindrical shell is one of the basic structural shapes. When a shell is subjected to homogeneous loading, e.g., uniaxial tension or compression, internal or external hydrostatic pressure, or pure shear, the shell maintains its shape. There is no change in curvature in either the circumferential or the longitudinal direction. Because of this geometric constraint imposed on cylindrical shells under homogeneous loadings, the induced stress distribution can be represented by simpler relations than those just outlined. By assuming no change in curvature (this can be represented by letting $\varkappa \equiv 0$), the total strain is now equal to the in-plane strain. This is obtained directly from Equation (12) by letting $\varkappa \equiv 0$. Strain is therefore homogeneous across the thickness of the shell, i.e., independent of z.

For cylindrical shells, the stress components for each layer are also constant, as given by Equation (13). Using Equation (20), one can immediately determine the in-plane, i.e., total strain caused by N_j,

$$\epsilon_i^o = A_{ij}^* \bar{N}_j \tag{25}$$

The stress components are:

$$\sigma_i^{(k)} = C_{ij}^{(k)} \left[A'_{jk} \bar{N}_k - \alpha_j^{(k)} T \right] \tag{26}$$

Being independent of z, this equation is considerably simpler than Equation (24).

The strength analysis of cylindrical shells subjected to a few frequently occurring loading conditions has also been programmed. The entire program is outlined in detail in Appendix A.

Post-Yielding Behavior

For most fiber-reinforced composites presently available, initial yielding is often dictated by the values of the transverse and shear strengths, which are significantly lower than the axial strength. The initial yielding introduces failures parallel to the fibers. These failures are audible during the loading and become visible soon after the theoretically predicted yield stress is attained.

The post-yielding behavior of cross-ply composites has been investigated previously.[2] For a cross-ply composite subjected to a uniaxial tensile load in the direction of the fibers of one of the constituent layers, additional load can be supported after initial yielding until ultimate fiber failure is induced. Thus, initial yielding does not necessarily determine the load-carrying capacity of a laminated composite. After one or more layers have yielded, the layers of the laminated composite which are still intact must be

investigated to ascertain whether or not they can support the prevailing externally applied load.

However, in the case of an angle-ply composite under uniaxial tension, the still intact layers cannot carry the existing load after initial yielding. For this reason, there is no post-yielding load-carrying capability.[2] Thus, under uniaxial tension applied along one of the material symmetry axes of the composite, cross-ply composites can carry additional load after the initial yielding but angle-ply composites cannot.

A general theory for the analysis of the post-yielding behavior of a laminated composite is difficult to formulate because the material is transformed from a continuum to a "discontinuum" on the microscopic scale. A theory will be proposed in this report, using some of the assumptions of the conventional netting analysis. It is assumed that, after initial yielding,* the unidirectional layers of a composite can carry tensile load only along the fiber axis. To maintain static equilibrium, load transverse to the fibers and distortional load must be carried by other internal agencies of the composite. Such agencies may be derived from filament crossovers in the case of a helical-wound structure, or from some end constraint typical of shell-type structures, e.g., at the shell-and-head junction.

An internal agency is necessary for the transfer of the externally applied loads to axial loads along the unidirectional fibers. Before initial yielding, this internal agency is achieved by the binding matrix. The entire composite is a continuum. After initial yielding, failure in the matrix and/or at the fiber-matrix interface is introduced. Fibers are apparently still intact. In the case of angle-ply composites under uniaxial loading, no internal agency

*A composite, after initial yielding occurs, is referred to as a "degraded" composite in Reference 2.

is operative after the initial failure. Complete failure of the composite occurs immediately after initial yielding. However, in the case of cross-ply composites, an internal agency is not needed for transferring the external load. Since some of the filaments are aligned parallel to the applied load, they can continue to carry load until filament failure is reached.

Filament-wound structures often acquire filament crossovers during winding with a helical pattern. This type of composite may be represented by an angle-ply with filament crossovers. The geometric distribution and the frequency of occurrence of filament crossovers for a given helical-wound tube depend on the helical angle, the width of the roving, the diameter of the tube, and other process parameters, which may include the characteristics of the winding machine. In the present investigation, it is assumed that the effect of filament crossovers introduces two factors:

(1) As an internal agency, filament crossovers provide additional load-carrying capacity to helical-wound composites. This strengthening of angle-ply composites is exhibited by higher effective transverse and shear strengths, designated as $\overline{Y}$ and $\overline{S}$, respectively.

(2) In contradiction to the strengthening effect above, filament crossovers will be sources of stress concentrations, since filaments can be subjected to direct abrasion among themselves. Therefore, crossovers will tend to reduce the axial strength X of the constituent layers.

Because of the existence of filament crossovers, it may be necessary to treat helical-wound composites differently than angle-ply composites. It may be possible for helical-wound composites to carry a higher load because of the internal agency generated by the crossovers. The ultimate load that the composite can carry will be governed by either the breakdown of the internal agency which is needed to transfer external loads or filament failure.

In conclusion, the post-yielding behavior of laminated composites is dictated by the ability of the filaments which are still intact to sustain continued loading. This is accomplished in cross-ply composites when subjected to uniaxial tension or internal pressure, for example, by having filaments aligned parallel to the applied load. The post-yielding capability can also be achieved by means of an internal agency in the composite, an example of which is due to the filament crossovers which exist in woven fabric and helical-wound structures. Angle-ply composites under uniaxial load do not have a post-yielding capability because fibers are not aligned in the direction of applied loads, nor is there an internal agency for load transfer. Assuming that an internal agency is available in a composite such that the externally applied load, N_i, i = 1, 2, 6, can be transferred to an axial load, N_f, in the unidirectional layers, one can derive the relation between the axial stress; N_f, of a unidirectional constituent layer and N_i as follows.

As shown in Figure 4a, the equilibrium of forces between the externally applied load, N_1, and the induced load, N_f, in the direction of the fibers must satisfy the relation:

$$\frac{N_f \cos \alpha}{A} = - \frac{N_1}{A \cos \alpha} \tag{27}$$

or

$$N_f = N_1/\cos^2\alpha = N_1/m^2 \tag{28}$$

In order to maintain equilibrium in the 2-direction, an internal force, N_{21}, must be:

$$\frac{N_{21}}{A \sin \alpha} = - \frac{N_f \sin \alpha}{A} \tag{29}$$

or

$$N_{21} = -N_f \sin^2 \alpha = -n^2 N_f = -n^2 N_1/m^2 \tag{30}$$

Similarly, in Figure 4b, the equilibrium of forces between the externally applied load, N_2, and the induced load, N_f, results in the condition:

$$N_f = N_2/n^2 \tag{31}$$

$$N_{12} = m^2 N_f = m^2 N_2/n^2 \tag{32}$$

In the case of an externally applied shear force, N_6, the equilibrium condition, as shown in Figure 4c must satisfy:

$$\frac{N_f}{A} = \pm \frac{N_6 \sin \alpha}{A \cos \alpha} \pm \frac{N_6 \cos \alpha}{A \sin \alpha} = \pm \frac{N_6}{Amn} \tag{33}$$

or

$$N_f = \pm N_6/mn \tag{34}$$

The internally induced load, N_{66}, in this case is zero because

$$\frac{N_{66}}{A} = \frac{N_6 \cos \alpha}{A \cos \alpha} - \frac{N_6 \sin \alpha}{A \sin \alpha} = 0 \tag{35}$$

Equations (28), (31), and (34) show the contribution of each externally applied load, N_1, N_2, and N_6, to the axial stress along the unidirectional layer with an orientation of α degrees from the 1-axis. The total axial stress is, by superposition:

$$N_f = \frac{N_1}{m^2} + \frac{N_2}{n^2} + \frac{N_6}{mn} \tag{36}$$

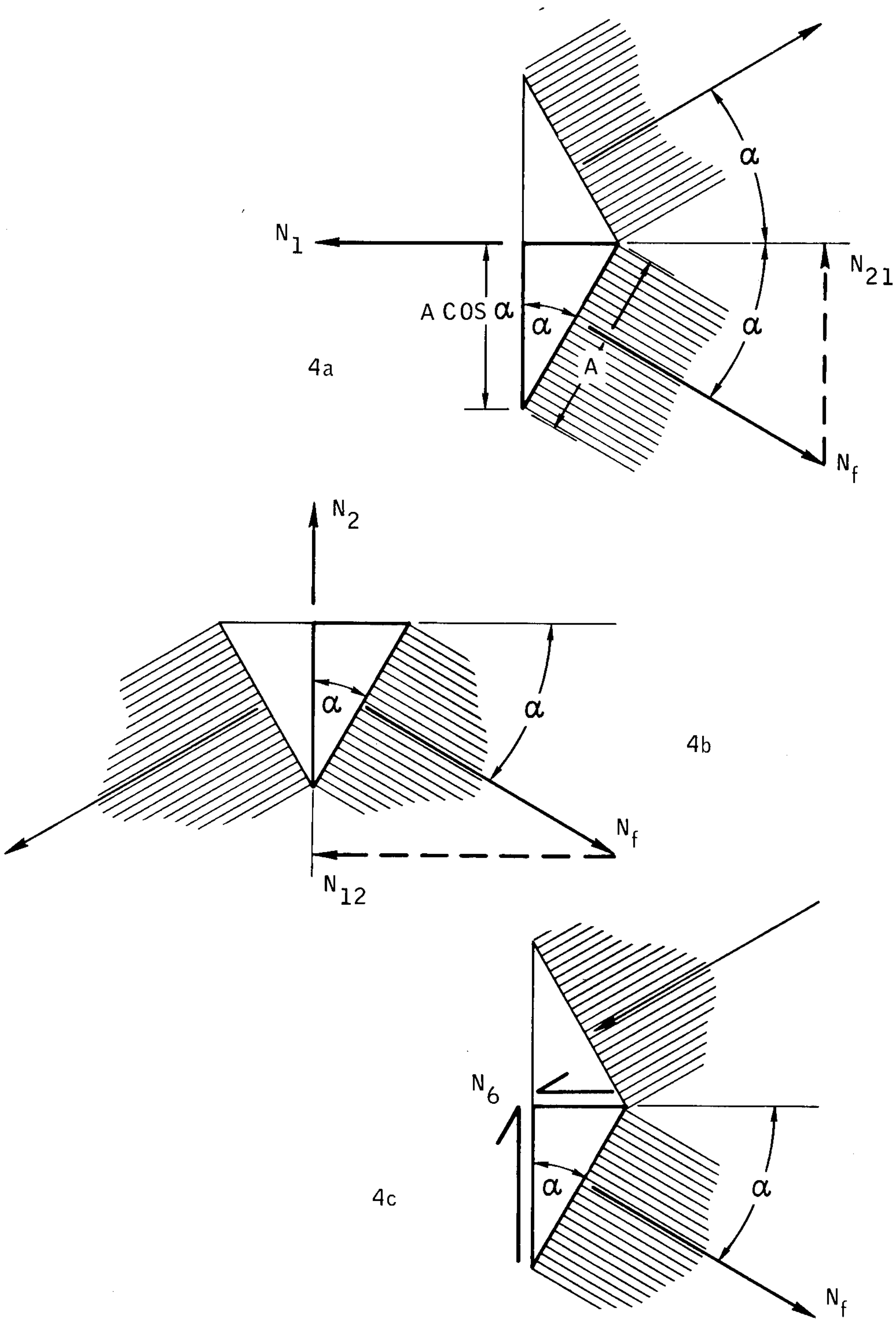


Figure 4. Netting Analysis - Notation

This equation gives the maximum load-carrying capacity of each unidirectional constituent layer of a laminated composite. The ultimate load is governed by the axial strength, X, of each unidirectional layer. It is, of course, assumed that some internal agency of the laminated composite, by virtue of the filament crossovers, is capable of supporting the internal forces N_{12} and N_{21} at least up to the axial strength of the constituent layers.

The validity of this analysis is limited to the capability of the internal agency to transfer the load. In particular, the filament crossovers in helical-wound tubes will be examined as a specific internal agency. As stated previously, the effect of crossovers may be characterized by effective transverse and shear strengths, $\overline{Y}$ and $\overline{S}$, higher than those of unidirectional composites, and by a reduction in the effective axial strength X, possibly caused by the abrasive action between filaments at crossover points. Presently, the exact change in magnitude of these effective strengths must be determined experimentally. Future investigations may provide a basis for the theoretical prediction of these values.

In the next two sections, detailed procedures for the determination of the load-carrying capacity of cross-ply and helical-wound tubes will be outlined. The theoretical results will be compared with experimental data, using E glass and epoxy as the constituent materials.

Cross-Ply Composites

In this paragraph, the deformation and ultimate strength of cross-ply composites are discussed. Theoretical predictions, using the strength analysis program outlined in Appendix A, are made. A sample problem is presented in detail and numerical results are tabulated. The theoretical results are then compared with experimental data.

A cross-ply composite consists of two systems of unidirectional constituent layers with adjacent layers oriented orthogonal to each other. There are two lamination parameters: (1) the total number of layers, n, (each layer may consist of one or more unidirectional plies of roving, all of which must have the same fiber orientation), and (2) the cross-ply ratio, m, which is defined as the ratio of the total thickness of all the layers oriented in one direction to the total thickness of the layers in the orthogonal direction. For laminated beams and plates, as reported in References 1 and 2, the cross-ply ratio is computed using the layers with 0 degree orientation, as measured from the reference coordinate system, as the first system of layers. In the case of cylindrical pressure vessels, which will be discussed in this paragraph, the cross-ply ratio is defined on the basis of the outermost layer as being in the first system of layers. If the outermost layer is a hoop winding, which is usually the case, then the cross-ply ratio is the ratio of the thickness of all the hoop windings to that of the longitudinal windings.

The deformation and ultimate strength of cross-ply specimens subjected to uniaxial tension has been reported previously.[1, 2, 7] However, a computational error in the calculation of the stress at initial yielding (the knee) has been discovered. The corrected theoretical result is as follows:

Cross-ply Ratio, m	Initial Yielding, N_I/h, ksi
0.25	7.9
1.00	13.7
2.50	17.6
4.00	19.1

These results have been computed using the following material properties, which are the same as those reported previously:

$$
\begin{aligned}
C_{11}^{(1)} &= C_{22}^{(2)} = 7.97 \times 10^{6} \text{ psi} \\
C_{12}^{(1)} &= C_{12}^{(2)} = 0.66 \times 10^{6} \text{ psi} \\
C_{22}^{(1)} &= C_{11}^{(2)} = 2.66 \times 10^{6} \text{ psi} \\
C_{66}^{(1)} &= C_{66}^{(2)} = 1.25 \times 10^{6} \text{ psi} \\
C_{16}^{(1)} &= C_{26}^{(1)} = C_{16}^{(2)} = C_{26}^{(2)} = 0 \\
\alpha_{1}^{(1)} &= \alpha_{2}^{(2)} = 3.5 \times 10^{-6} \text{ in./in./}^{\circ}\text{F} \\
\alpha_{2}^{(1)} &= \alpha_{1}^{(2)} = 11.4 \times 10^{-6} \text{ in./in./}^{\circ}\text{F} \\
\alpha_{6}^{(1)} &= \alpha_{6}^{(2)} = 0 \\
T &= -200^{\circ}\text{F (lamination temperature)} \\
n &= 3 \text{ (number of layers)}
\end{aligned}
\tag{37}
$$

In addition, the following strength data are used:

$$
\begin{aligned}
X = X' &= 150 \text{ ksi} \\
Y &= 4 \text{ ksi} \\
Y' &= 20 \text{ ksi} \\
S &= 6 \text{ ksi}
\end{aligned}
\tag{38}
$$

These material properties are required inputs in the strength analysis program outlined in Appendix A. The corrected theoretical results show better agreement with the experimental results, as can be seen in Figure 5 (which is Figure 6 of Reference 2 and Figure 3 of Reference 7 with the corrected initial yielding curve shown). The procedure for the determination of the post yielding stiffness and the ultimate load is also outlined in these references. Essentially, post-yield load carrying capability is possible for cross-ply composites because the filaments in the direction of the applied uniaxial load can carry the prevailing load. No internal agency for load transfer is required in this case. The ultimate load is obtained when the axial strength of the unidirectional layer is reached, i.e., when $X = 150$ ksi.

It is important to recognize that the value of the axial strength X is experimentally determined. It is not calculated from the fiber strength using the rule-of-mixtures equation, from which, for E glass, the computed axial strength would be $400 \times 2/3 = 266$ ksi (filament strength times percent filament volume).

Cross-ply pressure vessels will now be examined. A typical vessel is shown in Figure 6. The middle third of the vessel is the test section, the ends being built up from special aluminum fittings. The basic design of the vessel was developed at Aeronutronic under another research program. The longitudinal layers were laid up by hand and the hoop layers wound by machine. The rovings used were 20-end E glass preimpregnated with epoxy

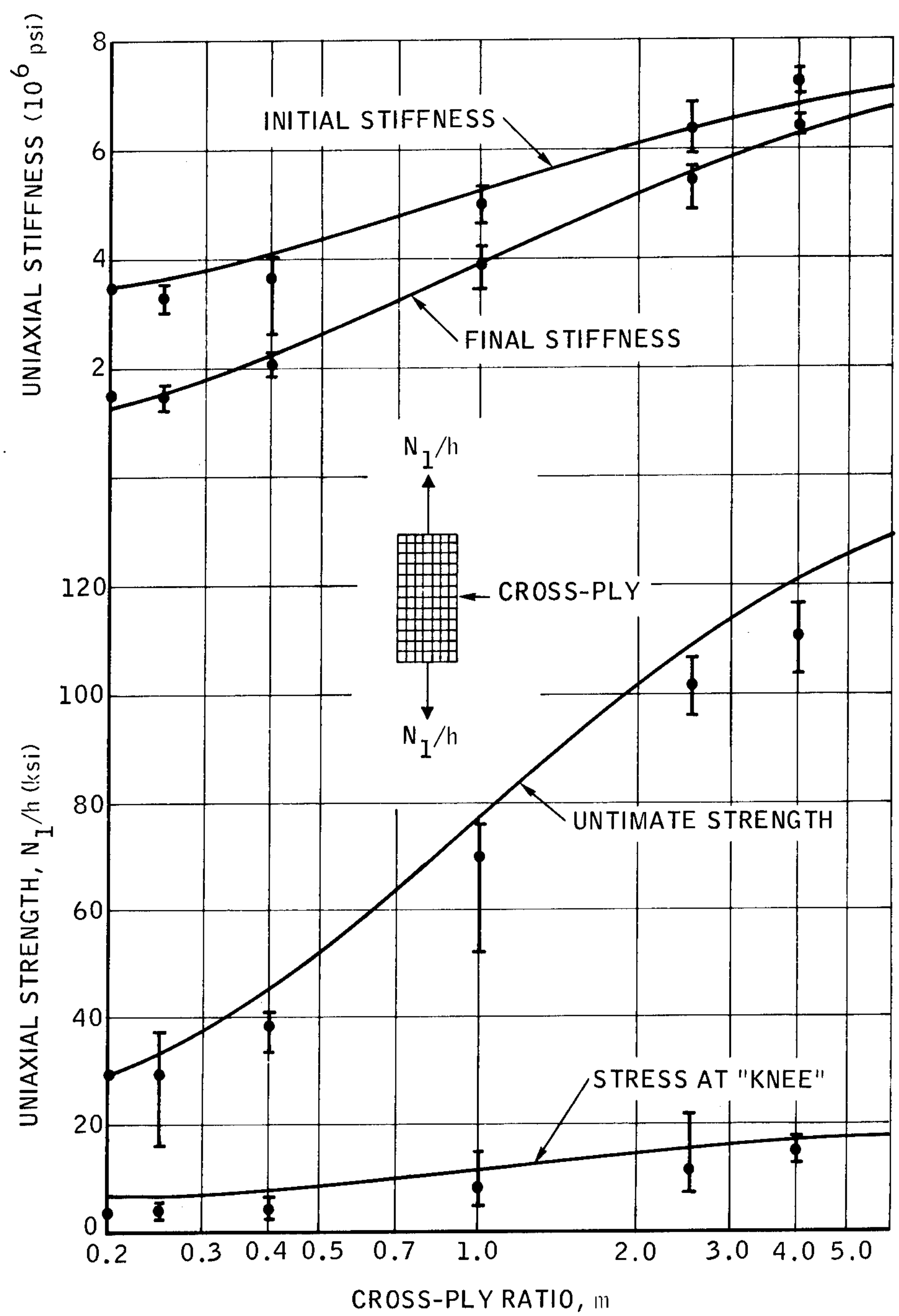


Figure 5. Glass-Epoxy Cross-ply Composites Subjected to Uniaxial Loads

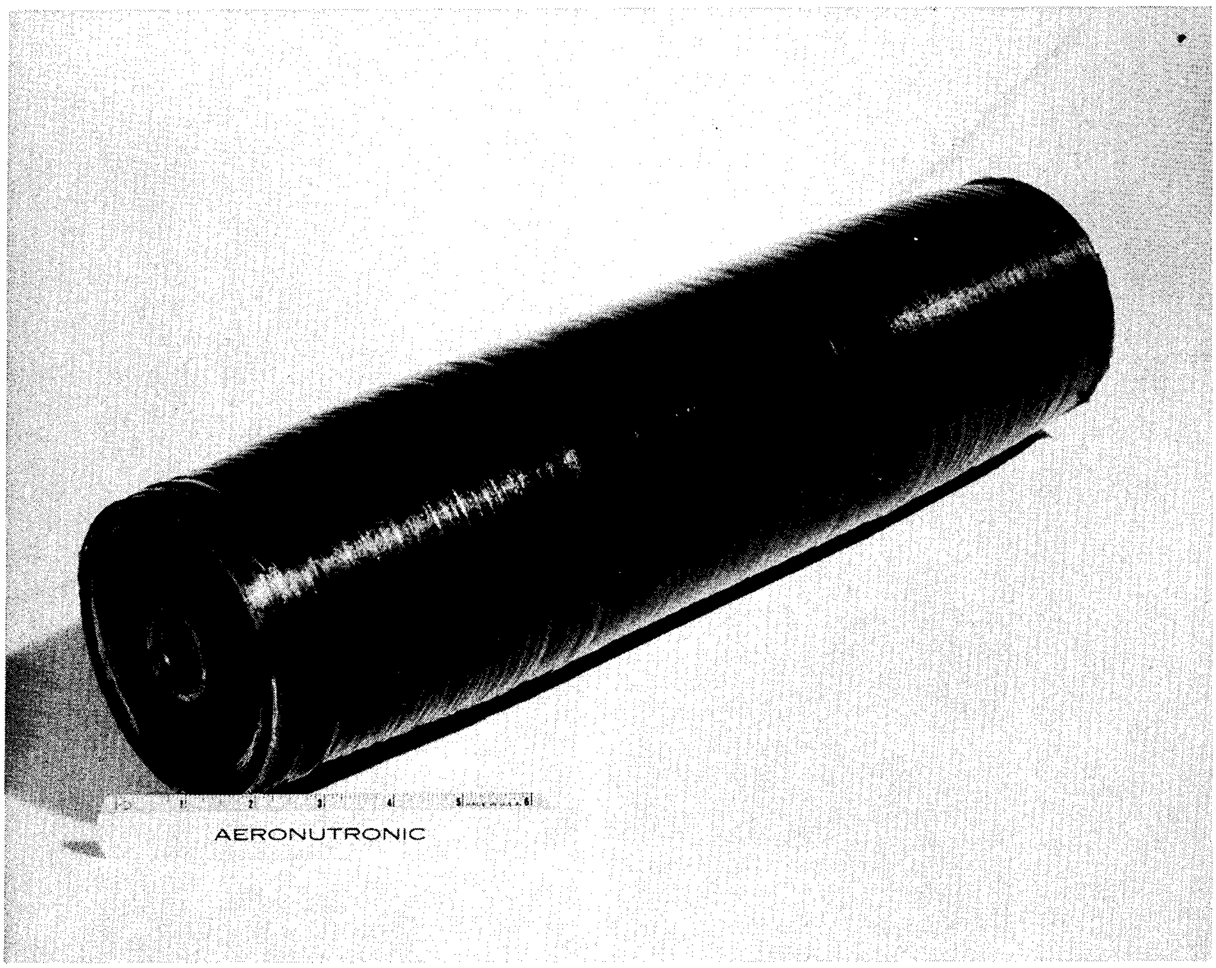


Figure 6. Cross-Ply Pressure Vessels

resin. Two-element strain gages were bonded to each pressure vessel with the elements oriented in the hoop and longitudinal directions. Internal pressurization was achieved using hydraulic oil and a pumping arrangement specifically designed for testing pressure vessels. Internal pressure and strains were recorded by a multi-channel continuous recorder. Using the material properties listed in Equations (37) and (38) in the program outlined in Appendix A, the results given in Table I were obtained for cross-ply ratios of 0.4, 1.0 and 4.0.*

TABLE I

CROSS-PLY PRESSURE VESSELS – INTERNAL PRESSURE

Cross-ply Ratio (m)	A^*_{11}	A^*_{12} (10^{-6} in/lb)	A^*_{22}	N_2/h (hoop stress at initial yielding)	Yielding Location
0.4	0.158	-0.025	0.244	9.3 ksi	Long.
1.0	0.191	-0.024	0.191	12.8 ksi	Long.
4.0	0.273	-0.026	0.147	14.6 ksi	Hoop

The numerical values of the A matrix are also given on pp 65, 67, and 69 of Reference 2 with the axes 1 and 2 interchanged. This change is necessary because of the differences in the definitions of the cross-ply ratio cited earlier in this section.

Using a reference coordinate system with the 1-axis in the longitudinal direction and the 2-axis in the hoop direction, strains along these axes can be computed using Equation (25):

$$\text{Longitudinal Strain} = \epsilon_1^{\circ} = A_{11}^{*} N_1 + A_{12}^{*} N_2$$
$$= \left(\frac{1}{2} A_{11}^{*} + A_{12}^{*} \right) N_2 \tag{39}$$

$$\text{Hoop Strain} = \epsilon_2^{\circ} = A_{12}^{*} N_1 + A_{22}^{*} N_2$$
$$= \left(\frac{1}{2} A_{12}^{*} + A_{22}^{*} \right) N_2 \tag{40}$$

where $2N_1 = N_2 = PR$ is assumed and P = internal pressure, R = radius.

Strain after initial yielding is obtained by the usual neeting analysis, which assumes that each unidirectional layer retains only its axial stiffness, E_{11}, the transverse stiffness and shear modulus being zero. The resulting relations, as shown in Equation (9-5) of Reference 1, are:

$$\frac{E_{11}h}{PR} \epsilon_1^{\circ} = \frac{1+m}{2} \tag{41}$$

$$\frac{E_{11}h}{PR} \epsilon_2^{\circ} = \frac{1+m}{m} \tag{42}$$

where h represents the total wall thickness of the pressure vessel.

Taking E_{11} as 7.8×10^6 psi, which is representative of an E glass - epoxy composite with a fiber volume of approximately 65 percent, the longitudinal and hoop strains, before and after initial yielding (the knee), are obtained from Equations (39) through (42). These are given in Table II.

TABLE II

LONGITUDINAL AND HOOP STRAINS OF CROSS-PLY VESSELS

Cross-ply Ratio (m)	Before Yielding		After Yielding	
	$\frac{E_{11}h}{PR}\epsilon_1^\circ$	$\frac{E_{11}h}{PR}\epsilon_2^\circ$	$\frac{E_{11}h}{PR}\epsilon_1^\circ$	$\frac{E_{11}h}{PR}\epsilon_2^\circ$
0.4	0.42	1.81	0.70	3.50
1.0	0.55	1.40	1.00	2.00
4.0	0.86	1.05	2.50	1.25

The burst pressure of the cross-ply vessels may be predicted as follows: First, the axial stress in the unidirectional composite at the initial yielding must be determined. Assuming that the outermost layer of all vessels is in the hoop direction (along the 2-axis), the stress components that represent the normal stress along the fibers are:

(1) Hoop layers (odd layers) : $\sigma_2^{(1)}$ or $\sigma_2^{(H)}$

(2) Longitudinal layers (even layers) : $\sigma_1^{(2)}$ or $\sigma_1^{(L)}$

where the superscripts designate the layers, and the subscripts the direction of the normal stresses. These stresses can be computed from Equation (26). In the present case, $2N_1 = N_2$, N_2 being equal to the lowest yield stress, since the computed yield stress for each constituent layer may be different.

As a sample problem, the case of m = 0.4 will now be outlined. The lowest initial yield stress for this case is $N_2 = 9.3$ ksi (from Table I). The yielding occurs in the longitudinal layer. The yield stress of the hoop layer would be $N_2 = 23.3$ ksi if the longitudinal layer could sustain a load equal to or higher than this value. The axial stresses in the longitudinal and hoop layers can be calculated from the stress coefficients, which are obtained

directly from the program outlined in Appendix A (or from page 65 of Reference 2 provided subscripts 1 and 2 are interchanged). Substituting $N_2 = 9.3$ ksi and $N_1 = N_2/2 = 4.65$ ksi, one can compute the axial stresses:

$$\begin{aligned}\sigma_2^{(H)} &= -0.095\,(4.65) + 1.92\,(9.3) - 0.0255\,(200) \\ &= 12.30 \text{ ksi}\end{aligned} \tag{43}$$

$$\begin{aligned}\sigma_1^{(L)} &= 1.239\,(4.65) - 0.0381\,(9.3) - 0.0062\,(200) \\ &= 4.17 \text{ ksi}\end{aligned} \tag{44}$$

For cross-ply composites, it is assumed that, after initial yielding, a complete uncoupling of constituent layers of the laminated composite is induced. Each layer will operate independently. This complete uncoupling has been reported in Reference 2 and appears reasonable for cross-ply composites in general because of the lack of an internal agency to bind or lock the laminates together. From Equations (43) and (44), each layer is axially stressed either to 12.30 or 4.17 ksi. Fiber failure will be induced if the axial stress reaches 150 ksi, which is the experimentally determined axial strength. Thus, the first layer (the odd or hoop layers) can sustain an additional axial stress of:

$$N_f^{(H)} = 150 - 12 = 138 \text{ ksi} \tag{45}$$

and the second layer:

$$N_f^{(L)} = 150 - 4 = 146 \text{ ksi} \tag{46}$$

In a completely uncoupled laminate,

$$N_f^{(H)} = E_{11}\,\epsilon_2^o, \qquad N_f^{(L)} = E_{11}\,\epsilon_1^o \tag{47}$$

Substituting these conditions into Equations (41) and (42) and solving for the additional hoop stress, N_2, that the pressure vessel can sustain beyond the initial yielding:

$$N_2^{(H)} = PR = \frac{m}{1+m} E_{11} \epsilon_2^{\circ} h = \frac{m}{1+m} N_f^{(H)} h \tag{48}$$

$$N_2^{(L)} = PR = \frac{2}{1+m} E_{11} \epsilon_1^{\circ} h = \frac{2}{1+m} N_f^{(L)} h \tag{49}$$

Using the values of Equations (45) and (46) and $m = 0.4$,

$$N_2^{(H)}/h = 0.286 \times 138 = 39.4 \text{ ksi} \tag{50}$$

$$N_2^{(L)}/h = 1.43 \times 146 = 209 \text{ ksi} \tag{51}$$

Thus, the burst strength is

$$N_2^{(H)}/h = 39.4 + 9.3 = 48.7 \text{ ksi} \tag{52}$$

and the fiber failure is induced in the hoop layers.

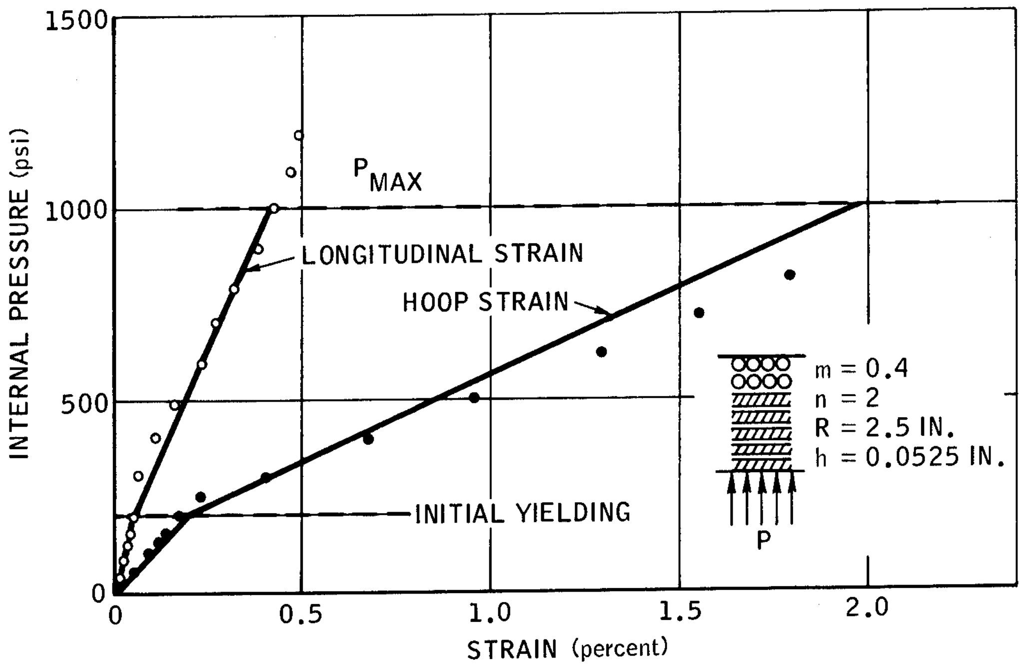
Similar calculations for other cross-ply ratios have also been computed and the results listed in Table III.

TABLE III

CROSS-PLY PRESSURE VESSELS

Cross-ply Ratio (m)	Initial Yielding (N_2/h)	Ultimate Strength (N_2/h)	Failure Location
0.4	9.3	48.7	Hoop
1.0	12.8	64.5	Hoop
4.0	14.6	56.8	Long.

The theoretical results listed in Tables II and III will now be compared with experimental data obtained for cross-ply pressure vessels. During pressurization, both hoop and longitudinal strains were recorded by a continuous strain recorder, along with the internal pressure. In the neighborhood of the predicted initial yielding, a cracking noise could be heard, this being attributed to a failure either in the matrix or at the fiber-matrix interface. Upon further pressurization, the recorded strains followed a secondary slope which agreed well with the theoretical prediction based on netting analysis. The observed burst pressures came within 20 percent of those predicted in Table III. Typical results of theory-versus-experiment for pressure vessels with cross-ply ratios of 0.4, 1.0, and 4.0 are shown in Figures 7, 8, and 9. In each of these figures, the number of layers equals two and three. According to the theory, there should be no differences between the two cases for pressure vessels because change of curvature does not occur. The stress in each layer does not vary across its thickness (radial direction). The experimental data, which are shown as dots, agree well with the theoretical predictions, not only at the burst pressure but also in predicting initial yielding and the primary and secondary slopes (the slopes before and after yielding). As stated in Reference 2, the conventional netting analysis is less exact than the present theory. The pressure-versus-strain relations are linear rather than bilinear in a netting analysis. Also, the ultimate burst pressure is computed using some value of glass strength corrected by the fiber volume ratio. For the glass used in the present



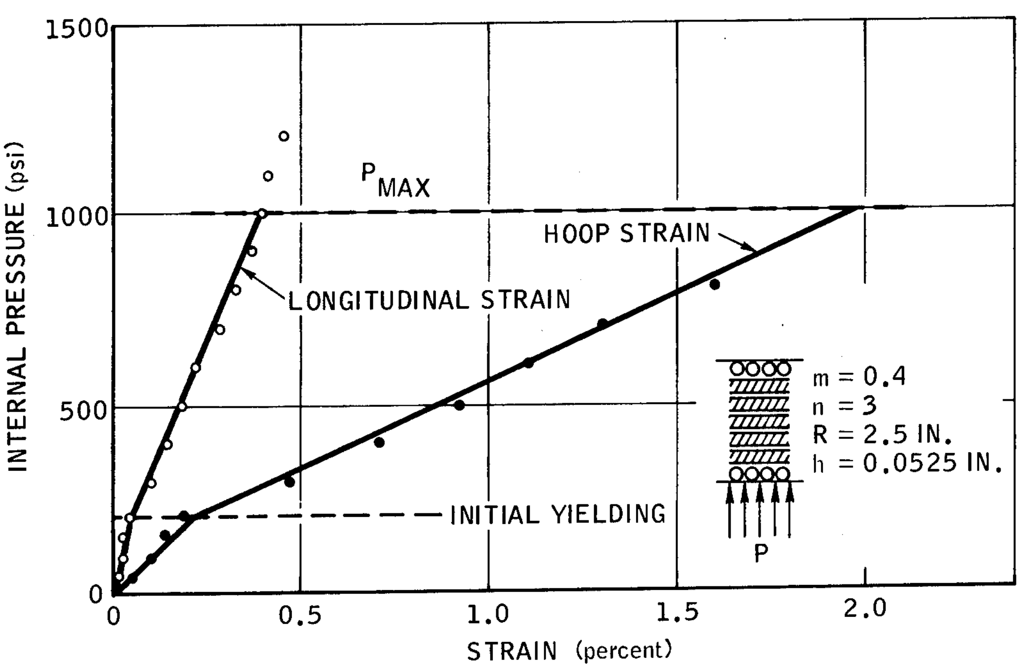


Figure 7. Glass-Epoxy Cross-Ply Pressure Vessels, m = 0.4

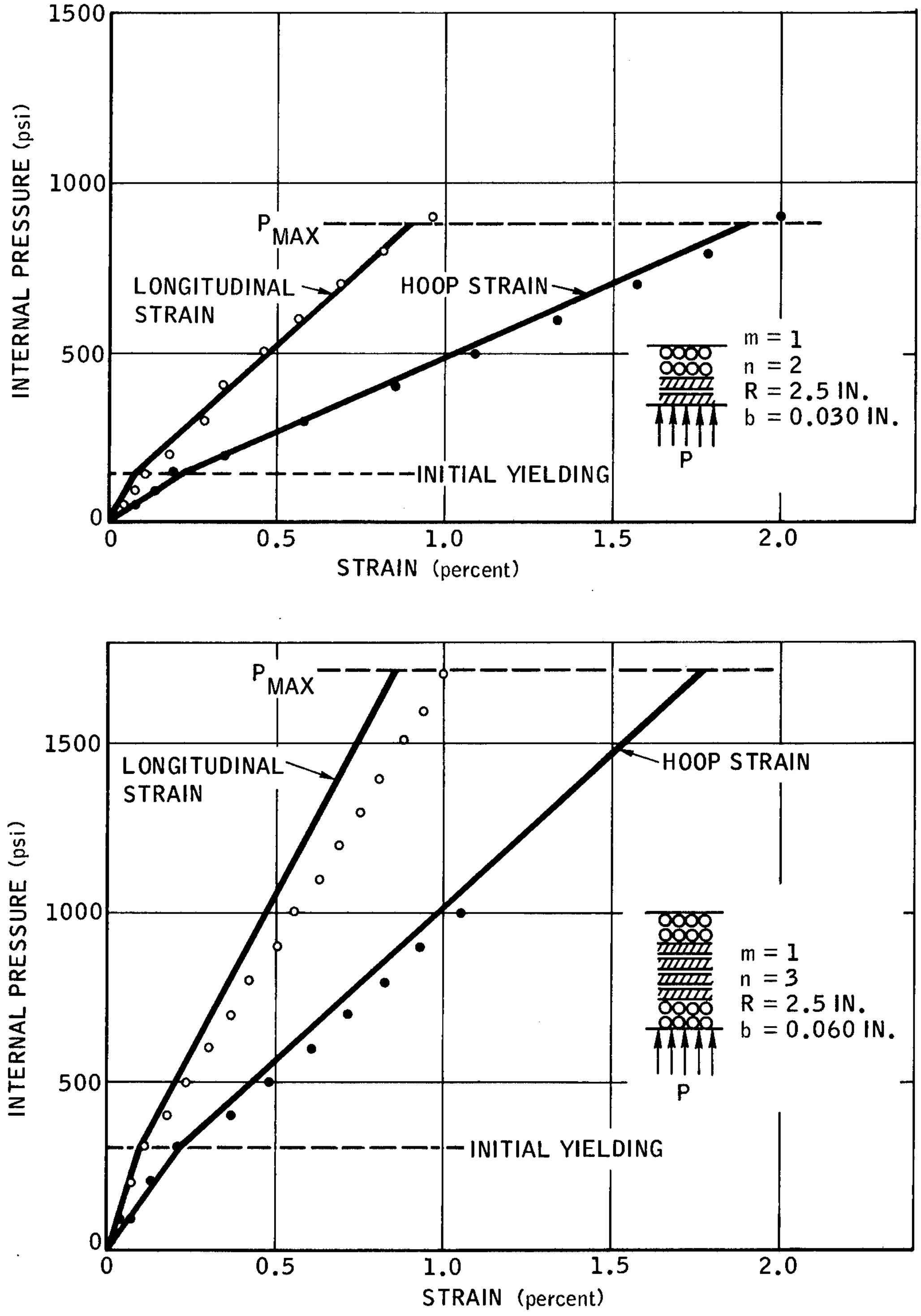


Figure 8. Glass-Epoxy Cross-Ply Pressure Vessels, m = 1.0

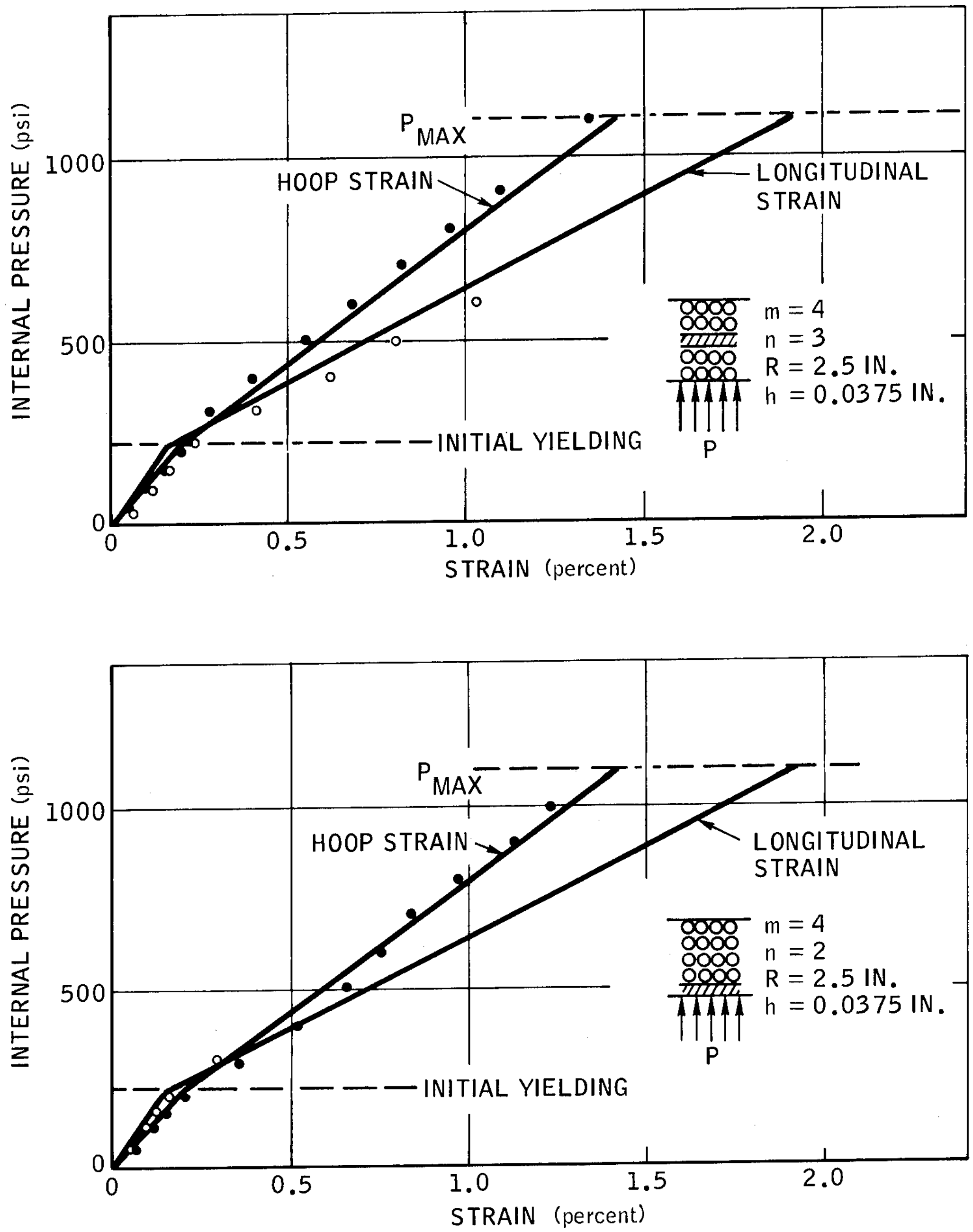
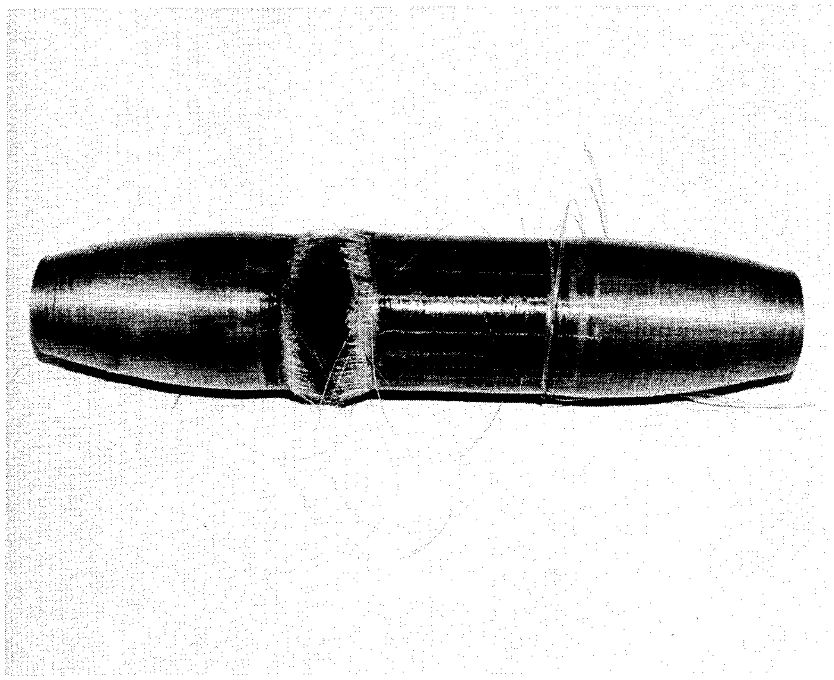


Figure 9. Glass-Epoxy Cross-Ply Pressure Vessels, m = 4.0

experiments, the strength is approximately 400 ksi. Using a volume ratio of 67 percent glass, the strength in the direction of the fibers would be approximately 270 ksi, which is considerably higher than the experimentally determined strength of 150 ksi. In fact, the factor between the theoretically predicted strength using a linear correction factor of the fiber volume and those actually measured is 270/150 = 1.8. It is, therefore, important to emphasize that the 150 ksi axial strength is a more realistic value, not only under unidirectional loading but also for the design of filament-wound composites subjected to biaxial loading.

For glass-epoxy systems, the initial yielding occurs at approximately 20 percent of the ultimate burst pressure. The exact level of the initial yielding can be predicted accurately for the present system and the present theory is equally applicable to other fiber-reinforced composites. Depending upon the relative values of the transverse strength and the axial strength, the level of the initial yielding will vary. In fact, an optimum composite material may very well be one in which the initial yielding, signifying failure of the matrix and/or the interface, coincides with the ultimate burst pressure, which in the case of cross-ply pressure vessels signifies fiber failure. Optimization can also be achieved such that both the longitudinal and hoop windings fail simultaneously. Using a netting analysis, the latter condition is satisfied if the cross-ply ratio is 2. According to the present theory, this ratio is dependent upon the basic properties of the constituent layers. Such properties include the elastic moduli and the axial, transverse, and shear strengths.

In Figure 10 are shown typical failures of cross-ply pressure vessels. In the upper vessel, a failure in the longitudinal layer was apparently initiated first. This vessel had a cross-ply ratio of 4. In the lower pressure vessel, hoop failure occurred first. This will be the case for cross-ply ratios of both 0.4 and 1.0.



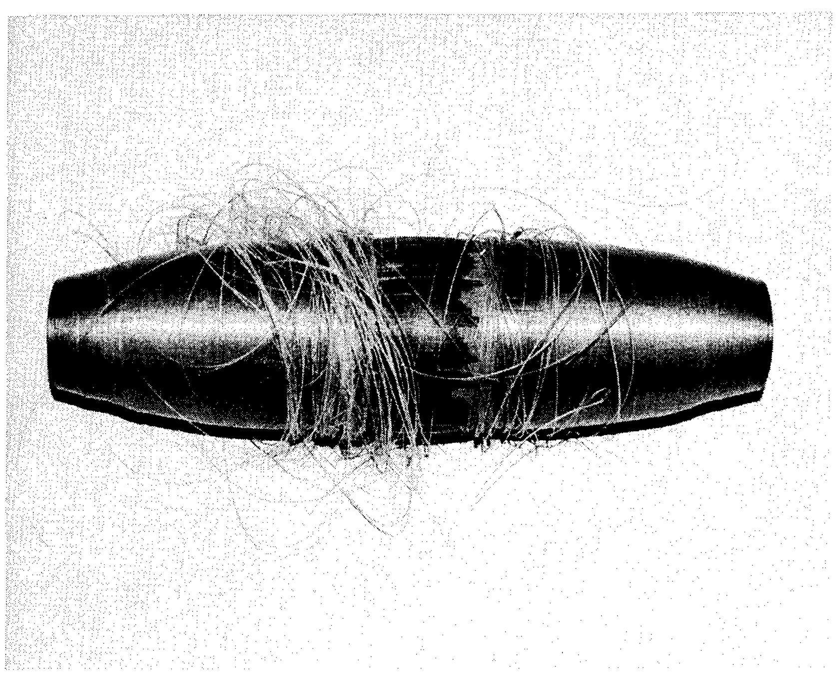


Figure 10. Typical Pressure Vessel Failures

Helical-Wound Tubes

The deformation and strength of helical-wound tubes subjected to homogeneous loadings will now be examined. Helical-wound tubes are of special interest for two reasons: (1) this is a very common method of fabrication of filamentary structures, and (2) the occurrence of filament cross-overs, which provide additional load-carrying capability after initial yielding because of filament crossovers, can be anticipated. The types of loadings that will be examined include uniaxial tension, uniaxial compression, pure torsion, and internal pressure. The strength analysis outlined in the previous paragraph, using both the continuum and discontinuum models, will be utilized. Experimental results will also be presented to demonstrate the degree of accuracy of the theoretical predictions of deformation and strength.

The filament-wound tubes fabricated during the present test program include 1-1/2, 3, and 5-inch I. D. tubes with helical angles from a low value of 27 degrees up to the maximum of 90 degrees. A few of the 1-1/2-inch tubes are shown in Figure 11 with the helical angles marked on each tube. The external load was applied to the tubes by means of end plugs, which were adhesive-bonded into the tubes. The uniaxial tension tests were performed as shown in Figure 12.

For uniaxial compression, the ends of the tubes were reinforced with additional hoop winding (over-wound) to prevent local buckling. The uniaxial compression tests were performed as shown in Figure 13. Torsion tests were conducted on the torsion machine shown in Figure 14. Internal pressurization was obtained in a manner similar to that employed in the case of cross-ply pressure vessels. For the 5-inch I. D. tubes, internal pressure only was applied.

As previously stated, the effect of filament crossovers may be characterized by higher values of transverse and shear strengths than for unidirectional composites. The exact amount of the increase must be determined experimentally at this time. Taking advantage of the strength

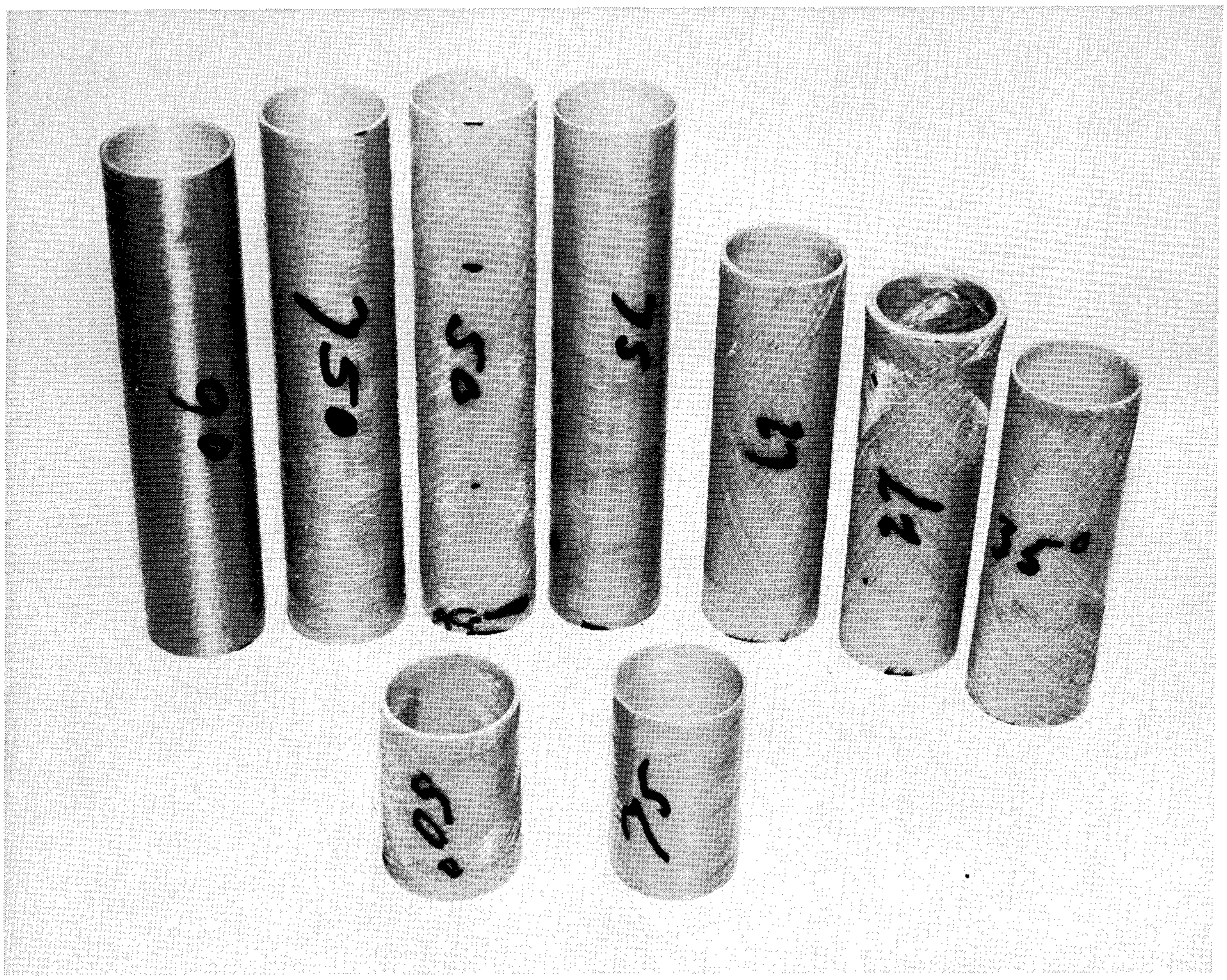


Figure 11. Helical-Wound Tubes, Glass-Epoxy

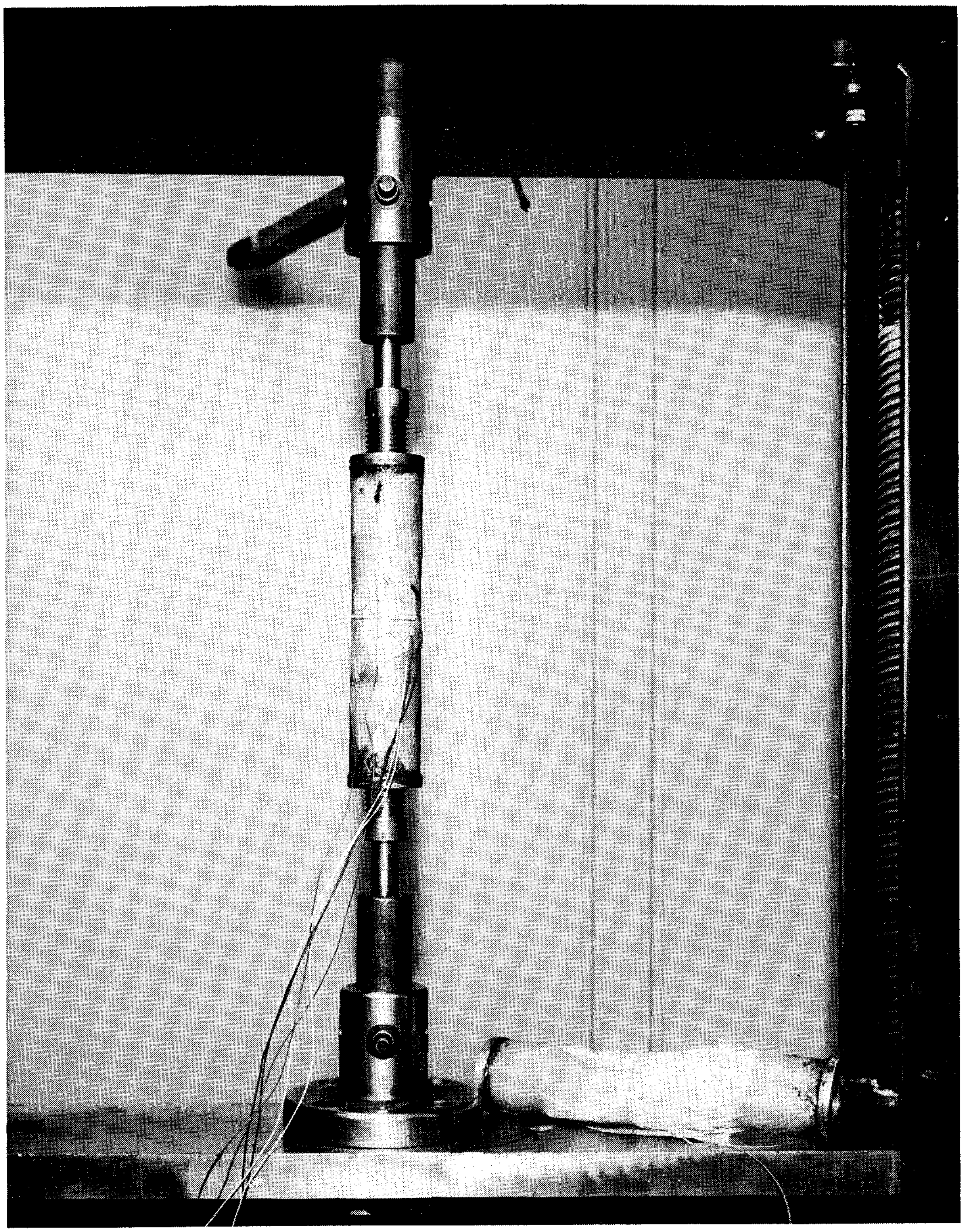


Figure 12. Uniaxial Tension Test

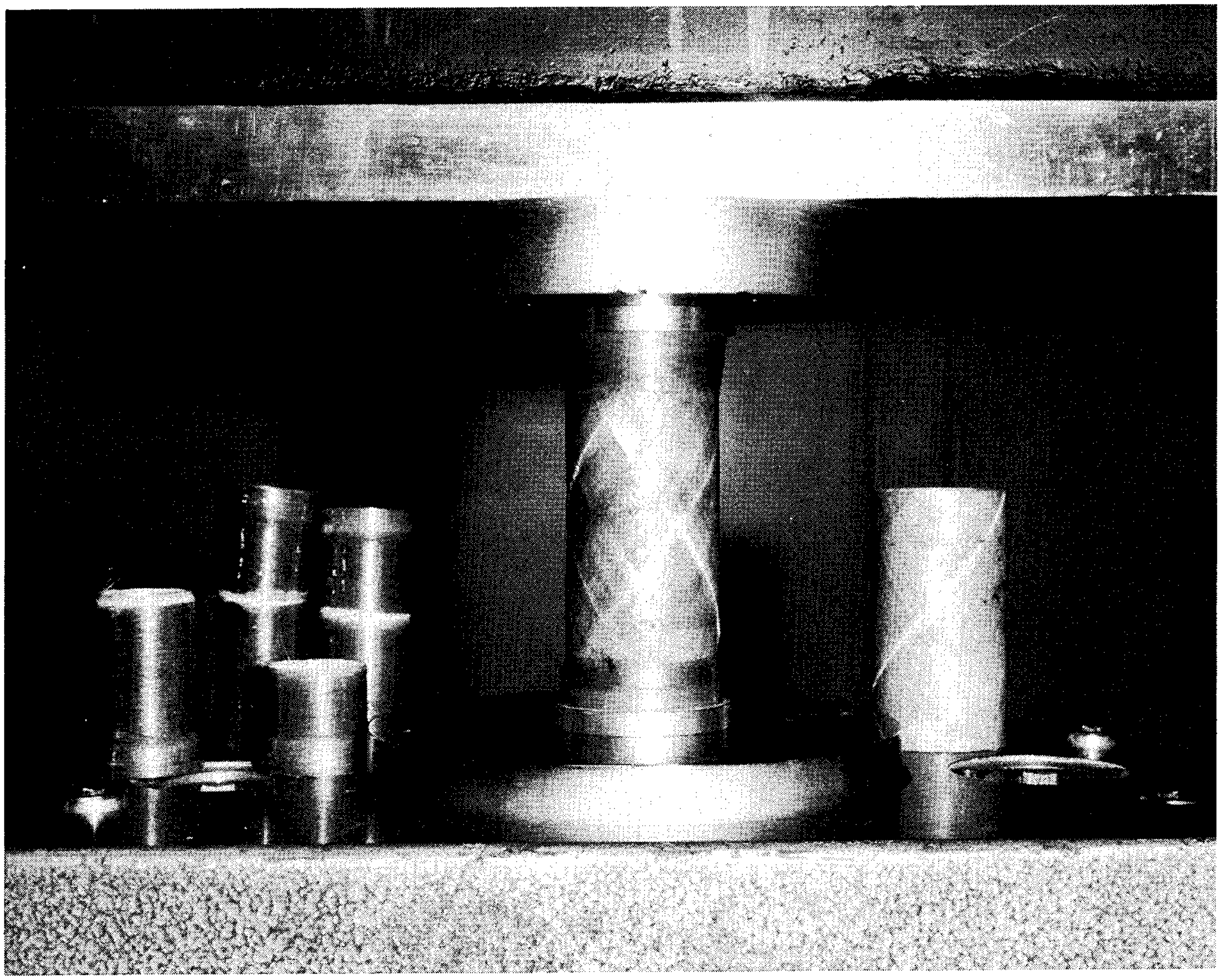


Figure 13. Uniaxial Compression Test

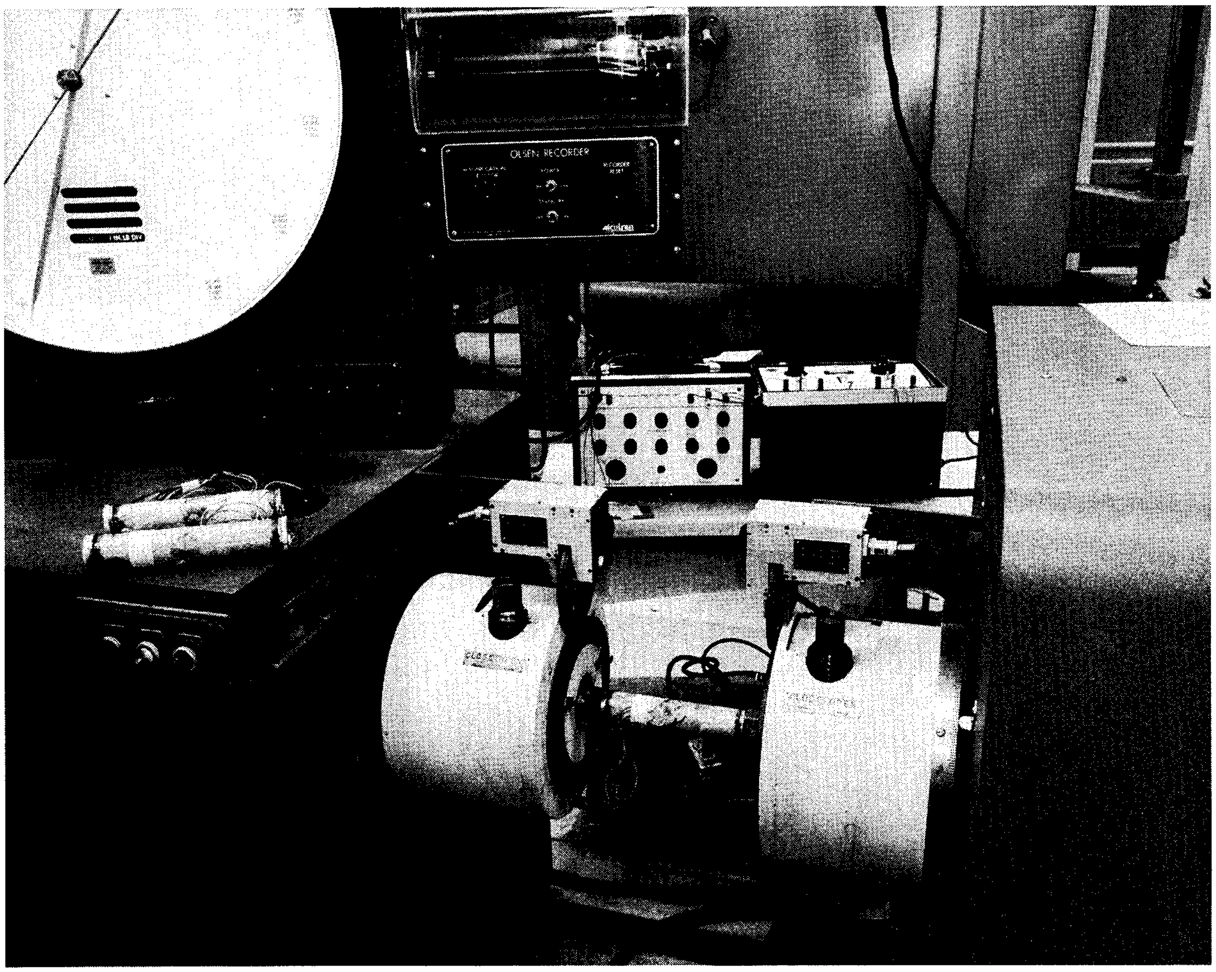


Figure 14. Torsion Test

analysis program outlined in Appendix A, a parametric study of the contribution of the principal strengths to the level of failure of the internal agency can be conducted.

In Figures 15, 16, 17, and 18, the effective stiffnesses and various strength criteria are given for helical angles between zero and 90 degrees. Appropriate experimental points are also shown in these figures.

The effective stiffness of helical-wound tubes can be readily determined from the A^* matrix in Equation (25). The numerical values of the matrix can be obtained using the elastic moduli of Equation (37) as inputs to the program outlined in Appendix A.

By assuming that the tensile and compressive moduli are equal, the uniaxial elongation or compression can be determined from A^*_{11}. The reciprocal of this value is plotted in Figures 15 and 16, which is equivalent to the axial stiffness. In Figure 17, the effective shear stiffness, the reciprocal of A^*_{66}, is shown. In Figure 18, the effective circumferential stiffness is shown as the ratio of the circumferential stress resultant to the measured circumferential strain. This is obtained using the following relation, where as before, the 1-axis is in the longitudinal direction and the 2-axis is in the circumferential or hoop direction:

$$E_{hoop} = 1/\left(\frac{1}{2} A^*_{12} + A^*_{22}\right) \tag{53}$$

Strain rosettes were bonded to the helical-wound tubes with elements oriented in the longitudinal and hoop directions and the tubes were subjected to uniaxial or internal pressure loadings. For the torsion tube, the rosettes were oriented at angles of ±45 degrees from the longitudinal axis. The effective stiffnesses of the tubes subjected to various loadings were computed from the recorded strains and are shown in Figures 15 through 18. They agree reasonably well with the theoretical predications of the program outlined in Appendix A, which are shown as solid lines.

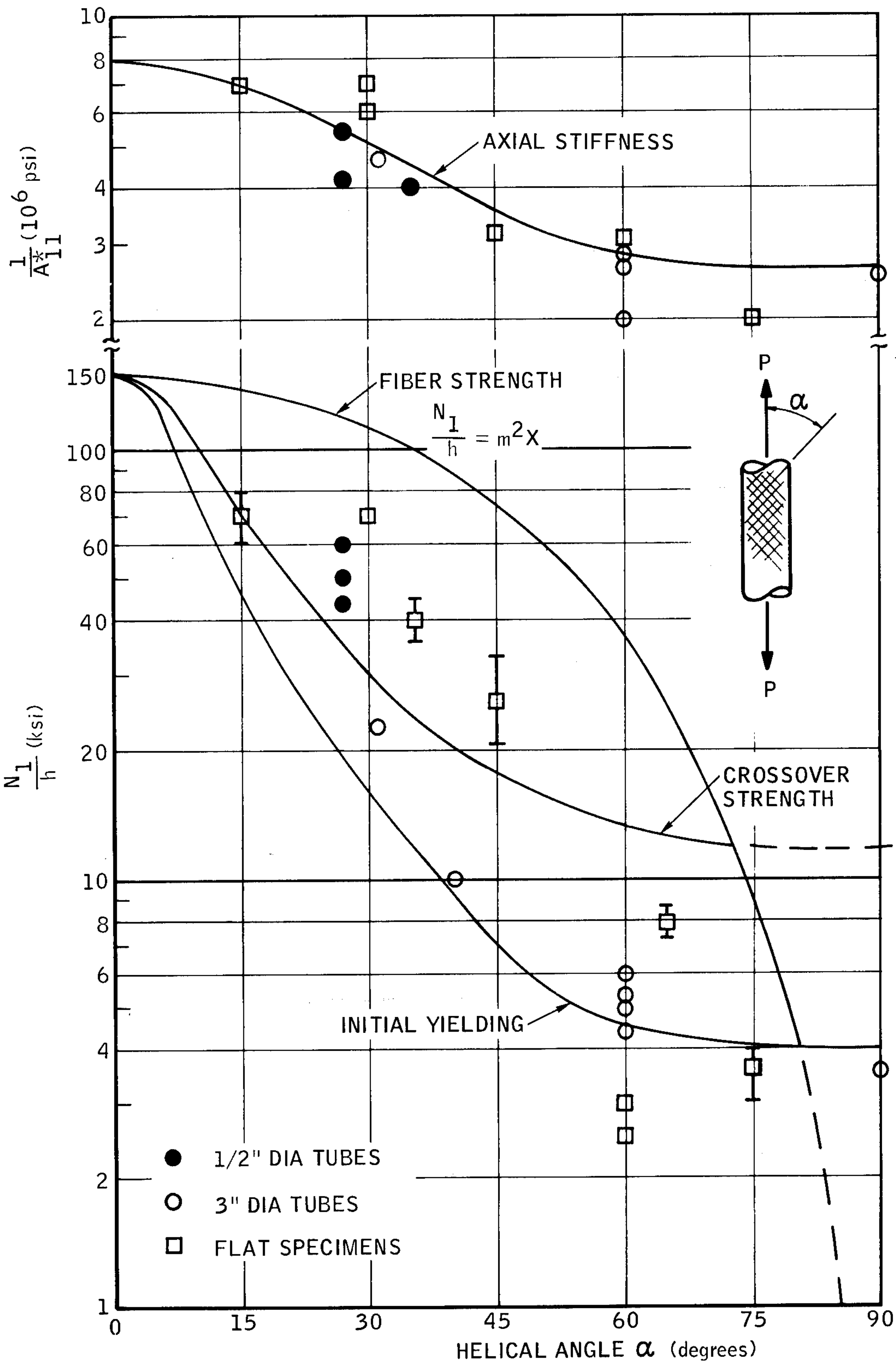


Figure 15. Uniaxial Tension Test, E Glass-Epoxy Helical-Wound Tubes

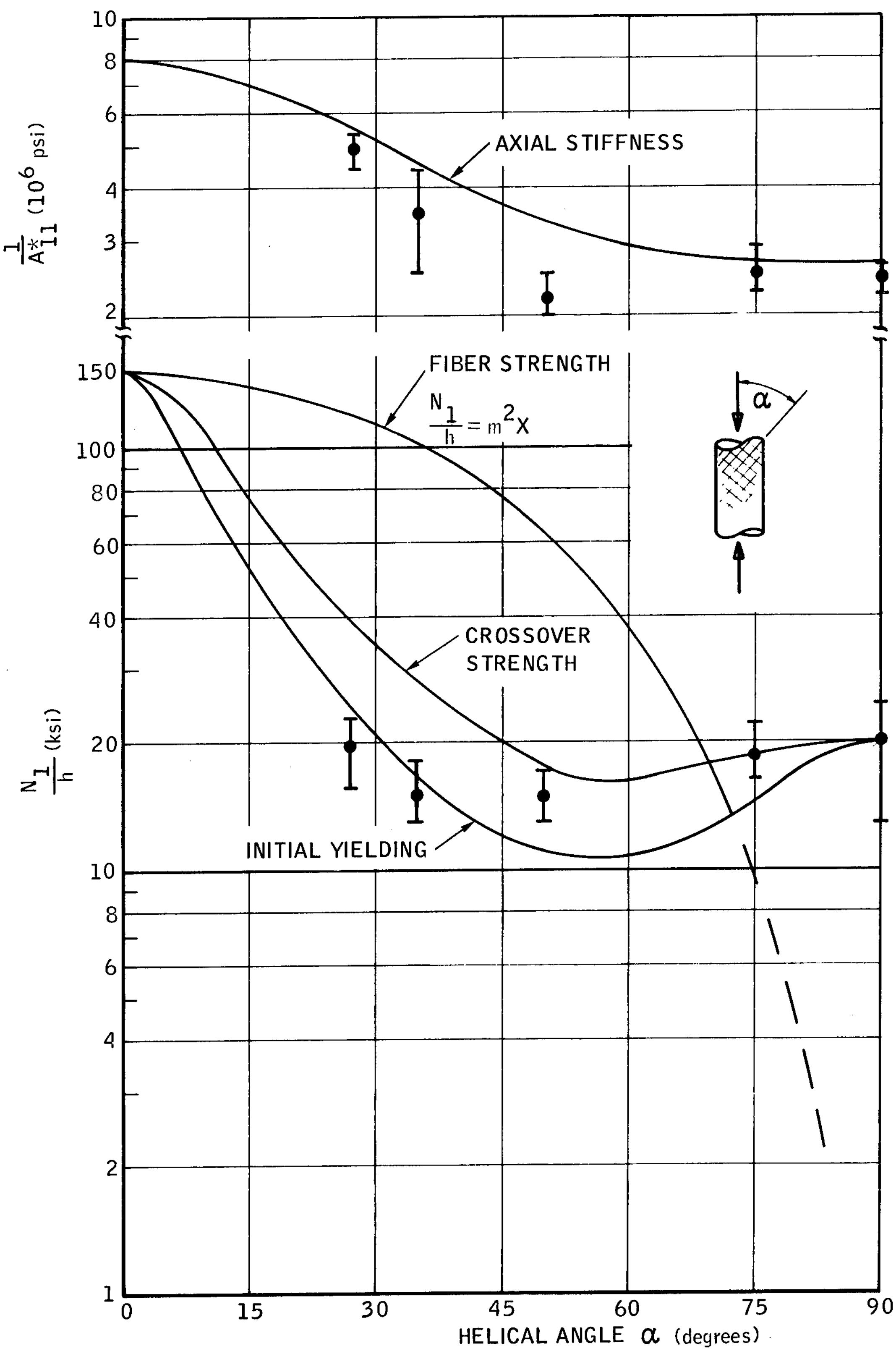


Figure 16. Uniaxial Compression Test, E Glass-Epoxy Helical-Wound 1-1/2 Inch Diameter Tubes

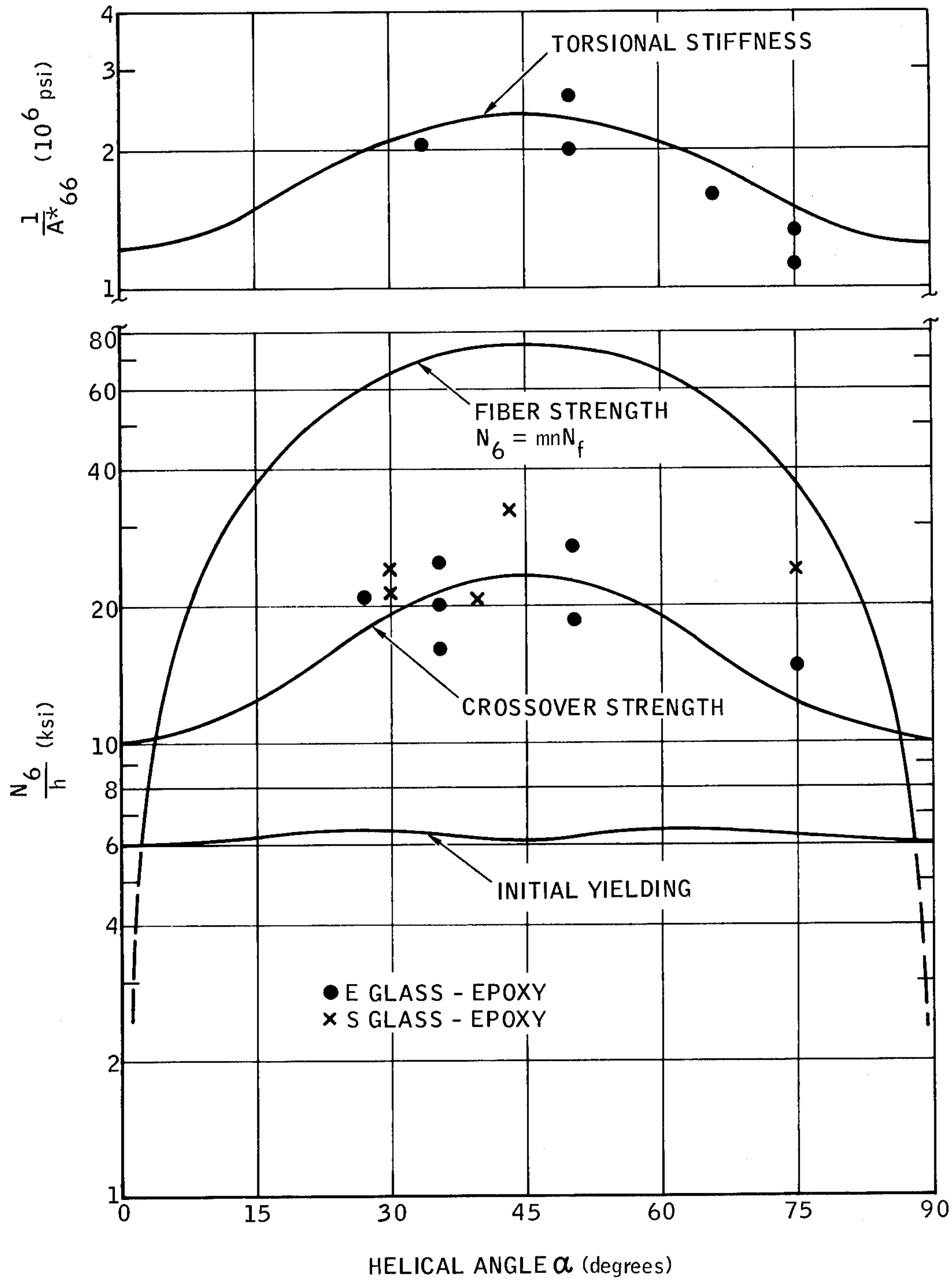


Figure 17. Pure Torsion Test, Glass-Epoxy Helical-Wound 1-1/2 Inch Diameter Tubes

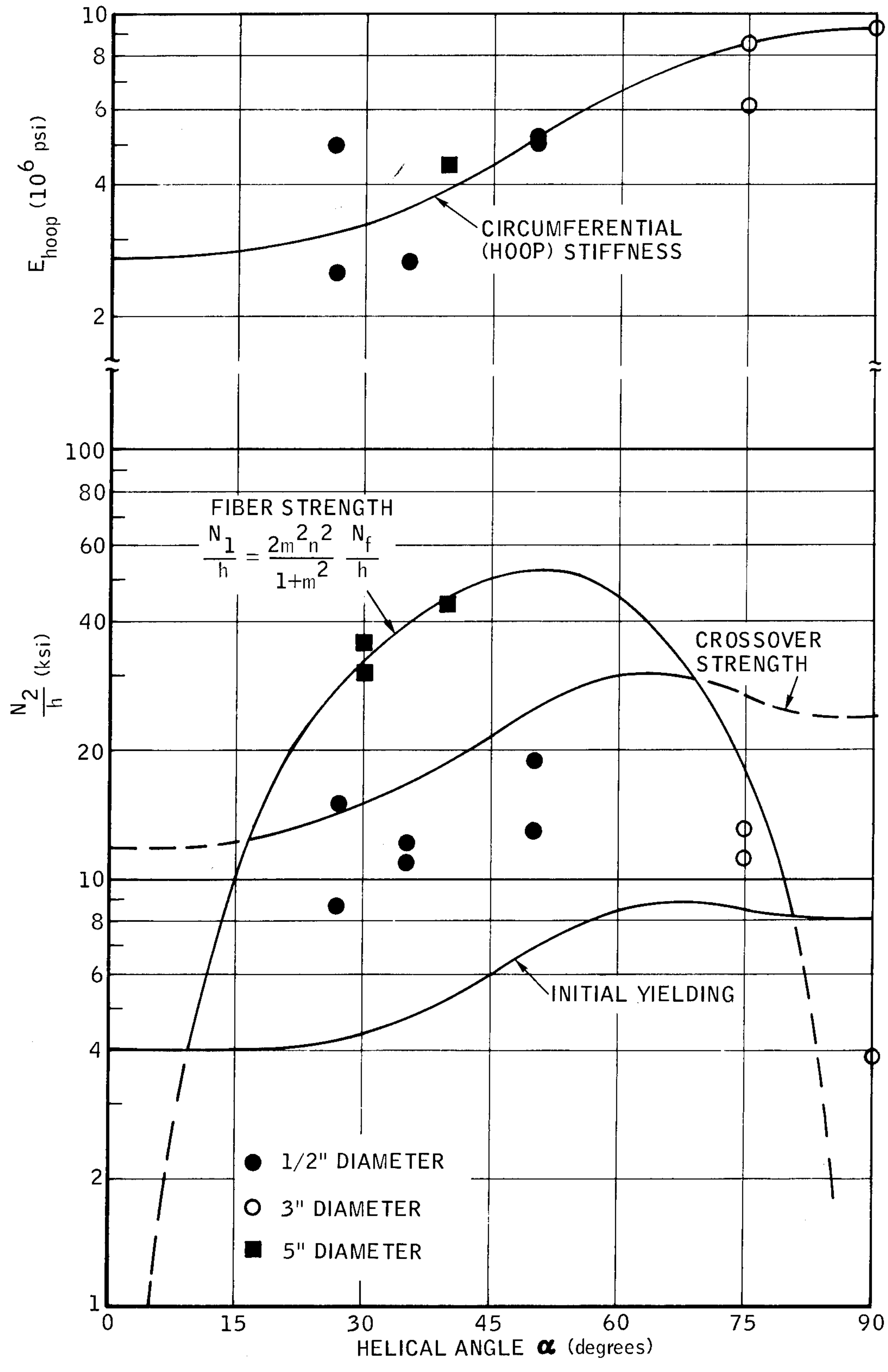


Figure 18. Internal Pressure Test, E Glass-Epoxy Helical-Wound Tubes

The results of the strength analysis are also shown in these figures. From the strength analysis, the various criteria for the determination of the load-carrying capacity of the helical-wound tubes can be determined.

Initial yielding was determined by using the constituent layer material constants given in Equations (37) and (38). The results of the computations are shown as solid lines and labeled "initial yielding" in Figures 15 through 18.

The strength criterion, assuming fiber failure, can be readily computed from Equation (36) using an axial strength of X = 150 ksi. The results of this computation for various loading conditions are shown as solid lines and labeled "fiber strength" in Figures 15 through 18.

The effect of crossovers can be accounted for by using effective transverse and shear strengths higher than those of the unidirectional composites. These higher strengths can be attributed to the additional reinforcement of the filament crossovers, similar to that occurring in woven fabrics. The exact amount of this increase can be experimentally determined. For the present, it requires a parametric study using the strength analysis outlined in Appendix A. Various transverse and shear strengths must be tried and the results that fit the experimental observations, as shown in Figures 15 through 18, can be considered appropriate. Consistent values of the effective strengths for various loading conditions must exist, since the effective strengths are treated as intrinsic characteristics of the material. Based upon experimental observation, an effective transverse strength of 12 ksi and an effective shear strength of 10 ksi appear to give reasonable results. They are shown as solid lines in Figures 15 through 18 and labeled "crossover strength". In all cases, for intermediate helical angles, the crossover strength criterion falls between the initial yielding and the ultimate strength based upon fiber failure. In the actual testing, initial yielding signifies the point where cracking in the matrix and/or interface becomes audible and visible. Because of the crossovers, complete uncoupling between the constituent layers is prevented until such time as the crossovers can no longer act as an effective internal agency to

perform the necessary load transfer. Beyond the crossover strength, the composite material will cease to be a continuum. In the case of a pressure vessel, excessive leakage through the wall is observed and the helical-wound tube cannot sustain additional pressure.

In the case of uniaxial tensile loading, the crossover strength signifies a complete departure from a continuum and continued loading will cause the fiber axes to rotate (a tendency to reduce the helical angle) and the load cannot be increased. The helical-wound tube behaves like an elastic-perfectly plastic material, permitting a large increase in strain at a constant stress.

The actual failure under uniaxial compressive loading occurred between the initial yielding and the crossover strength. The failure mechanism involved some buckling of fibers on the microscopic scale. There was no gross buckling. Away from one or two helical failure lines along which this microscopic buckling had occurred, the helical-wound tube remained essentially intact. There was no indication that crossover points had failed. For this reason, the actual compressive strength was lower than that predicted by the crossover strength. The failure mechanism under pure torsion also involved local buckling. But areas of matrix and interface failures were much more extensive than for compression. Crossover failures apparently had occurred. The experimentally determined ultimate load agreed with the theoretical prediction.

In order to establish the validity of filament crossovers as an internal agency for load transfer, a comparison has been made between the behavior of helical-wound tubes under tension and flat specimens cut from panels made by slitting and flattening out helical-wound tubes before curing. This comparison demonstrates that the increase in strength of helical-wound composites is derived from the crossovers rather than the external constraint provided by the end plugs bonded to a particular helical-wound tube. The flat specimens have cut fibers, whereas in the helical-wound tubes, the filaments are continuous and anchored at the end plugs. Experimental results demonstrate that the ultimate load for both the flat

specimens (data shown as squares in Figure 15) and the helical-wound tubes (data shown as dots in Figure 15) are identical. This leads to the conclusion that crossovers do, in fact, behave as an internal agency for load transfer, even when the filaments are not continuous, as in the case of the flat specimens. The circles in Figures 15 and 18 represent data obtained by testing 3 inch I.D. helical-wound tubes. The distribution of crossovers for these tubes is different than for the 1-1/2 inch I.D. tubes, the number of crossovers being fewer. The strength effect of the crossovers is apparently lower, thus making the strength of the 3 inch I.D. tubes not much different from that predicted by the initial yielding criteria. Of all the specimens tested, as shown in Figures 15 through 18, fiber tensile failures were induced only in the 5 inch I.D. pressure vessels, the data shown as solid squares in Figure 19. In the case of uniaxial tensile and compressive loadings, the failures did not involve breaks in the fibers. This experimental result is in agreement with the theoretical prediction of the netting analysis, in which a higher load is required (corresponding to 150 ksi fiber stress) for fiber failures to occur. In the case of torsion, the failure mechanism involved fiber buckling and again the compressive strength along the fiber axis was not reached.

Helical-wound tubes under tensile loading exhibited a linear stress-strain relationship up to the initial yielding. This is shown in Figure 20, where both the axial and hoop strains of a 3 inch I.D. tube were recorded. The effective stiffnesses, as measured by A^*_{11} and A^*_{12}, were in excellent agreement with the theoretical predictions. The solid lines shown in this diagram are the reciprocals of A^*_{11} and A^*_{12}, and represent the results obtained from the computer program outlined in Appendix A, using the data of Equations (37) and (38). A 1-1/2 inch I.D. helical-wound tube, with a helical angle of 27 degrees, was also tested. The axial strain readings indicated a considerable amount of time-dependent effect. This inelastic behavior is very pronounced after initial yielding occurs. The stress-strain relation obtained is shown in Figure 21. The theoretically predicted axial stiffness is shown as a solid line and the actual strain as recorded by a hand-operated strain recorder, is shown as a dotted line. The degree of inelasticity depended upon the time required to make the

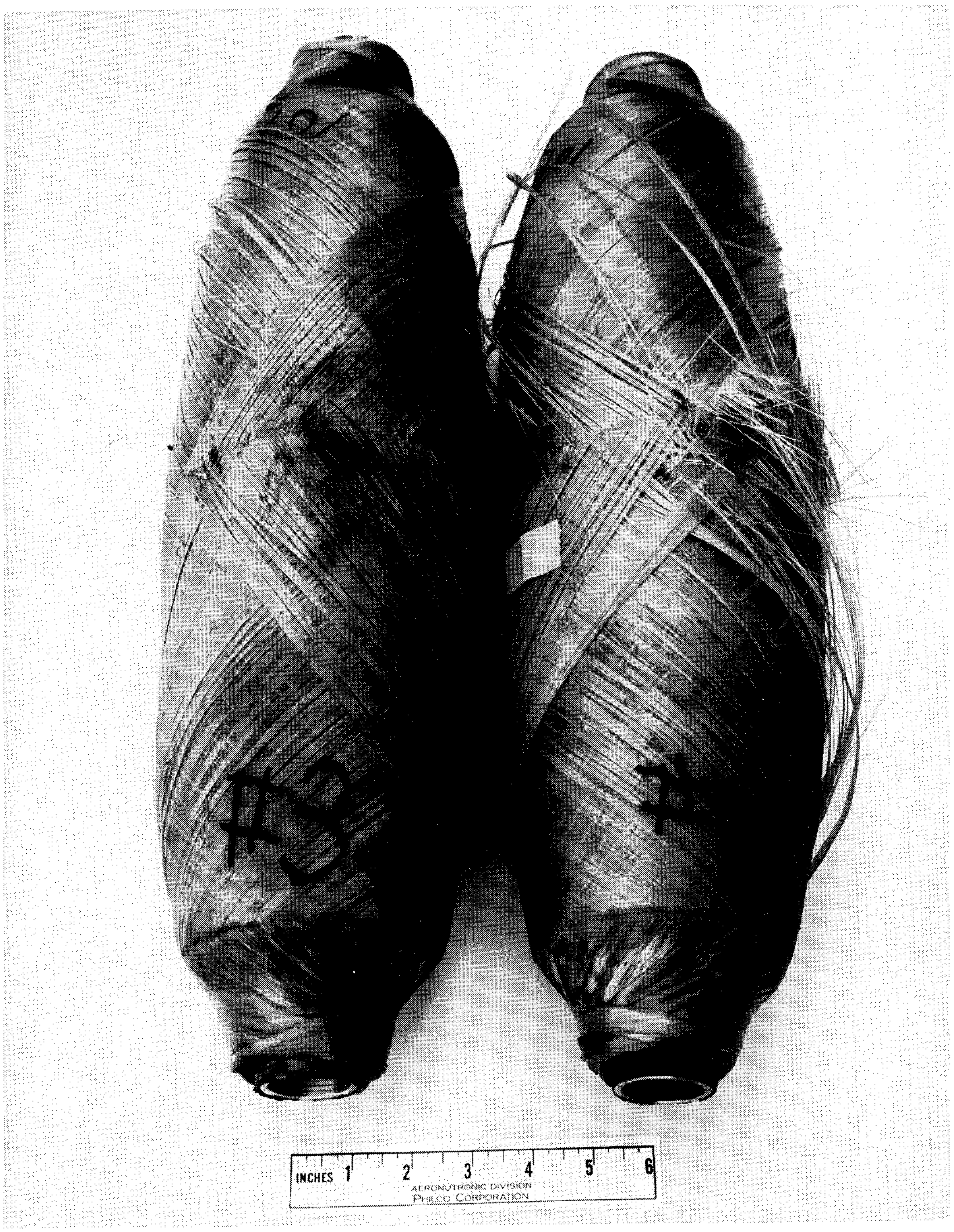


Figure 19. Helical-Wound Tubes After Failure

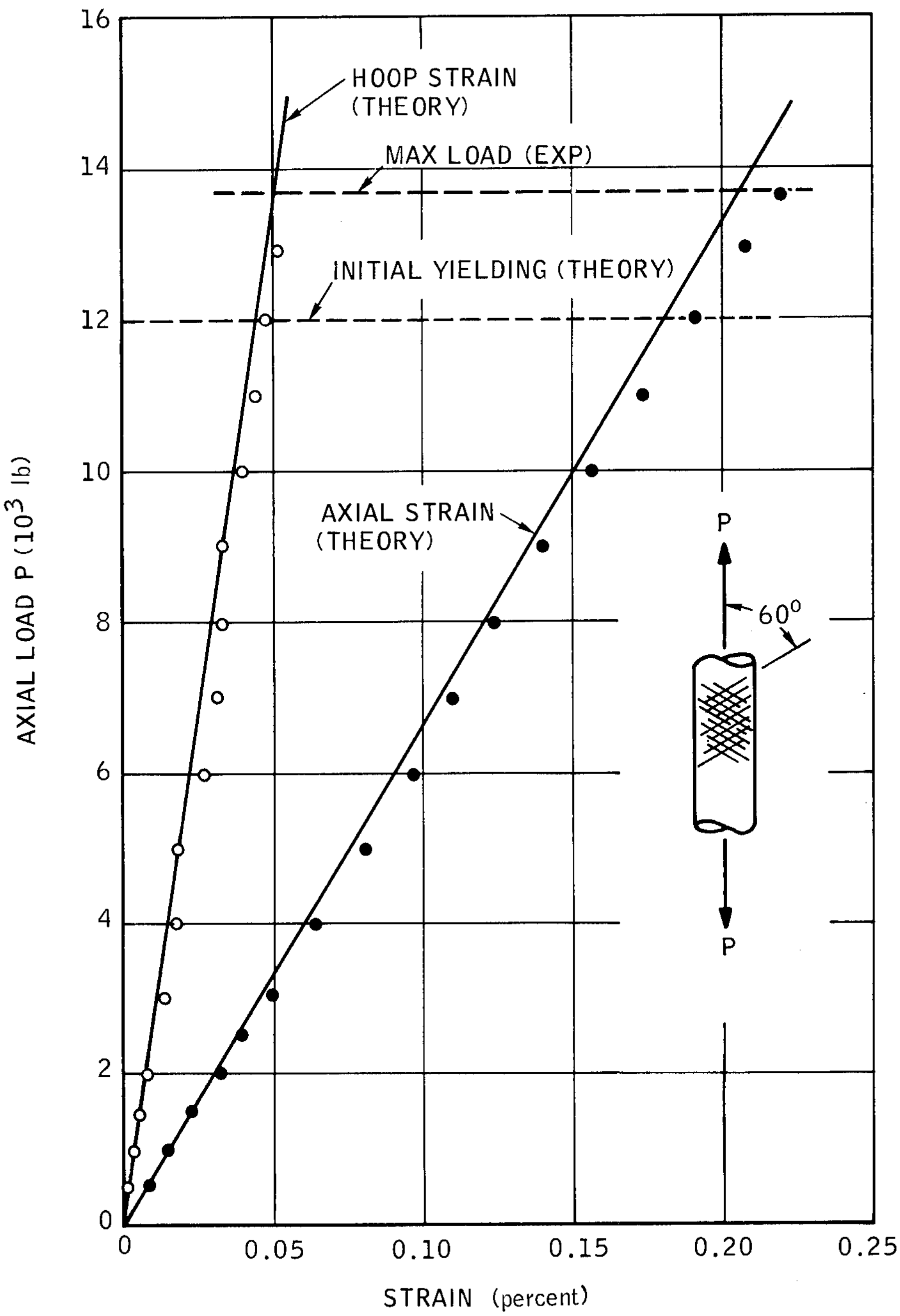


Figure 20. Uniaxial Tension Test of a 3 Inch Diameter Glass-Epoxy Helical-Wound Tube

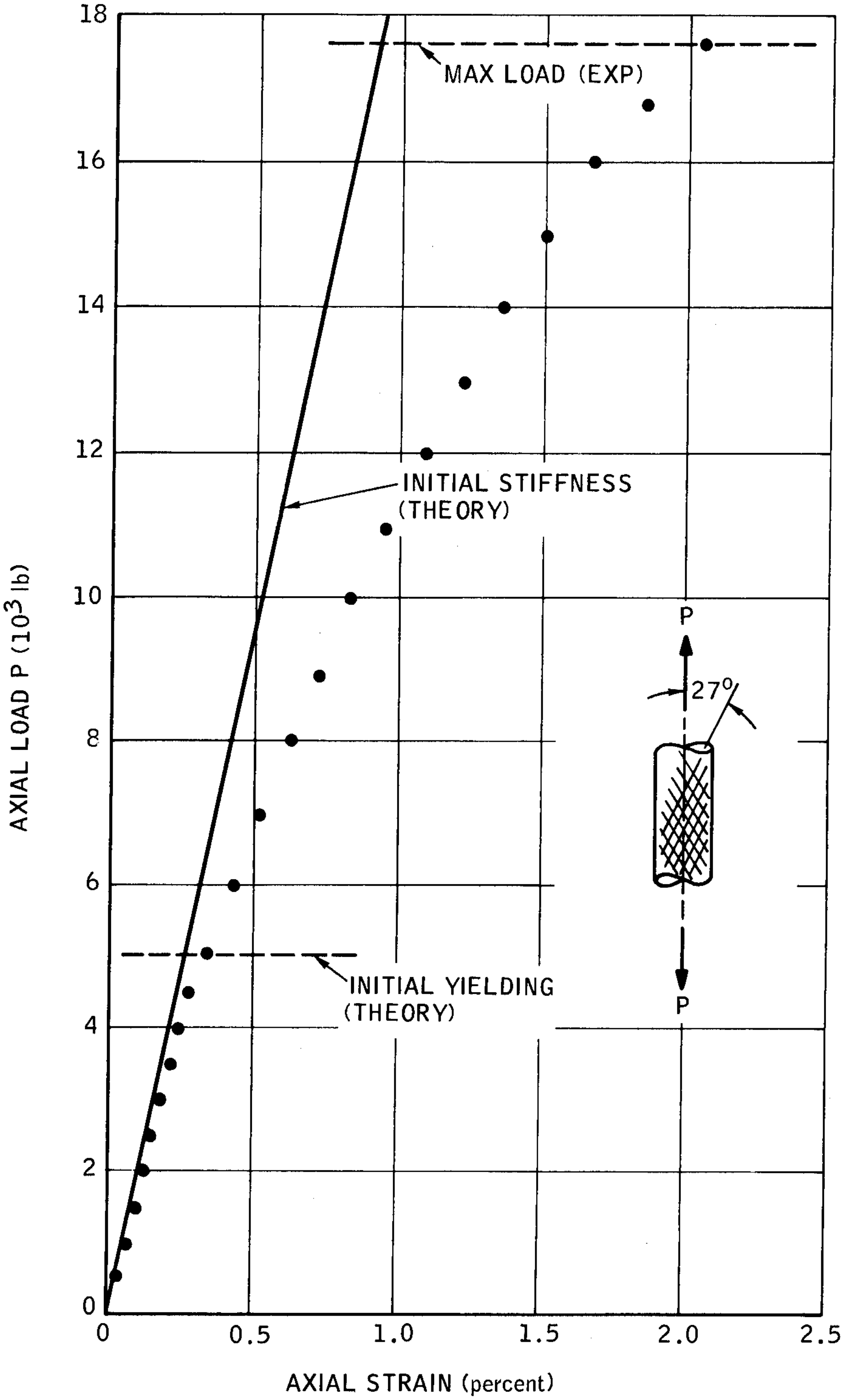


Figure 21. Uniaxial Tension Test of a 1-1/2 Inch Diameter Glass-Epoxy Helical-Wound Tube

strain recording at each load level. It is, of course, anticipated that the actual strain reading will be different as the rate of loading and the time required for the strain recording are changed.

The stress-strain relationships obtained for typical compression tests also exhibited a degree of nonlinearity very similar to that shown in Figure 21.

In torsion tests, inelastic behavior becomes apparent after initial yielding, as shown in Figure 22. The initial slope agrees very well with that predicted by the theory.

In Figure 23, a typical pressure versus strain relation for a pressure vessel subjected to internal pressure is shown. Again, the theoretically predicted slope, represented by the solid line, corresponds closely to the experimental observation. The ultimate pressure was reached when excessive leaking occurred. This pressure corresponds to the crossover strength as predicted by using the effective transverse and shear strengths. No fiber failure was induced in this case. This can be explained by the fact that the internal agency could not support the pressure required to cause fiber failure. In the case of the 5 inch I.D. pressure vessels (data shown as solid squares in Figure 19), a very heavy rubber liner was installed inside the pressure vessel. This liner prevented leakage through the wall after the crossover strength was exceeded and internal pressure could be increased to induce fiber failures. The pressure at which fiber failure occurred agreed with that predicted by the simple netting analysis.

In conclusion, helical-wound tubes tested in the present program had various patterns of filament crossovers, which provided post-yielding load-carrying capability. The crossovers, however, did not have sufficient strength to transfer external load necessary to cause fiber failures. The only exceptions to this, apparently, were the 5 inch I.D. pressure vessels subjected to internal pressurization. The implication is that the intrinsic strength of the fibers is not fully developed in helical-wound tubes under a general loading condition. Thus, higher filament strengths may not be

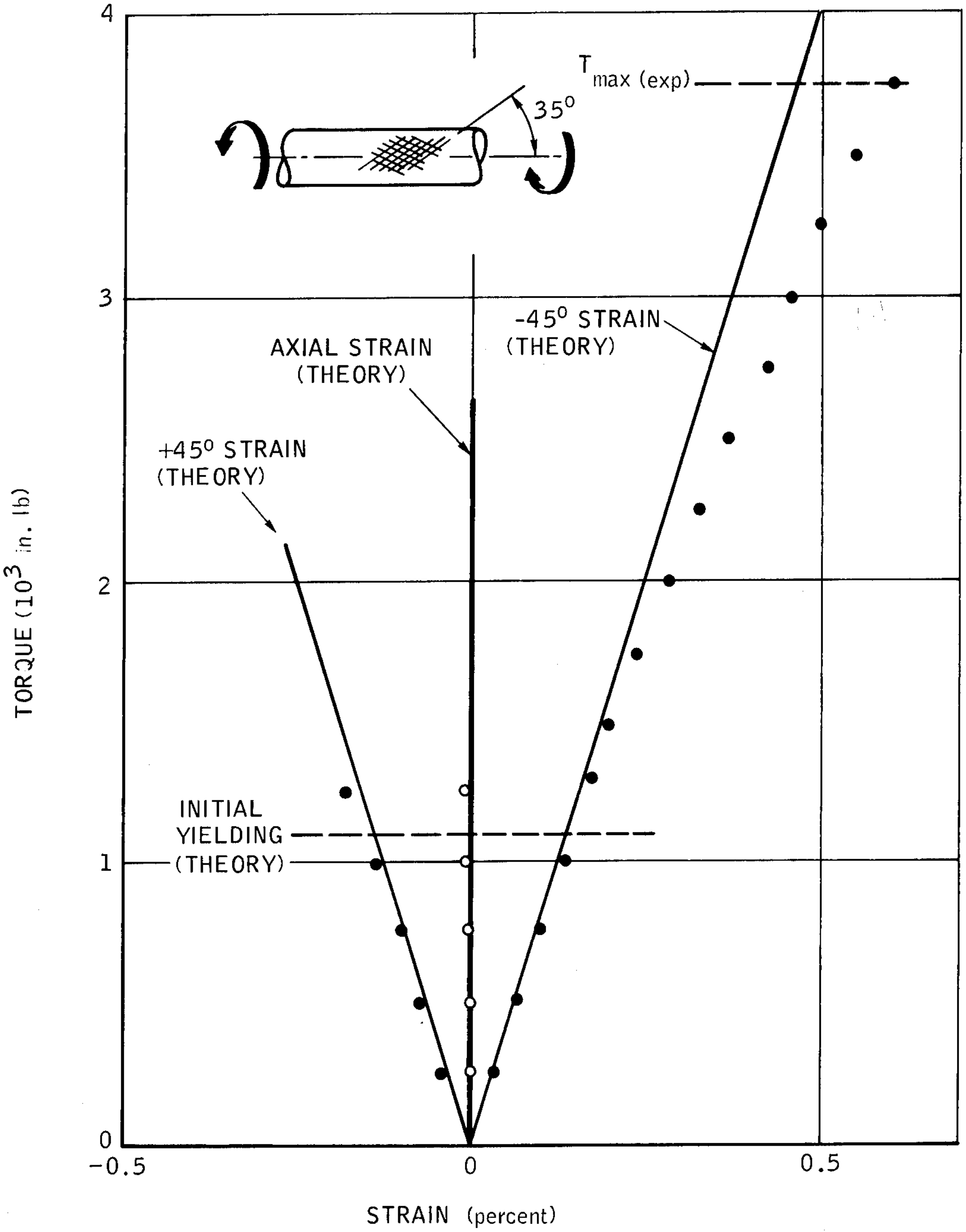


Figure 22. Torsion Test of a 1-1/2 Inch Diameter Glass-Epoxy Helical-Wound Tube

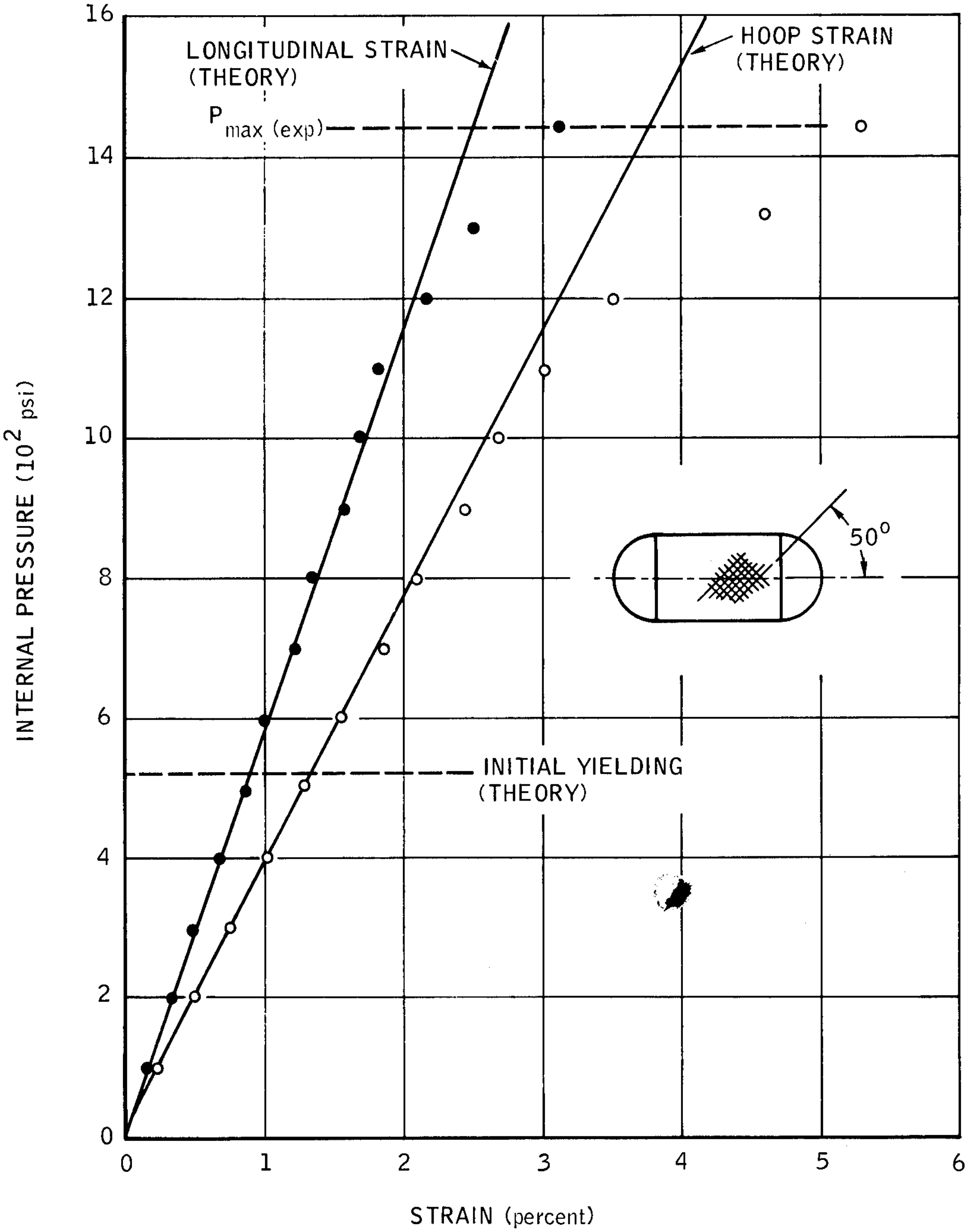


Figure 23. Internal Pressure Test of a 1-1/2 Inch Diameter Glass-Epoxy Helical-Wound Tube

necessary for many structural applications, particularly those involving tensile and compressive loads and pure torsion.

Based upon available experimental data, one could very well construct curves using one-half of the values predicted by the netting analysis. A simple explanation would be that the crossovers induce stress concentrations of a factor of about two, and that the experimental data in the case of tension, torsion, and internal pressure closely follow this prediction. However, this curve-fitting technique is not reasonable to the extent that none of these loadings induce fiber failures as assumed in the netting analysis. The failure mechanisms are associated with the breakdown of the internal agency and it is believed that the theory proposed here on the basis of crossover strength is more directly applicable.

SECTION 3

LONGITUDINAL SHEAR LOADING

Introduction

As discussed in detail in previous investigations,[2,7] and utilized in Section 2, a strength analysis of composite materials requires a knowledge of the stiffness properties E_{11}, E_{22}, and G of the unidirectional composite, as well as its strength properties X, Y, and S. In previous investigations,[2] these values were experimentally determined.

In this and the next section, methods will be presented for analytically predicting the values of E_{22}, G, Y, and S, based upon the constituent material properties of the unidirectional composite, as well as geometrical considerations such as filament shape, packing arrangement, and volume percent.

The material properties G and S, the composite shear modulus, and composite shear strength, respectively, can be evaluated by considering a longitudinal shear loading, as will be discussed in this section.

The material properties E_{22} and Y, composite transverse modulus and composite transverse strength, respectively, are obtained from a transverse normal loading, as discussed in Section 4.

The axial properties of a unidirectional composite, E_{11} and X, and specific problems associated with their analytical prediction, are discussed in Reference 8.

Description of Problem

To obtain a meaningful solution for the distribution of stresses within the filaments and matrix of a composite material, the problem must be accurately formulated. That is, the actual physical behavior must be correctly represented on the micromechanical scale.

Because of the complex stress state to be solved for, a theory of elasticity approach must necessarily be utilized. A strength of materials solution is not applicable because realistic assumptions as to strain distributions cannot be formulated. Since it can be assumed that no variations of stress in the direction of the unidirectional filaments occur when a longitudinal shear loading is applied to the composite, the problem is two-dimensional.

To treat the problem analytically, assumptions must be made as to filament packing arrangement and geometry of the individual filaments. The method of solution to be used is based upon the existence of certain symmetry conditions. A rectangular filament packing array is assumed, as shown in Figure 24. The individual filament cross-sections are assumed to be symmetrical about each of the coordinate axes, x and y. Within this restriction, the filaments can be of arbitrary shape, i. e., circular, elliptical, diamond, square, rectangular, hexagonal, etc.

Having established the assumptions of rectangular packing and symmetric filaments, the problem can be formulated exactly (within the usual assumptions of the theory of linear elasticity). This is perhaps the key point of the analysis to be presented.

Because of this assumed symmetry, a fundamental or repeating unit, as indicated by the dashed lines of Figure 24, can be isolated and analyzed, being typical of the entire composite. When the composite is subjected to longitudinal shear loads applied at a distance from the element being analyzed, in the directions indicated by the average values $\overline{\tau}_{zx}$ and $\overline{\tau}_{zy}$ in Figure 25, a complex shear stress distribution will be induced. This is

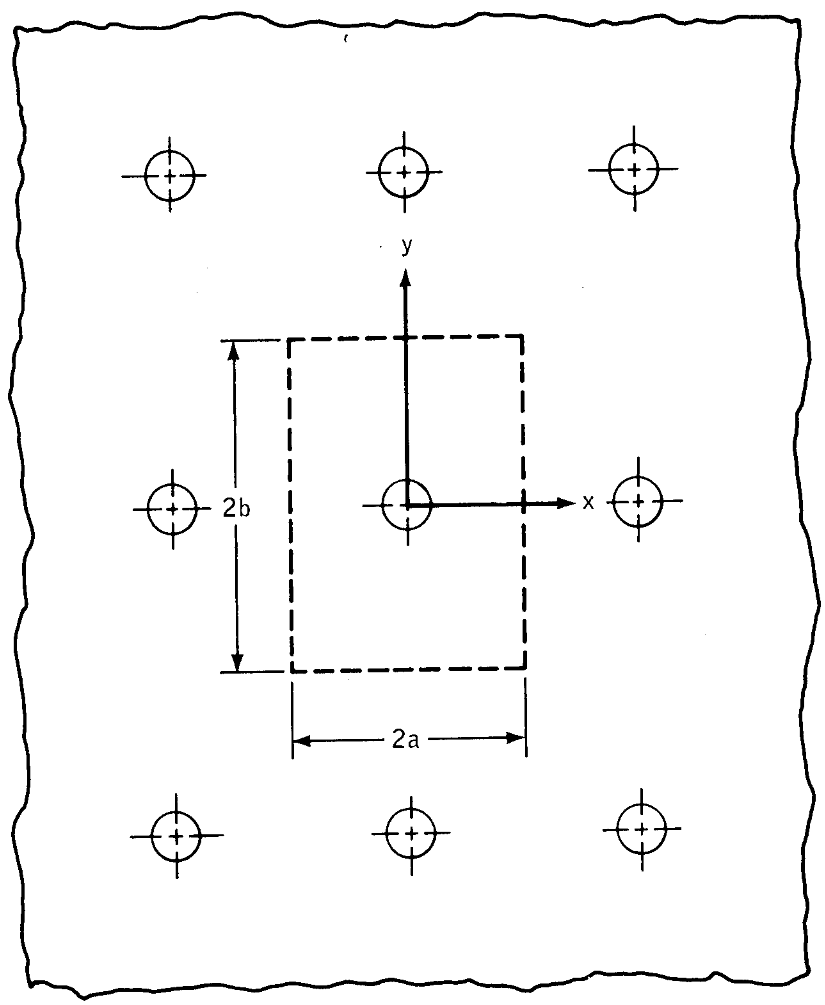


Figure 24. Composite Containing a Rectangular Array of Filaments Imbedded in an Elastic Matrix

the result of the dissimilar material properties of the filaments and matrix and also because of interactions between the filament being analyzed and adjacent filaments.

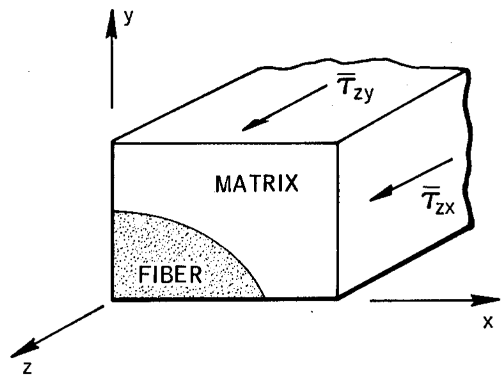


Figure 25. First Quadrant of the Fundamental Region - Longitudinal Shear Loading

However, because of symmetry, each average longitudinal shear stress τ_{zx} and τ_{zy}, when applied separately, will cause a uniform axial displacement of the boundary of the fundamental region on which it acts. Thus, the problem can be formulated as a displacement boundary value problem, interactions between adjacent filaments being automatically and accurately taken into account.

Method of Analysis

The problem of longitudinal shear loading is defined by a displacement field of the form

$$u = v = 0 \qquad w = w\ (x, y) \tag{54}$$

For such a system the only nonvanishing stress components are:

$$\tau_{zx} = G \frac{\partial w}{\partial x}, \qquad \tau_{zy} = G \frac{\partial w}{\partial y} \tag{55}$$

where G is the shear modulus of the material.

The equilibrium equations in the x and y directions are identically satisfied, equilibrium in the z direction requiring that

$$G\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) = 0 \tag{56}$$

Consider an infinite elastic body containing a rectangular array of cylindrical elastic inclusions oriented parallel to the z axis (see Figure 24). Because of the necessity of establishing certain symmetry conditions in the solution, the individual inclusions must have two axes of symmetry, these axes being oriented parallel to the x and y axes. Within this restriction, the inclusions can be of arbitrary shape.

It will be assumed that the inclusions, which have a shear modulus G_f, are perfectly bonded to the matrix, which has a shear modulus G_m.

The spacings of the inclusions in the x and y directions are taken as 2a and 2b, respectively. The dimensions of the inclusions are arbitrary within the physical limits imposed by these spacings.

The body is assumed to be loaded at infinity by uniform shear stresses, $\overline{\tau}_{zx}$ and $\overline{\tau}_{zy}$, each of arbitrary magnitude.

The stresses in the composite medium can be analyzed by isolating a fundamental region in the x-y plane consisting of a rectangular element of dimensions 2a by 2b (see Figure 24) containing an inclusion. The average shear stresses $\overline{\tau}_{zx}$ and $\overline{\tau}_{zy}$ acting on the sides of the rectangle will be chosen as the arbitrary loading parameters.

Because of the assumed double periodicity of the inclusion geometry and inclusion spacing, the displacement field must satisfy the requirement

$$w\ (x, y) = -w\ (-x, -y) \tag{57}$$

It normally is desired to solve the shear problem for a given set of shear loading conditions, i. e., specifying $\overline{\tau}_{zx}$ and $\overline{\tau}_{zy}$, rather than for given boundary displacement conditions. However, it is much simpler to solve the problem when expressed in terms of displacements as, for example, in Equations (55) and (56). Thus, the procedure will be to first solve the problem for a specified uniform displacement, w_1^*, along the side x = a of the fundamental region, the boundary condition on the other three straight sides being, from symmetry conditions:

$$G \frac{\partial w_1^*}{\partial y} = 0 \text{ along } y = 0 \text{ and } y = b$$

$$w_1^* = 0 \text{ along } x = 0 \tag{58}$$

Having solved this problem, defined as Problem 1, the average shear stress $\overline{\tau}^*_{zx}$ corresponding to this specified displacement, w_1^*, is determined by first calculating τ^*_{zx} at each node point on the boundary x = a and then taking the average value.

Assuming that it was desired in the original problem to solve for the case of a specified average shear loading $\overline{\tau}_{zx}$, along x = a, the values of displacements $w_1(i, j)$ and the stresses $\tau_{zx}(i, j)$ and $\tau_{zy}(i, j)$ at each node point (i, j) in the array corresponding to this loading are obtained by multiplying the results above by the ratio

$$f_1 = \frac{\overline{\tau}_{zx}}{\tau^*_{zx}} \tag{59}$$

Thus, a solution for the case of specified average shear loading τ_{zx} along the boundary x = a and zero shear along the boundary y = b has been obtained (Problem 1).

This same procedure is then repeated to obtain a solution for the case of a specified average shear loading $\overline{\tau}_{zy}$ along the boundary y = b and zero shear along the boundary x = a (defined as Problem 2), i. e., specify a uniform displacement, w_2^*, along the boundary y = b, and solve the displacement boundary problem using the boundary conditions:

$$G \frac{\partial w_2^*}{\partial x} = 0 \text{ along } x = 0 \text{ and } x = a$$

$$w_2^* = 0 \text{ along } y = 0 \tag{60}$$

After calculating an average shear stress $\overline{\tau}_{zy}^*$ along y = b, all stress and displacement values calculated above are multiplied by the ratio

$$f_2 = \frac{\overline{\tau}_{zy}}{\overline{\tau}_{zy}^*} \tag{61}$$

to obtain the solution for the case of a specified average shear loading $\overline{\tau}_{zy}$ along the boundary y = b and zero shear along the boundary x = a (Problem 2).

In solving the two individual problems outlined, it is necessary to establish continuity conditions at the interface between the inclusion and the matrix. These conditions, which are identical in both problems, are:

(1) continuity of displacement across the interface

$$w_f = w_m \tag{62}$$

(2) continuity of shear stress across the interface

$$G_f \frac{\partial w}{\partial n} = G_m \frac{\partial w}{\partial n} \tag{63}$$

where n is in a direction normal to the interface boundary and the subscripts f and m represent filament and matrix, respectively.

The effective shear moduli of the composite material are determined as follows:

<u>x - direction</u>

$$G_x = \frac{\bar{\tau}_{zx}}{w_1\,(a, o)/a} = \frac{a\,\bar{\tau}_{zx}}{w_1\,(a, o)} \tag{64}$$

<u>y - direction</u>

$$G_y = \frac{\bar{\tau}_{zy}}{w_2\,(o, b)/b} = \frac{b\,\bar{\tau}_{zy}}{w_2\,(o, b)} \tag{65}$$

Having obtained a solution for each of the two problems outlined, i. e., $\bar{\tau}_{zx}$ specified, $\bar{\tau}_{zy} = 0$ and $\bar{\tau}_{zy}$ specified, $\bar{\tau}_{zx} = 0$, the solution of the general problem of combined shear loading is obtained by superposition.

Solution Technique

A relaxation method of solution of the two problems outlined in the previous paragraph has been formulated using a finite difference representation. The method of solution is presented in Appendix B, along with a complete description of the digital computer program developed, a computer program listing, and a sample problem. The program is written in Fortran IV programming language for the Philco 2000 digital computer. The program can, of course, be readily converted for use on other computer systems.

Several unique numerical analysis techniques and computer programming methods were developed during the course of this investigation. These are discussed in Appendix B.

Presentation of Results

The primary goal of the present investigation has been to develop a method of determining the distribution of stresses in a composite and the composite stiffness, rather than to make extensive parametric studies. However, typical results obtained for several filament geometries and packing densities are shown in Figure 26. The computer solution calculates stresses and displacements throughout the region, as indicated in the sample problem of Appendix B. In Figure 26, only the effective composite shear modulus, G, and the stress concentration factor, SCF, i. e., the ratio of the maximum induced shear stress to the applied stress, are shown. A glass-epoxy system was assumed, using $G_f = 4.0 \times 10^6$ psi and $G_m = 0.2 \times 10^6$ psi.

The results given for square fibers in a diamond packing were obtained by a transformation of the coordinate axes through an angle of 45 degrees from the case of square fibers in a square array. It is interesting that the diamond packing, for $v_f = 70$ percent, yields the highest composite shear modulus (1.92×10^6 psi) without inducing a high stress concentration (SCF = 2.46).

In Figure 27 are shown typical results obtained for circular fibers and various composite systems. The reinforcing factor, G/G_m, i. e., the ratio of the composite shear modulus to the shear modulus of the, matrix, is plotted against the ratio of the shear moduli of the constituents, G_f/G_m, with percent fiber volume as a parameter. A few typical combinations of constituent materials are indicated. As can be seen, the composite shear modulus increases significantly as the filament packing density is increased.

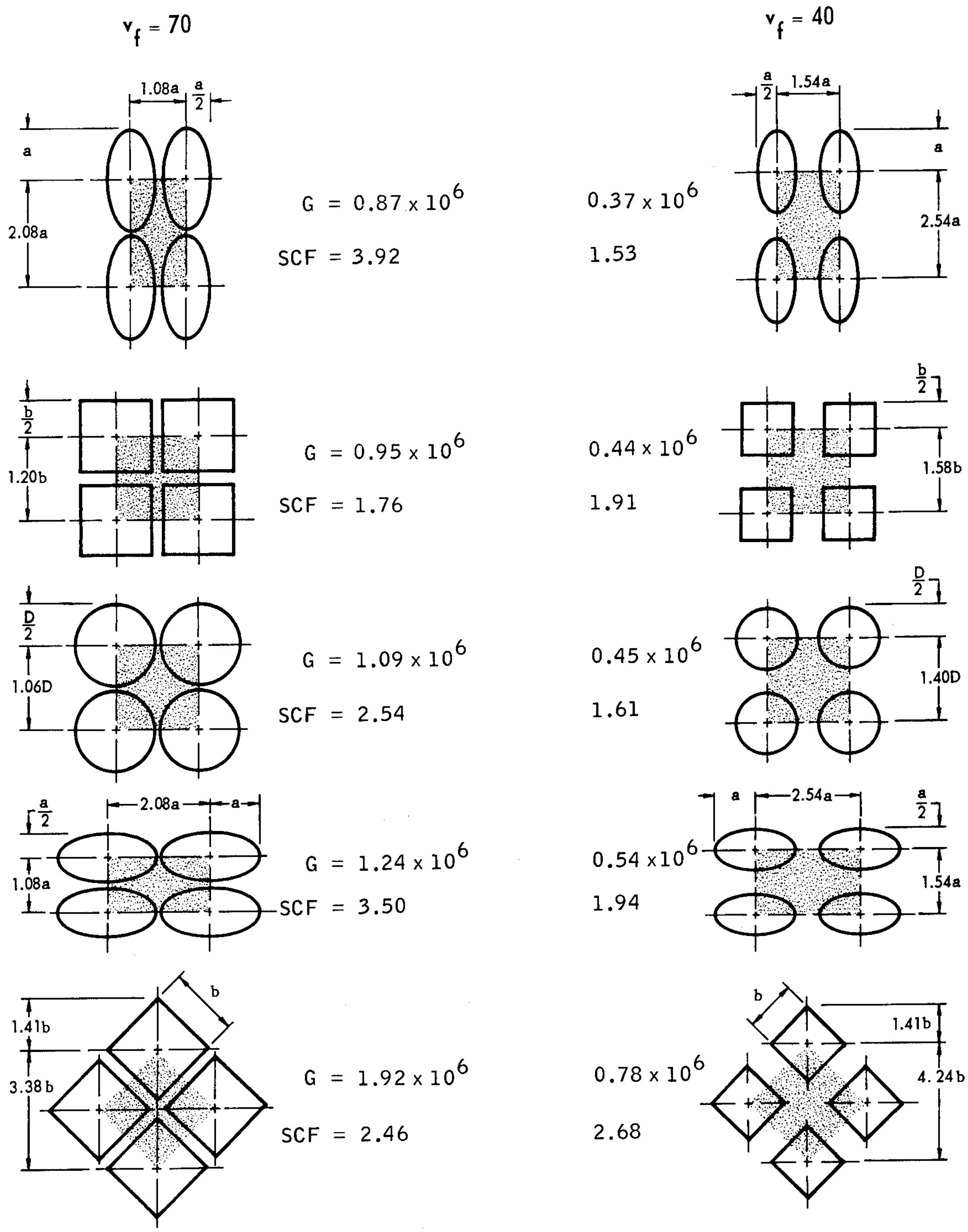


Figure 26. Shear Modulus (G) and Stress Concentration Factor (SCF) for Glass-Epoxy Composites Subjected to an Applied Shear Stress

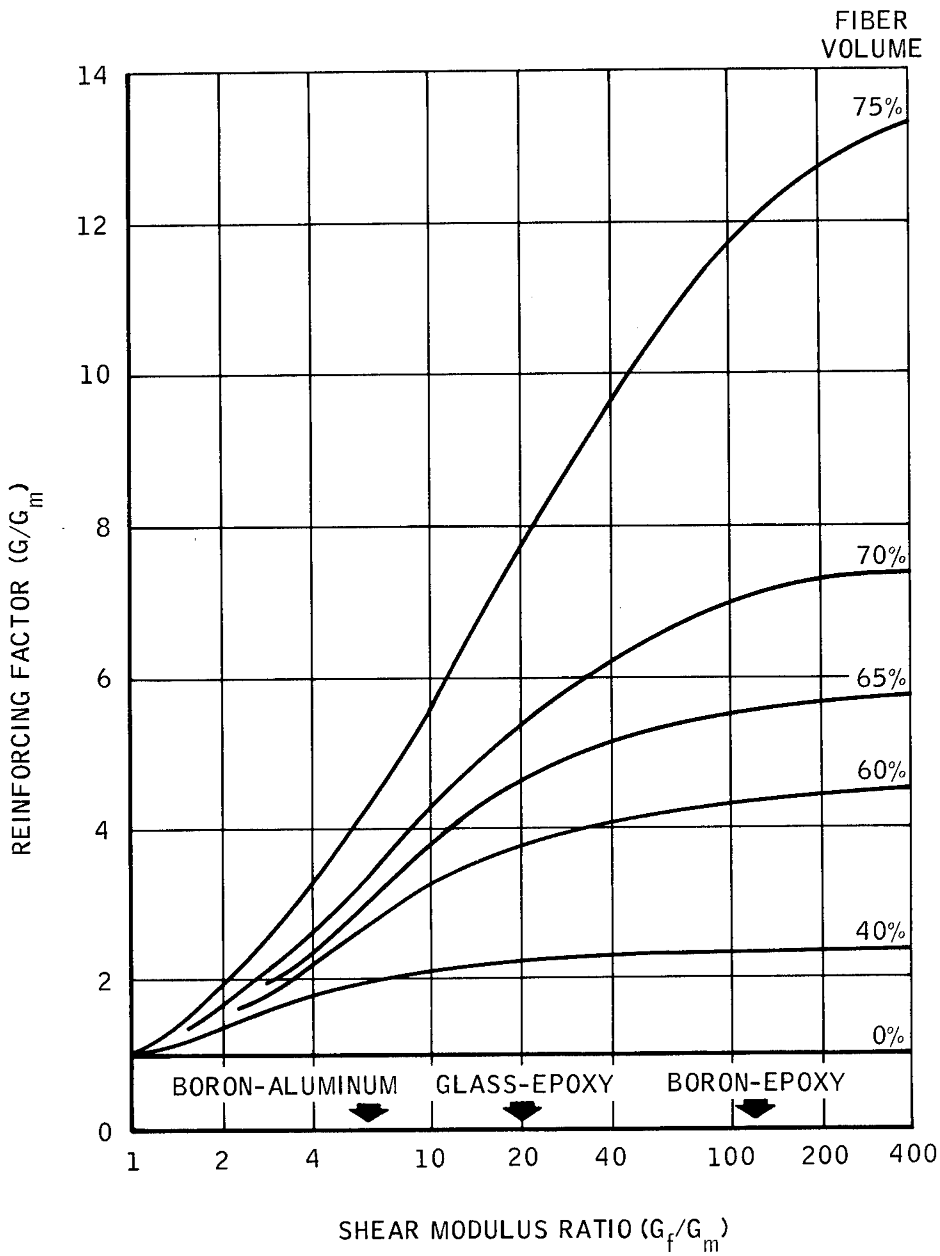


Figure 27. Composite Shear Modulus for Circular Fibers in a Square Packing Array

Based upon available experimental data, the theoretical predictions presented in Figure 27 are reasonably accurate. For example, for a fiber volume of 70 percent, and an epoxy shear modulus of 0.2×10^6 psi, the following values are obtained:

	Composite Shear Modulus	
	Predicted	Experimental
Glass-epoxy composite	1.1×10^6 psi	1.2×10^6 psi
Boron-epoxy composite	1.4×10^6 psi	1.5×10^6 psi

To show the specific influence of the matrix material on the composite shear modulus, another plot is shown in Figure 28, in which a particular fiber shear stiffness is assumed and held constant ($G_f = 24 \times 10^6$ psi was used, which is typical, for example, of boron filaments). Composite shear modulus, G, is plotted against matrix shear modulus, G_m, with percent fiber volume as a parameter. Various potential matrix materials are indicated on the abscissa. The range of attainable composite shear moduli for each matrix material is clearly shown.

The significance of these results to materials design is discussed in greater detail in Section 5 of this report.

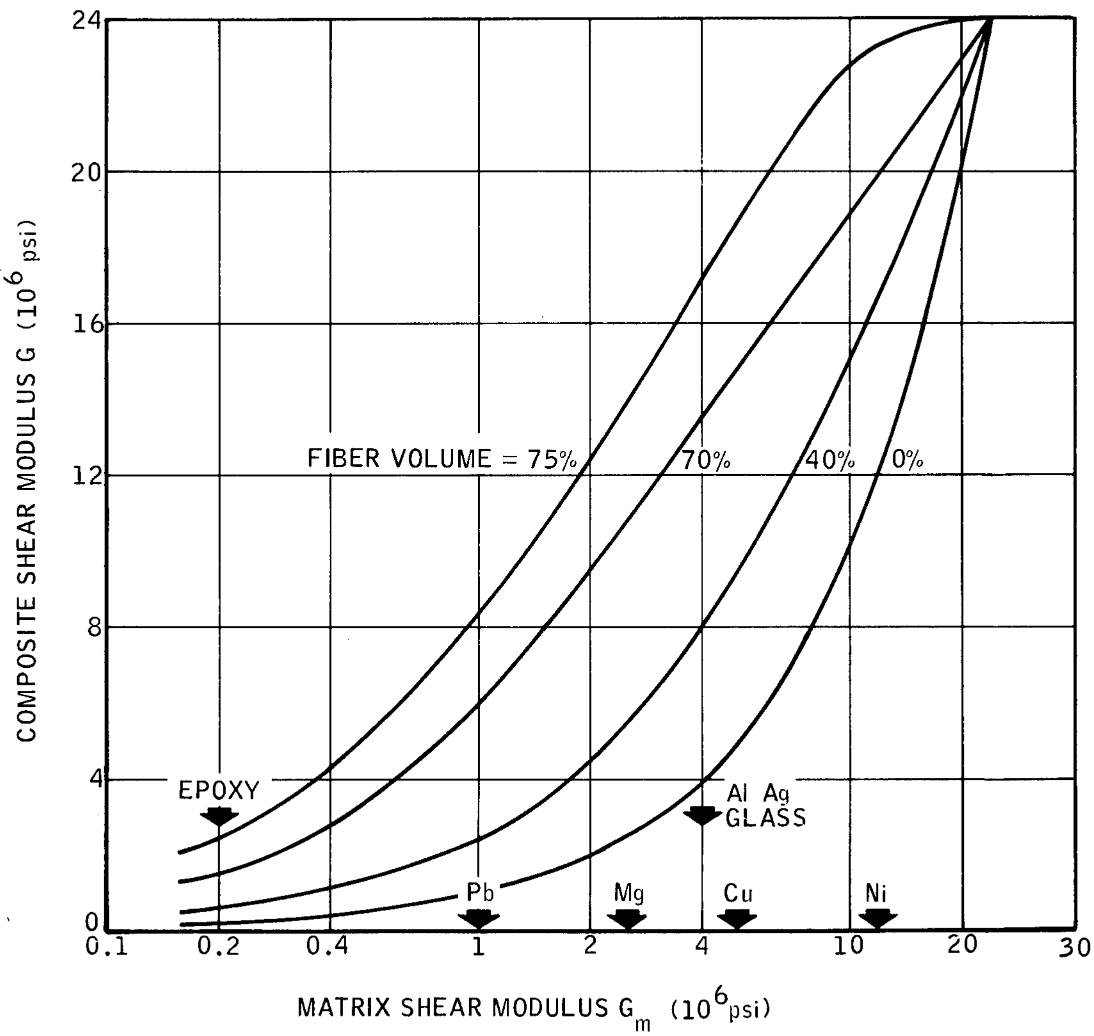


Figure 28. Composite Shear Modulus for Boron Fibers as a Function of Matrix Shear Modulus and Fiber Volume

SECTION 4

TRANSVERSE NORMAL LOADING

Introduction

The need for detailed investigations of the stresses developed in individual fibers and the surrounding matrix of a unidirectional composite material was discussed in the first two paragraphs of Section 3, longitudinal shear loading being considered.

A transverse normal loading will be analyzed in this section. The basic principles of the formulation of the problem are essentially the same as for a longitudinal shear loading condition. However, the details of the formulation and the numerical solution required are considerably more complex. This is primarily because of the fact that two dependent displacement variables, u and v, occur, whereas for longitudinal shear loading, only a single dependent variable, axial displacement w, exists.

The basic formulation of the problem follows that developed by Aeronutronic consultant, Dr. H. B. Wilson, Jr., for the case of a doubly periodic array of rigid inclusions in an elastic matrix.[9]

As in Section 3, to treat the problem analytically, assumptions must be made as to filament packing arrangement and the geometry of the individual filaments. Because the method of solution to be used is based upon the existence of certain symmetry conditions, a rectangular filament packing array has been assumed, as shown in Figure 29. The individual filament cross sections are assumed to be symmetrical about each of the coordinate axes, x and y. Within this restriction, the filaments can be of arbitrary

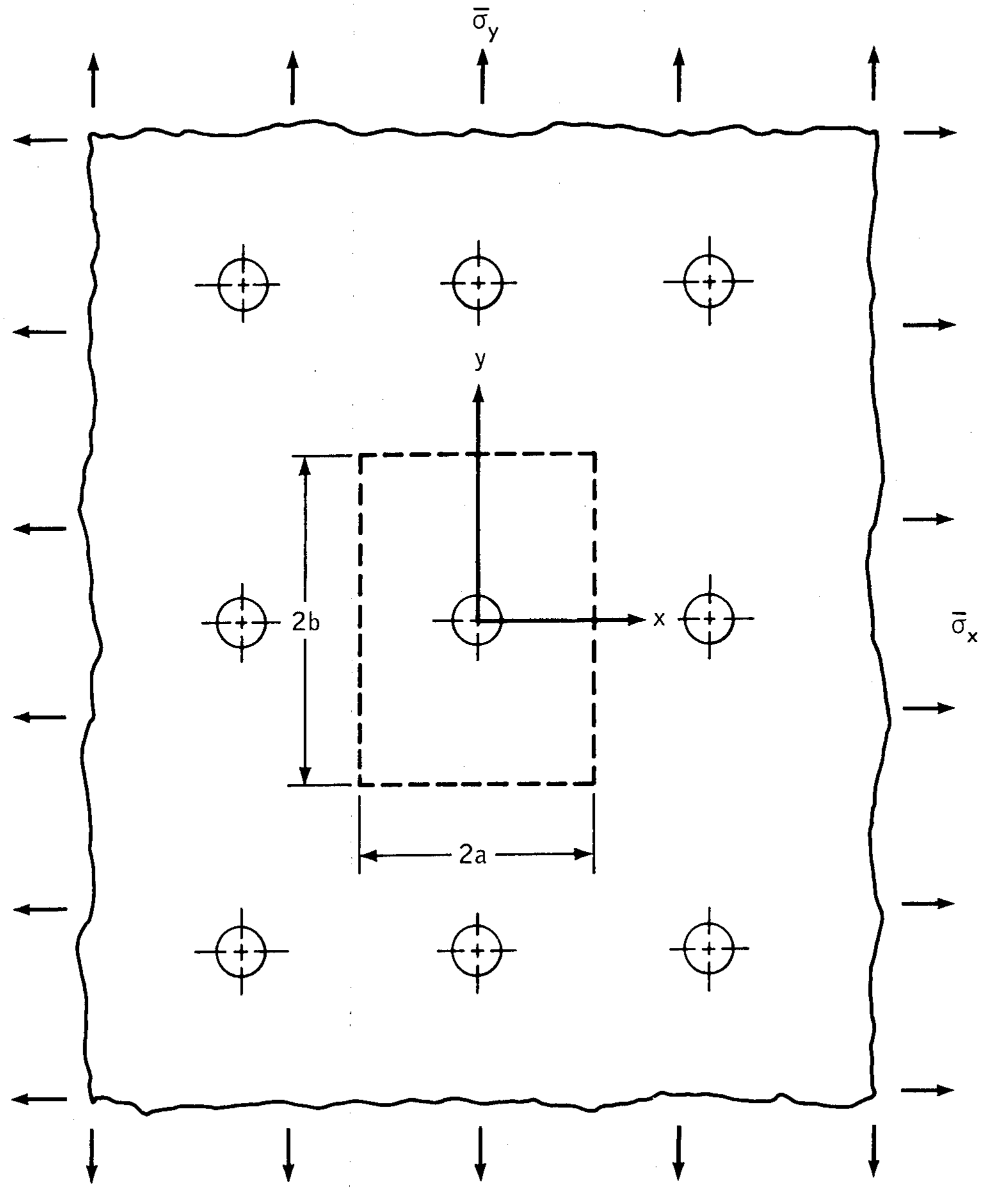


Figure 29. Composite Containing a Rectangular Array of Filaments Imbedded in an Elastic Matrix and Subjected to Uniform Transverse Normal Stress Components at Infinity

shape, i.e., circular, elliptical, diamond, square, rectangular, hexagonal, etc.

Having established the assumptions of rectangular packing and symmetric filaments, the problem can be formulated exactly (within the usual assumptions of the theory of linear plane elasticity). As in the longitudinal shear problem, this is perhaps the key point of the method of analysis.

The concepts of two-dimensional plane elasticity can be applied to the problem of transverse loading, since no variations of stress will occur in the direction of the unidirectional filaments. Either a condition of plane stress or plane strain can be assumed.

Because of the assumed symmetry, a fundamental or repeating unit, as indicated by the dashed lines of Figure 29, can be isolated and analyzed, being typical of the entire composite. When the composite is subjected to transverse normal loads applied at a distance from the element being analyzed, as indicated by $\bar{\sigma}_x$ and $\bar{\sigma}_y$ in Figure 29, a complex state of stress is induced in the composite. This is the result of the dissimilar material properties of the filaments and matrix and also because of interactions between the filament being analyzed and adjacent filaments. The stress distribution along the sides of the fundamental region will not be uniform, although the average of the normal stresses along the sides must equal the average applied stresses, $\bar{\sigma}_x$ and $\bar{\sigma}_y$, from equilibrium considerations.

However, because of symmetry, the originally rectangular fundamental region remains a rectangle when transverse normal loads are applied, i.e., the normal component of displacement of each point on a boundary of the fundamental region is identical. Thus, the problem can be formulated in terms of displacements, interactions between adjacent filaments, which induce the nonuniform stresses at the boundaries of the fundamental region, being automatically and correctly taken into account.

Method of Analysis

The composite material is assumed to consist of a rectangular array of unidirectionally oriented elastic inclusions, e.g., reinforcing filaments, in an infinite elastic matrix, as shown in Figure 29. The inclusions are assumed to be perfectly bonded to the matrix and spaced a distance of 2a apart in the x direction and 2b apart in the y direction. By assuming a regular packing arrangement, a fundamental or repeating unit can be isolated, as indicated by the dashed lines in Figure 29. Because of the necessity of establishing certain symmetry conditions in the solution, the inclusions will be assumed to have two axes of symmetry, these axes being oriented parallel to the x and y axes of the fundamental unit. Within this restriction, the inclusions can be of arbitary shape.

The body is assumed to be loaded at infinity by uniform normal stresses $\bar{\sigma}_x$ and $\bar{\sigma}_y$ in the x and y coordinate directions, respectively, as shown in Figure 29. These stresses may each be of arbitrary magnitude in tension or compression. The influence of thermal stresses induced by a uniform temperature change T in the composite material, e.g., residual stresses induced during cooling from the composite curing temperature, has also been included.

Because of the double periodicity of the inclusion geometry and inclusion spacing, only one quandrant of the fundamental region need be considered, as indicated in Figure 30.

The problem can be treated as one of plane elasticity, either a condition of plane stress or plane strain being assumed, as appropriate.

It is normally desired to solve the problem for a specified loading configuration, i.e., for given values of $\bar{\sigma}_x$ and $\bar{\sigma}_y$, rather than for specified boundary displacements. However, it is simpler to formulate the problem in terms of displacements and subsequently evaluate stresses.

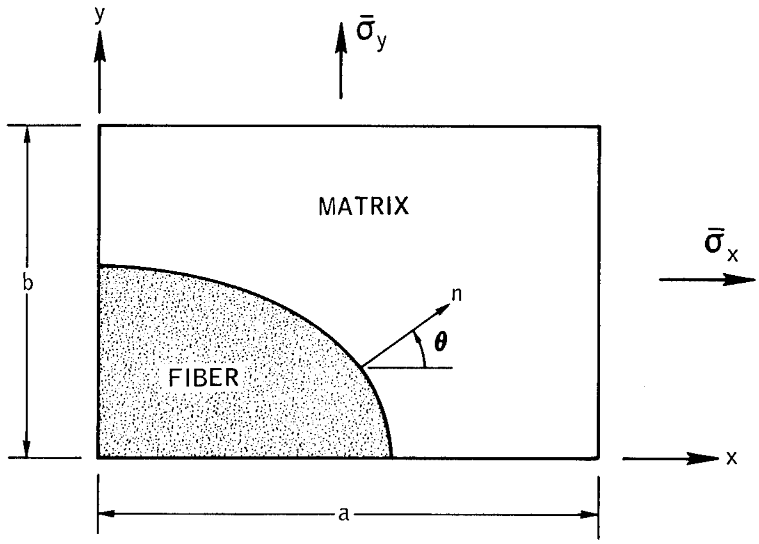


Figure 30. First Quadrant of the Fundamental Region

In terms of displacements u and v in the x and y cordinate directions, respectively, the equilibrium equations to be satisfied are:

x - direction

$$G\left[(A+1)\frac{\partial^2 u}{\partial x^2} + A\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 v}{\partial x \partial y}\right] = 0 \qquad (66)$$

y - direction

$$G\left[\frac{\partial^2 u}{\partial x \partial y} + A\frac{\partial^2 v}{\partial x^2} + (A+1)\frac{\partial^2 v}{\partial y^2}\right] = 0 \qquad (67)$$

where

$$A = \begin{cases} \dfrac{1-\nu}{1+\nu} & \text{plane stress} \\ 1-2\nu & \text{plane strain} \end{cases}$$

$$G = \text{Shear Modulus} = \frac{E}{2(1+\nu)}$$

$$E = \text{Modulus of Elasticity}$$

$$\nu = \text{Poisson's ratio}$$

The stress-displacement equations are of the form:

$$\begin{aligned} \sigma_x &= B\left(\frac{\partial u}{\partial x} + C\,\frac{\partial v}{\partial y}\right) - F \\ \sigma_y &= B\left(C\,\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) - F \\ \sigma_z &= D\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) - H \\ \tau_{xy} &= G\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) \end{aligned} \qquad (68)$$

where

	PLANE STRESS	PLANE STRAIN
B	$\frac{E}{(1+\nu)(1-\nu)}$	$\frac{(1-\nu)E}{(1+\nu)(1-2\nu)}$
C	ν	$\frac{\nu}{1-\nu}$
D	0	$\frac{\nu E}{(1+\nu)(1-2\nu)}$

	PLANE STRESS	PLANE STRAIN
F	$\frac{\alpha E T}{1 - \nu}$	$\frac{\alpha E T}{1 - 2\nu}$
H	0	$\frac{\alpha E T}{1 - 2\nu}$

Because of the assumed symmetry about each of the coordinate axes, the original rectangular unit of Figure 30 will remain rectangular when subjected to transverse loads, i.e., no shear stresses exist along the rectangular boundaries of the element. This shear stress condition, along with the specification of a uniform normal displacement of each side of the rectangular unit, is adequate to define the required boundary conditions.

In addition to the prescribed boundary conditions, stress and displacement continuity conditions must be satisfied at the inclusion-matrix interface. Defining n as the direction normal to the interface at any point and θ as the direction of the normal as measured from the positive x-axis (see Figure 30), the continuity conditions are:

$$
\begin{aligned}
u_f &= u_m \\
v_f &= v_m \\
\sigma_{n_f} &= \sigma_{n_m} \\
\tau_{n\theta_f} &= \tau_{n\theta_m}
\end{aligned}
\tag{69}
$$

where the subscripts f and m represent filament and matrix, respectively, σ_n the normal stress at the interface, and $\tau_{n\theta}$ the shear stress tangent to the interface.

Although displacement boundary conditions are utilized in the solution, it is normally desired to specify average normal stresses to be acting in a

practical application. Thus, the problem must be solved in three steps and these steps suitably combined to provide the desired solution. The first step consists of assuming $T = 0$, i.e., zero temperature change, and solving the boundary value problem defined by the following boundary conditions (see Figure 30):

$$
\begin{aligned}
\tau_{xy} &= 0 \quad \text{along all four rectangular boundaries} \\
u &= 0 \quad \text{along } x = 0 \text{ (points remain on the coordinate axis because of symmetry)} \\
u &= 1 \quad \text{along } x = a \text{ (arbitrarily specified unit displacement)} \\
v &= 0 \quad \text{along } y = 0 \text{ (points remain on the coordinate axis because of symmetry)} \\
v &= 0 \quad \text{along } y = b \text{ (specified displacement condition)}
\end{aligned}
\tag{70}
$$

These conditions, along with the interface continuity equations (Equation 69), are sufficient to define the problem. A finite difference numerical relaxation technique has been developed to solve this problem and is presented in detail in Appendix C.

The second step in the complete solution is to solve another boundary value problem identical with the first except specifying

$$
\begin{aligned}
u &= 0 \quad \text{along } x = a \\
v &= 1 \quad \text{along } y = b
\end{aligned}
\tag{71}
$$

Again, a solution is obtained, using the relaxation technique developed.

The third step consists of imposing the desired temperature change T, specifying all the boundary displacements of Equation (70) to be zero, and obtaining a relaxation solution.

These three separate solutions are then suitably combined to obtain a complete solution for the desired combination of imposed transverse loads and temperature change. The method of combining solutions is shown schematically in Figure 31.

In the process of combining solutions, the effective elastic modulus and effective coefficient of thermal expansion of the composite material, in each of the two coordinate directions, are also calculated. These steps are also indicated in Figure 31.

The complete solution for a specified filament geometry, filament packing arrangement, temperature change, and loading condition thus provides the following information:

(1) Both u and v displacements at all node points throughout the matrix and filament, including those on the interface.
(2) All normal and shear stress components in the coordinate directions at each node point.
(3) The magnitudes and directions of the principal stresses at each node point.
(4) An evaluation of the von Mises yield criteria at each node point.
(5) The effective elastic modulus of the composite in each coordinate direction.
(6) The effective coefficient of thermal expansion of the composite in each coordinate direction.

The details of the numerical solution established, using a finite difference relaxation technique, are given in Appendix C along with a complete description of the digital computer program developed.

Discussion of Results

A typical problem solution is presented in Appendix C, showing the form in which results are obtained. As can be seen, a complete stress distribution is available, as well as the evaluation of a yield criterion. Since

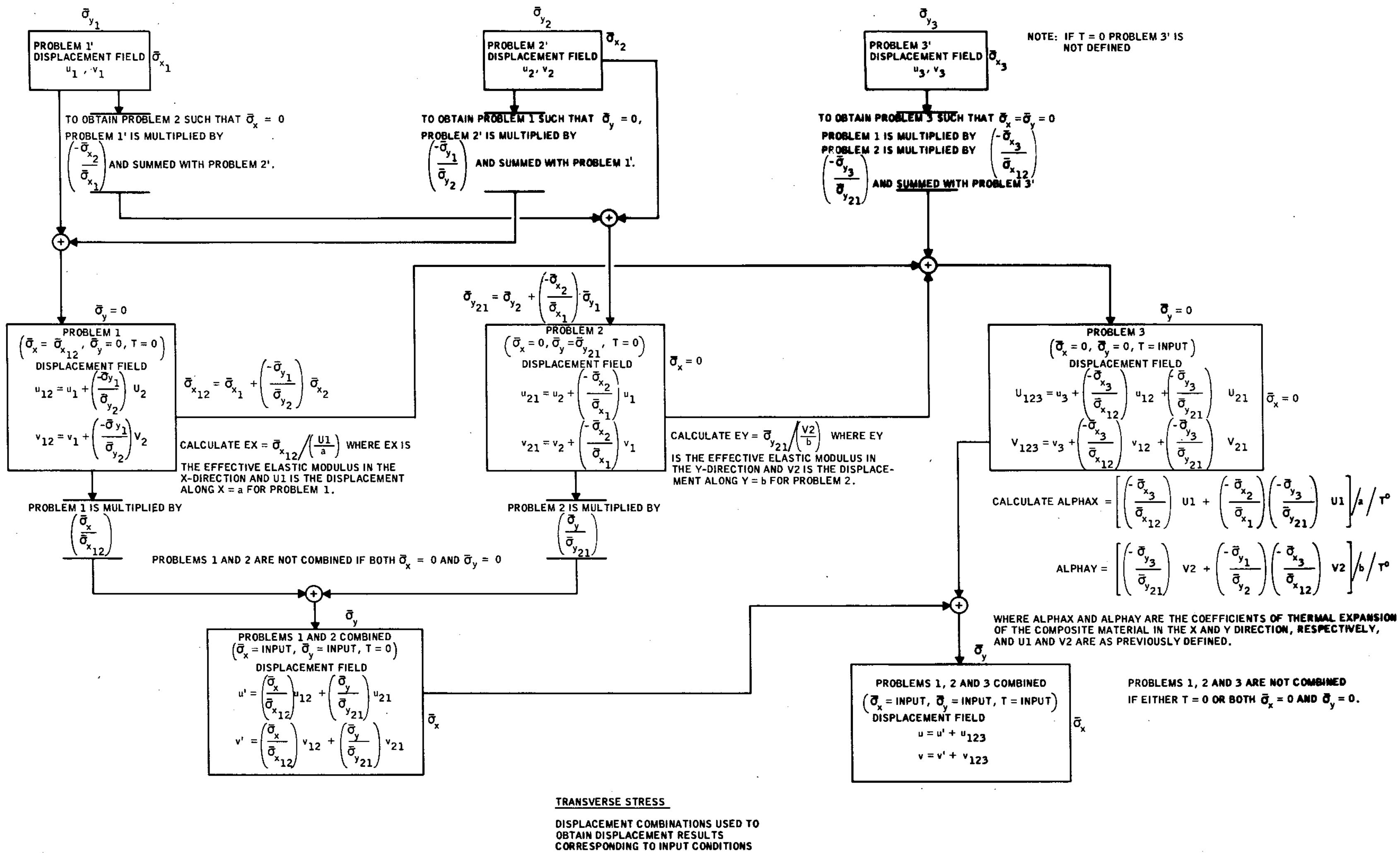


Figure 31. Method of Combining Problems 1, 2, and 3 to Obtain Desired Solution

the primary purpose of the present investigation has been to develop a method of solution rather than to make detailed parametric studies, only a selected number of composite configurations have been numerically evaluated to date. Now that a solution is available, it will be possible to make detailed parametric studies of material behavior.

Two plots of typical behavior are presented, however, to show the utility of the method of solution. Figure 32 is a plot of the transverse reinforcement obtained as a function of the stiffness ratio (E_f/E_m) of the constituent materials for various filament volume ratios (ν_f). Circular filaments in a square array have been assumed. Stiffness ratios for three typical composite systems are specifically indicated. As can be seen, the composite transverse stiffness (E_{22}) is increased significantly as the filament volume percent increases. As the composite filament packing becomes more dense, i. e., as the filaments are moved closer together, interactions between adjacent filaments become important, the present analysis taking these interactions into account. The contribution of filament stiffness (E_f) can be seen by comparing reinforcing factors at various filament volume percents for the two familiar epoxy composite systems indicated, i. e., glass-epoxy and boron-epoxy. Particularly for the higher filament packing densities, use of the higher modulus boron results in a considerably higher composite transverse modulus.

To show the contribution of the matrix stiffness, E_m, to composite transverse stiffness, E_{22}, more directly, another plot is given in Figure 33. Again circular filaments in a square array have been used and a filament modulus of 60 x 10^6 psi (typical, for example, of boron) has been assumed. As expected, the composite transverse stiffness, E_{22}, increases as either the matrix stiffness, E_m, or the fiber volume, v_f, is increased.

A detailed study of the influence of filament geometry and non-square packing arrangements, an interpretation of the yield criterion as it relates local stress states to the composite strength, and the establishment of optimum configurations for specific applications will all be fruitful areas of additional investigation, using the analysis developed.

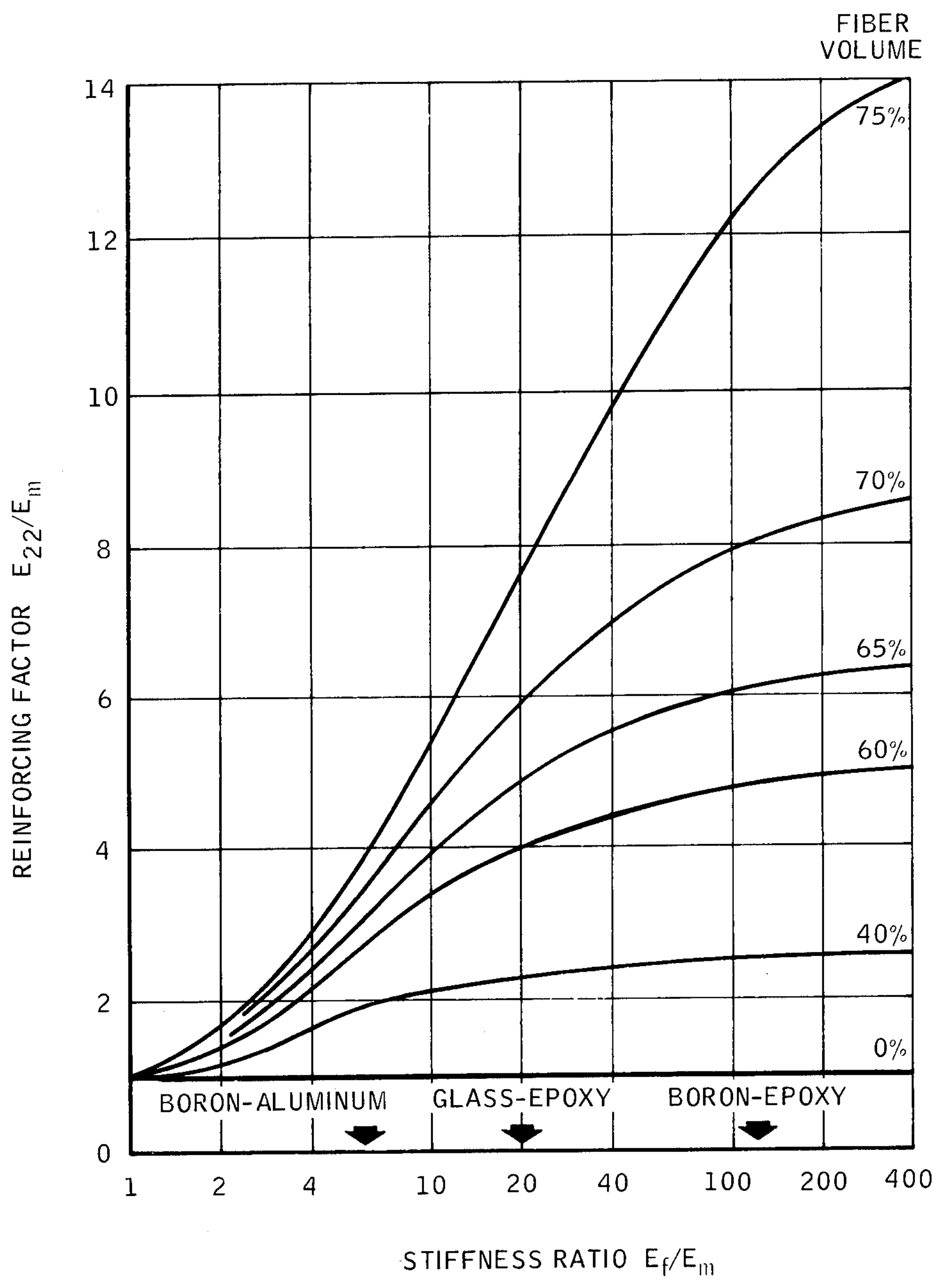


Figure 32. Composite Transverse Stiffness for Circular Fibers in a Square Array

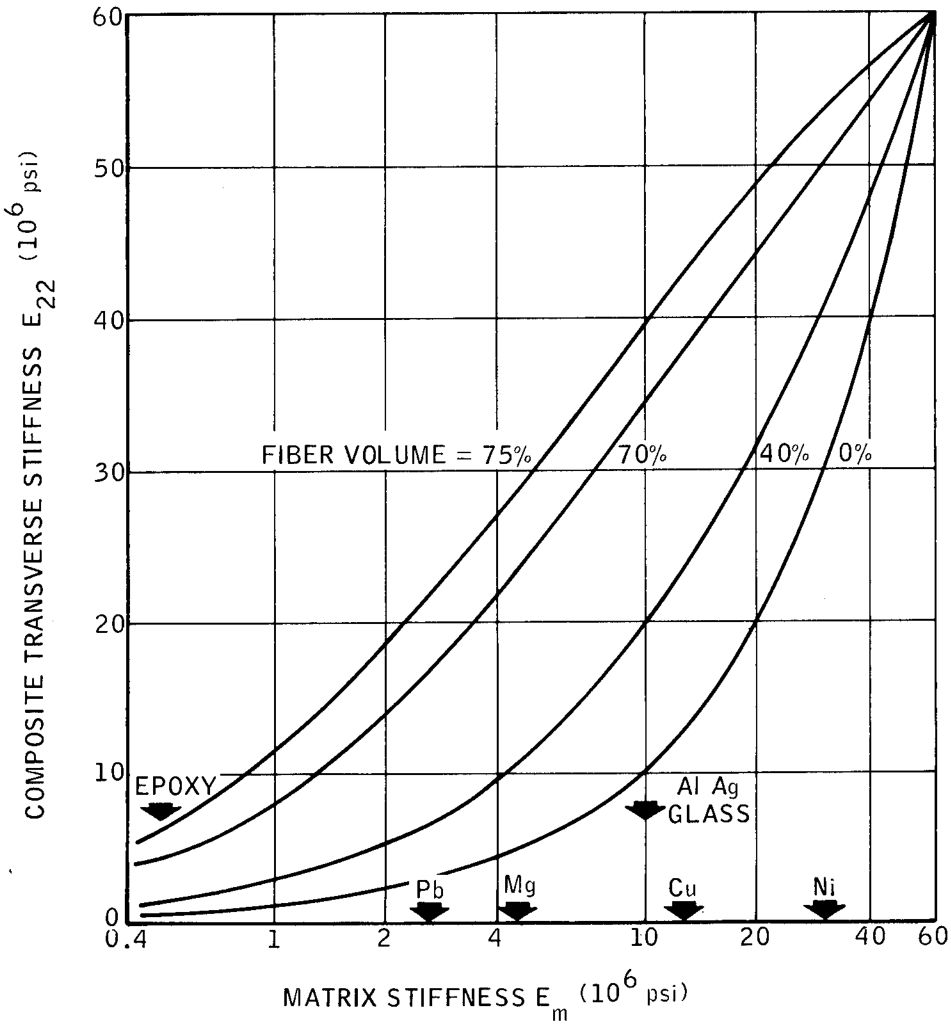


Figure 33. Composite Transverse Stiffness for Boron Fibers as a Function of Matrix Shear Modulus and Fiber Volume

SECTION 5

CONCLUSIONS

In this report, a theoretical basis for the determination of the deformation and load-carrying capacity of laminated and helical-wound composites subjected to complex loadings has been outlined. With the aid of the strength analysis program outlined in Appendix A, parametric studies of the contribution of the intrinsic properties to the structural behavior of filamentary structures can be conducted. The relative importance of each of the mechanical properties, such as elastic moduli and principal strengths, can be quantitatively determined. This information can be used in the selection and design of composite materials for the purpose of achieving an optimum design for a given structural application.

Based on information available thus far, it appears that the elastic deformation of both unidirectional and laminated composites can be predicted with reasonable accuracy, i.e., within 20 percent. In the case of load-carrying capacity, both cross-ply and angle-ply composites, subjected to uniaxial or multiaxial loading, are also predictable within the same level of accuracy as that of the elastic deformation. The ultimate load-carrying capacity of helical-wound tubes requires further investigation. In this report, an attempt has been made to assess the effect of filament crossovers on the load-carrying capacity of helical-wound tubes. A strength criterion based on the ability of the crossovers to transfer the externally applied load to a load parallel to the fibers provides a reasonable prediction of the load-carrying capacity. This is achieved by assuming some increase in the effective transverse and shear strengths and a reduction in the axial

strength. These adjustments to the principal strengths are taken to be independent of the helical-angle and other lamination parameters.

Insofar as guidelines for materials design are concerned, several specific points will be outlined in this section. The implications of the present discussion may have an influence on the thinking associated with determining desired properties of the constituent materials, as well as establishing geometric shapes and arrangements leading to optimum composite materials design.

Stiffness Ratios

The ratio of the stiffnesses of the fiber and matrix constituents, E_f/E_m, has a direct bearing on the composite material behavior. The numerical value of this ratio is approximately 20 for glass-epoxy and 120 for boron-epoxy. In the case of a uniaxial loading along the fibers of a unidirectional composite, this stiffness ratio signifies the relative stress ratios between the fibers and the matrix. A higher ratio implies that a higher proportion of the externally applied load is being carried by the fibers. Based on the rule-of-mixtures relation, a linear relationship between the stiffnesses of the constituent materials and the axial stiffness E_{11} exists. The stiffness ratio of the constituents, however, does not make a linear contribution to the transverse stiffness E_{22} and shear modulus G, as in the case of axial stiffness. In the numerical results presented in Sections 3 and 4, the contribution of the stiffness ratio to the composite elastic moduli levels off after a certain value. As the stiffness ratio exceeds a value of approximately 100, a further increase does not significantly affect the composite elastic moduli. In fact, the composite moduli will remain finite even when the stiffness ratio approaches infinity, which represents the case of rigid fibers.

Since the elastic moduli of a unidirectional composite involve four independent parameters, the stiffnesses of unidirectional and laminated composites can be controlled by varying one or all of these moduli. Which particular modulus parameter will produce the greatest change can be

determined using the information contained in this report. For example, an increase in the fiber stiffness, say in changing from glass to boron, will have the greatest effect on E_{11}. In this particular example, the axial stiffness increases from 8×10^6 to 40×10^6 psi. The boron filaments, however, do not induce a significant increase in the transverse stiffness or shear modulus. The increases in these moduli are nominal, e.g., E_{22} increases from 2.6×10^6 to 4.0×10^6 psi and G increases from 1.2×10^6 to 1.6×10^6 psi. Thus, the increase caused by the substitution of boron for glass filaments is significant only in the case of E_{11}.

However, a higher matrix stiffness will induce a much greater increase. For example, as shown in Figures 28 and 33, a boron-nickel composite may have a shear modulus of 16×10^6 psi and a transverse stiffness of 40×10^6 even at a comparatively low fiber volume of 40 percent. This is significantly higher than for the boron-epoxy system.

In conclusion, the ratio of the stiffnesses of the constituent materials will have differing influences on the gross elastic moduli. There is no "rule-of-thumb" that can be established at this time to determine the most effective way of achieving higher stiffness in a laminated composite. This has to be determined for each individual case, and other considerations such as strength, fiber volume and fiber cross-sectional shape must all be taken into account.

The effect of the stiffness ratio E_f/E_m on the principal strength will now be investigated. The axial strength of a unidirectional composite is dictated by the fiber strength, which can be expressed in terms of the average and the standard deviation of the fiber strength, $\bar{\sigma}$ and s, respectively, the fiber volume v_f, and a factor β, which is a measure of the matrix effectiveness in "shear transfer."[8] The relation is:

$$X = \beta v_f \sigma_B \tag{72}$$

where σ_B is defined as the bundle strength and can be computed from $\bar{\sigma}$ and s. The stiffness ratio E_f/E_m has no effect on the fiber volume and the bundle strength. The matrix effectiveness β measures the gross effect of the interface strength and the stress concentration around a broken fiber. The stiffness ratio will have a definite effect on the stress concentration and a possible effect on the interface strength. As shown in Reference 8, β can vary between 1 and 2 for the case of perfect interfacial bond. If the bond strength is zero, β will remain equal to 1 regardless of the stiffness ratio. Thus, qualitatively, β approaches 1 as the stiffness ratio approaches infinity.

The effect of E_f/E_m on the transverse and shear strengths, Y and S, may be correlated with the stress concentration around fibers. The higher the stiffness ratio, the higher the stress concentration factor. From this viewpoint, a lower stiffness ratio may yield higher values of Y and S.

Fiber Volume

Composites can be classified into two broad categories with respect to fiber volume v_f.

(1) Dense Composites. Composites containing a fiber volume of 50 percent or higher will be classified as dense composites. Significant interactions among the fibers are present. Most glass-epoxy and boron-epoxy composites now in use are in this category.

(2) Dilute Composites. Composite containing a fiber volume of less than 50 percent will be classified as dilute composites. The mechanical interaction among the fibers is relatively small. The behavior of a dilute composite on the microscopic scale may be represented by the solution of the problem of a single inclusion in an infinite matrix domain. This type of composite is normally associated with those utilizing metal matrices.

It is commonly believed that a higher loading of the fibers, that is, a higher fiber volume, will necessarily lead to higher performance of the composite. Based on the present work, this "rule-of-thumb" is by no means conclusive. Again, one should analyze the influence of the fiber volume on the various mechanical properties on the macroscopic scale. These properties include the gross elastic moduli and the principal strengths.

Insofar as the axial stiffness E_{11} is concerned, a higher fiber volume will give a higher composite axial stiffness. The axial stiffness is linearly proportional to the fiber volume. As far as the transverse stiffness and shear modulus are concerned, a higher fiber volume will increase these gross elastic moduli but the amount of increase is not linear. The quantitative relations between fiber volume and E_{22} or G can be seen in the diagrams of Sections 3 and 4.

Both the fiber volume and the stiffness ratio discussed previously have a strong influence in the determination of the final gross effective moduli. It is therefore necessary to examine both the fiber volume and the stiffness ratio simultaneously. This again can be achieved by using the diagrams in Sections 3 and 4. In the case of axial stiffness, a simple linear relationship is adequate and the contribution of each constituent material and the fiber volume can be determined directly from the rule-of-mixtures equation.

The influence of fiber volume on the axial strength is not very well understood. The role of the matrix as a mechanism to isolate fiber breaks is not defined other than by the use of an experimentally determined factor β. It may well be true that a dilute composite provides a more effective means of isolating fiber breaks than a dense composite. This will presumably give a higher value of β and, therefore, a higher axial strength than anticipated. The problem becomes one of a trade-off between the amount of matrix required to effectively isolate fiber breaks and utilizing the properties of the fibers in a given composite. Insofar as transverse shear strength is concerned, dilute composites are also more favorable

than dense composites because the interaction among the fibers is reduced. A more favorable stress distribution results in the case of a dilute composite. This may provide higher transverse and shear strengths than a dense composite with equal constituent material properties.

Fiber Cross Section

Noncircular fibers have been investigated in this report. However, further studies will be necessary before definite conclusions can be made. In this report, methods of analyses have been outlined and digital computer programs presented for the determination of the composite elastic moduli and stress distributions around noncircular fibers. A detailed study can be carried out in the future for the evaluation of the relative merits of various fiber shapes.

In Figure 26, the effective shear modulus for various fiber cross sections for unidirectional glass-epoxy composites are shown. The moduli for circular inclusions with fiber volumes of 70 and 40 percent are 1.09×10^6 and 0.45×10^6 psi, respectively. When the fiber cross section is changed to a 2:1 ellipse, the shear moduli for the dense composite ($v_f = 70$) are 1.24×10^6 and 0.87×10^6 psi along the major and minor axes, respectively. The effective modulus of an elliptical inclusion is greater along the major axis and less along the minor axis than for a circular inclusion. As a comparison, the product of the two shear moduli is approximately equal to the square of the shear modulus of a composite containing circular inclusions. In this sense, the increase along the major axis is offset proportionally by a decrease along the minor axis. The same relationship holds for the case of a dilute composite ($v_f = 40$).

Of the shapes studied, the circular fiber has the lowest stress concentration factor for a given fiber volume. If the stress concentration factor can be related to the shear strength of the composite, the circular fiber should give a higher shear strength than the other shapes studied under this program. The behavior of noncircular fibers under the action of transverse loading will presumably follow closely the previous

conclusions. Both the elastic moduli and the stress concentration factor will vary as the fiber shape changes. Quantitative information, however, is not final at this stage.

The cross-sectional shape of the fibers will influence the axial stiffness and strength since the fiber volume and the contribution of the matrix will vary. No mathematical study has yet been made on the effect of the binding matrix as a vehicle to isolate fiber failures. However, as the fiber shape deviates from a circle, the ability of the matrix to heal fiber breaks may decrease because of the stress concentration induced, e.g., at the sharp corners of rectangular fibers or at the small radius of curvature at the end of the major axis in the case of elliptical fibers. The β-factor in Equation (72) will tend to approach unity, which is the lower bound of the axial strength.

Filament Crossovers

Filament crossovers have been treated as an internal agency contributing to the post-yielding, load-carrying capability of helical-wound tubes. The influence of crossovers has been quantitatively shown by increases in the effective transverse and shear strengths, and a decrease in the axial strength. Thus, crossovers perform two functions: (1) they lock the laminated composite together as an integral unit, thereby providing additional load-carrying capacity beyond initial yielding, and (2) they induce stress concentrations, possibly because of the abrasive action among filaments. The net effect of the crossovers is to provide a strength level to helical-wound tubes that usually falls between that corresponding to initial yielding and the strength based on fiber failures. The test results of this program indicated that most helical-wound tubes will fail according to the strength level predicted by the locking capability of the crossovers. This level, for intermediate helical angles, is higher than the initial yielding but is lower than the strength predicted by a netting analysis. The influence of crossovers is apparently insufficient to transfer the external load necessary to cause fiber failures. On the basis that the strongest composites will be those governed by the fiber strength, i.e., fibers fail, the glass-epoxy

helical-wound tubes tested under the present program fell short of the optimum combination. Fiber failure was induced only in the 5 inch ID pressure vessels.

A number of S glass helical-wound tubes were also made and tested in torsion. The axial strength of the S glass is approximately one-third higher than that of the E glass. The increased axial strength of the S glass did not produce any increase in the ultimate shear strength of the tubes subjected to torsion. The test data for the S glass tubes are shown as crosses in Figure 17. From this figure, one can see that the ultimate torque that the tubes carried did not differ much from that of the E-glass tubes. This experimental observation is in agreement with the theoretical prediction of the strength analysis of Appendix A, where a variation of the axial strength of the constituent layer from 50 to 150 ksi did not induce any significant change in the predicted torsional strength.

The optimum strength of a helical-wound tube may be arrived at by selecting the proper axial strength of the unidirectional composite and the crossover strength required to transfer external loads. If the externally applied load on a tube cannot induce fiber failures, it appears unnecessary to use higher strength fibers, since the higher strength cannot be realized because of the lack of an adequate internal agency.

Future Research

Two areas of additional investigation appear to be very important at this time. One area deals with the characterization of filament crossovers. From the theoretical standpoint, this study will reduce the amount of empiricism that is necessary in the present strength analysis. In particular, the distribution and pattern of the crossovers as a function of various process parameters, such as the diameter of the tube and the width of the roving, should be included in addition to the helical angle. These parameters will change the effective strength values which, in the present program, are assumed to be constant.

Another area which is of equal urgency is the investigation of the inelastic behavior of unidirectional and laminated composites. When external loading induces a stress level beyond the initial yielding, time-dependent effects become very significant. Some of the experimental results presented in this report were obtained by assuming time-independent material properties. This idealization should be examined more critically in the future. Assuming that the deformation and strength of structures can be predicted with reasonable accuracy, it will be an interesting investigation to consider optimizing materials for various structural applications. The contribution of the constituent materials to the eventual structure can now be determined, using the stiffness and strength analyses covered in this report. The results of this parametric study will have a definite impact on the objectives of materials scientists. The desired properties of both the fibers and the matrix can be described in terms of general guidelines. These guidelines may replace the present "rules-of-thumb," which basically rely on the limited validity of netting analysis.

Finally, extensive experimental measurements are needed in order to conclusively establish the results presented in this report. Only with sufficient experimental evidence, can designers of filamentary structures proceed with structural analyses and syntheses with confidence.

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APPENDIX A

STRENGTH ANALYSIS OF LAMINATED COMPOSITES

A.1 INTRODUCTION

The Fortran program, Strength Analysis of Laminated Composites, is written in two parts. The first part, identified by MN CM, i.e., Main-Composite Materials, determines the coefficient matrices, and the second part, identified by PARTWO, i.e., Subroutine PARTWO, deals with the yield criteria. This program is written in Fortran IV programming language and has been used on the Philco 2000 digital computer, a 32K system.

MN CM is used in the stress analysis of a plate, cylinder, or pressure vessel to compute,

(1) the composite moduli A, B, D, A*, B*, H*, D*, A', B' and D'.

(2) the thermal forces and moments defined by

$$(N_i^T, M_i^T) = \int_{-h/2}^{h/2} C_{ij}\, \alpha_j\, T\,(1, z)\, dz$$

for a constant temperature T across the laminated composite.

(3) the coefficients for each N_i, M_i, and T in the stress relation

$$\sigma_i^{(k)} = C_{ij}^{(k)} \Big\{ (A'_{jk} + z B'_{jk}) N_k + (B'_{jk} + z D'_{jk}) M_k + \Big[(A'_{jk} + z B'_{jk}) N_k^T + (B'_{jk} + z D'_{jk}) M_k^T - \alpha_j^{(k)} \Big] T \Big\}$$

for a plate, and

$$\sigma_i^{(k)} = C_{ij}^{(k)} \Big\{ A_{jk}^* N_k + \Big[A_{jk}^* N_k^T - \alpha_j^{(k)} \Big] T \Big\}$$

for a cylinder or pressure vessel,

from input values of $C_{ij}^{(k)}$, $\alpha_j^{(k)}$ and h_k ($k = 1, \ldots n$), where n is the total number of layers of the laminated composite. The derivation of these equations is discussed in Section 2.

A.2 DETERMINATION OF COEFFICIENT MATRICES

The first part of the Strength Analysis program, MN CM, is used to determine the coefficient matrices.

It is assumed that each unit layer is homogeneous. Thus, matrices A, B, and D, whose elements are defined as

$$(A_{ij}, B_{ij}, D_{ij}) = \int_{-h/2}^{h/2} C_{ij} (1, z, z^2)\, dz \quad (i, j = 1, 2 \text{ and } 6)$$

are computed from the relations

$$A_{ij} = \sum_{k=1}^{n} C_{ij}^{(k)} \left(h_{k+1} - h_k \right)$$

$$B_{ij} = 1/2 \sum_{k=1}^{n} C_{ij}^{(k)} \left(h_{k+1}^2 - h_k^2 \right) \quad (i, j = 1, 2 \text{ and } 6)$$

$$D_{ij} = 1/3 \sum_{k=1}^{n} C_{ij}^{(k)} \left(h_{k+1}^3 - h_k^3 \right)$$

Matrices A^*, B^*, H^* and D^* are computed from matrices A, B and D as

$$A^* = A^{-1}$$

$$B^* = -A^{-1} B$$

$$H^* = BA^{-1}$$

$$D^* = D - BA^{-1} B$$

Matrices A', B' and D' are computed from matrices A^*, B^*, H^* and D^* as

$$A' = A^* - B^* D^{*-1} H^*$$

$$B' = B^* D^{*-1}$$

$$D' = D^{*-1}$$

The coefficients of the thermal forces are computed from the relations

$$N_i^T = \int_{-h/2}^{h/2} C_{ij}\ \alpha_j\ T dz$$

$$= \left\{ \sum_{k=1}^{n} C_{ij}^{(k)}\ \alpha_j^{(k)} \left(h_{k+1} - h_k\right) \right\} T \qquad \begin{array}{rl} k &= 1..n \\ i, j &= 1, 2 \text{ and } 6 \end{array}$$

and the coefficients of the thermal moments are computed from the relations

$$M_i^T = \int_{-h/2}^{h/2} C_{ij}\ \alpha_j\ T z dz$$

$$= \left\{ 1/2 \sum_{k=1}^{n} C_{ij}^{(k)}\ \alpha_j^{(k)} \left(h_{k+1}^2 - h_k^2\right) \right\} T \qquad \begin{array}{rl} k &= 1..n \\ i, j &= 1, 2 \text{ and } 6 \end{array}$$

For a cylinder or pressure vessel it is assumed that $\kappa = 0$, and thus the stress components for each layer are given as

$$\sigma_i^{(k)} = C_{ij}^{(k)} \left\{ A_{jk}^{*} N_k + \left(A_{jk}^{*} \int C_{k\ell}\ \sigma_\ell\ dz - \alpha_j^{(k)} \right) T \right\}$$

$$= C_{ij}^{(k)} \left\{ A_{jk}^{*} N_k + \left(A_{jk}^{*}\ N_k^T - \alpha_j^{(k)} \right) T \right\} \begin{array}{ll} \text{superscript } k & = 1...m \\ i, j, k & = 1, 2 \text{ and } 6 \end{array}$$

From these relations the coefficients of N_1, N_2, N_6 and T are computed for the stress components of each layer.

For a plate the stress components at the surface of each layer

$$\sigma_i^{(k)} = C_{ij}^{(k)} \Big\{ (A'_{jk} + zB'_{jk})\, N_k + (B'_{jk} + zD'_{jk})\, M_k$$

$$+ \Big[(A'_{jk} + zB'_{jk}) \int C_{k\ell} \quad \alpha_\ell \, dz$$

$$+ (B'_{jk} + zD'_{jk}) \int C_{k\ell} \quad \alpha_\ell \, z dz - \alpha_j^{(k)} \Big] T \Big\}$$

$$= C_{ij}^{(k)} \Big\{ (A'_{jk} + zB'_{jk})\, N_k + (B'_{jk} + zD'_{jk})\, M_k$$

$$+ \Big[(A'_{jk} + zB'_{jk})\, N_k^T + (B'_{jk} + zD'_{jk})\, M_k^T - \alpha_j^{(k)} \Big] T \Big\}$$

where

superscript k = 1...n
and subscripts i, j, k = 1, 2 and 6

From these relations, the coefficients of N_1, N_2, N_6, M_1, M_2, M_6 and T are computed for the stress components at the surface of each layer.

A.2.1 INPUT PARAMETER DEFINITIONS

Parameter	Definition
N	N is the total number of layers
THTA	THTA, defined for angle-ply composites, is the fiber orientation or lamination angle (degrees).

Parameter	Definition
LPP	LPP defines the particular case under consideration. LPP = 1 implies a cylinder or pressure vessel. LPP = 2 implies a plate.
J	J is a format control which defines the heading to be printed. J = 1 implies cross-ply J = 2 implies angle-ply J = 3 implies general laminate
RM	RM is the cross-ply ratio (total thickness of the odd layers divided by that of the even layers)
LKL	LKL is a format control which defines the heading to be printed. LKL = 0 implies all layers intact LKL = 1 imples all layers degraded
MATRIX H	H(K) is the thickness of the kth layer (in.)
C_{11}, C_{12}, C_{22}, C_{61}, C_{62}, C_{66}, ELEMENTS OF MATRIX C	C(I, J, K) is the C_{ij} element (psi) of the anisotropic stiffness matrix C for the kth layer.
MATRIX ALPHA	ALPHA (I, K) is the ith element, i = 1, 2 and 6, (in. /in. /^{o}F) of the thermal expansion matrix for the kth layer.
MATRIX THETA	THETA (K) is the fiber orientation or lamination angle (radians) for the kth layer.

A.2.2 INPUT DATA CARD LISTING

Card No.	Parameter	Data Field	Format
1	N	1-2	I2
	THTA	3-7	F5.2
	LPP, J	8, 9	I1
	RM	10-21	F12.6
	LKL	22	I1
2 to P	H	1-72	F12.6

Note: Card No. $P = 2 + \left[\frac{N-1}{6}\right]$ where N is the total number of layers and [ ] represents the greatest integer function.

Card No.	Parameter	Data Field	Format
P + 1 to Q	C	1-72	E12.6

Note: Card No. $Q = (P + 1) + (N-1)$

Card No.	Parameter	Data Field	Format
Q + 1 to R	ALPHA	1-72	E12.6

Note: Card No. $R = (Q + 1) + \left[\frac{N-1}{2}\right]$

Card No.	Parameter	Data Field	Format
R + 1 to S	THETA	1-72	E12.6

Note: Card No. $S = (R + 1) + \left[\frac{N-1}{6}\right]$

A.2.3 OUTPUT OF PROGRAM

(1) Repeated Input Data.

(2) Coordinates of the layer surfaces (in.)

(3) A, the in-plane stiffness matrix (10^{+6} lb/in.)

A^*, the intermediate in-plane matrix (10^{-6} in./lb)

A', the in-plane compliance matrix (10^{-6} in./lb)

B, the stiffness coupling matrix (10^{+6} lb)

$B^* = -A^*B$, the intermediate coupling matrix (in.)

B', the compliance coupling matrix (10^{-6} 1/lb)

$H^* = BA^*$, the intermediate coupling matrix (in.)

D, the flexural stiffness matrix (10^{+6} lb-in.)

D^*, the intermediate flexural matrix (10^{+6} lb-in)

D' the flexural compliance matrix (10^{-6} 1/lb-in.)

Coefficients of the thermal forces (lb/in./deg F)

Coefficients of the thermal moments (lb/deg F)

(4) For a plate:

The coefficients of N_1, N_2, N_6 (1/in.), M_1, M_2, M_6 (1/in.^{2}) and temperature (lb/in./$^\circ$F) for stress components SIGMA 1, 2 and 6 for each layer surface.

For a cylinder or pressure vessel:

The coefficients of N_1, N_2, N_6 (1/in.) and temperature (lb/in.^{2}/$^\circ$F) for stress components SIGMA 1, 2 and 6 for each layer.

A.2.4 SUPPORTING SUBROUTINES

(1) Subroutine PARTWO:

Description is outlined in Paragraph A.3

(2) Subroutine RW MATS:

This Fortran IV subroutine computes the inverse of a matrix B from the linear matrix equation BX = C where C is the identity matrix and X is the matrix where the inverse is stored.

(3) Aeronutronic Library Subroutine F4MAMU:

This Fortran IV subroutine computes the real matrix product C = AB in floating point single precision arithmetic.

(4) Aeronutronic Library Subroutine F4MSB:

This Fortran IV subroutine computes the difference of real matrices A and B where the matrix difference A-B replaces matrix B.

Note: MN CM can be used without entering Subroutine Partwo. This is effected by the data control card KQR defined in Paragraph A. 3. 1. In this case matrix THETA is not used in the computation; hence, this data card may either be blank or contain any arbitrary numbers formatted E12. 6.

A. 3 YIELD CRITERIA

Subroutine PARTWO determines those values of N_i and/or M_i which satisfy the yield condition defined in Section 2.

For a cylinder or pressure vessel, the stress components, $\sigma_i^{(k)}$, for each layer can be written

$$\sigma_i^{(k)} = L_i^{(k)} N_1 + P_i^{(k)} N_2 + Q_i^{(k)} N_6 + R_i^{(k)} T$$

where the coefficients $L_i^{(k)}$, $P_i^{(k)}$, $Q_i^{(k)}$ and $R_i^{(k)}$ have been computed in MN CM. Subroutine PARTWO considers the cases

1. $N_1 \neq 0, N_2 = N_6 = 0$
2. $2N_1 = N_2, N_6 = 0$
3. $N_6 \neq 0, N_1 = N_2 = 0$

For a plate, the stress components, $\sigma_i^{(k)}$, for each layer surface can be written

$$\sigma_i^{(k)} = I_i^{(k)} N_1 + J_i^{(k)} N_2 + S_i^{(k)} N_6 + U_i^{(k)} M_1 + V_i^{(k)} M_2$$

$$+ W_i^{(k)} M_6 + Z_i^{(k)} T$$

where the coefficients $I_i^{(k)}$, $J_i^{(k)}$, $S_i^{(k)}$, $U_i^{(k)}$, $V_i^{(k)}$, $W_i^{(k)}$ and $Z_i^{(k)}$ have been computed in MN CM.

Subroutine PARTWO considers the cases

$$\left.\begin{array}{l}
1.\ N_1 \neq 0,\ N_2 = N_6 = M_i = 0 \\
2.\ N_2 \neq 0,\ N_1 = N_6 = M_i = 0 \\
3.\ N_6 \neq 0,\ N_1 = N_2 = M_i = 0 \\
4.\ M_1 \neq 0,\ N_i = M_2 = M_6 = 0 \\
5.\ M_2 \neq 0,\ N_i = M_1 = M_6 = 0 \\
6.\ M_6 \neq 0,\ N_i = M_1 = M_2 = 0
\end{array}\right\} \quad i = 1, 2 \text{ and } 6$$

For the above cases, $\sigma_i^{(k)}$ reduces to an expression in 2 variables, one of the variables always being T.

The terms $\sigma_i^{(k)}$, which are defined in the 1-2 plane, where 1 and 2 represent the coordinate axes of the externally applied stress components, are transformed into the x-y plane, x and y being the material symmetry axes, by the relation

$$\begin{bmatrix} \sigma_x^{(k)} \\ \sigma_y^{(k)} \\ \sigma_s^{(k)} \end{bmatrix} = \begin{bmatrix} m^2 & n^2 & 2mn \\ n^2 & m^2 & -2mn \\ -mn & mn & m^2 - n^2 \end{bmatrix} \begin{bmatrix} \sigma_1^{(k)} \\ \sigma_2^{(k)} \\ \sigma_6^{(k)} \end{bmatrix}$$

where $m = \cos\theta$, $n = \sin\theta$ and θ = the fiber orientation or lamination angle (radians) of the kth layer. Thus $\sigma_x^{(k)}$, $\sigma_y^{(k)}$, and $\sigma_s^{(k)}$ are also expressions in 2 variables.

The yield condition for each quadrant in the $\left(\frac{\sigma_x}{X}, \frac{\sigma_y}{Y}\right)$ plane is given as

$$\text{Quadrant 1: } \left(\frac{\sigma_x}{X}\right)^2 - \frac{1}{r_1}\left(\frac{\sigma_x}{X}\right)\left(\frac{\sigma_y}{Y}\right) + \left(\frac{\sigma_y}{Y}\right)^2 + \left(\frac{\sigma_s}{S}\right)^2 = 1$$

$$\text{Quadrant 2: } \left(\frac{\sigma_x}{X'}\right)^2 - \frac{1}{r_2}\left(\frac{\sigma_x}{X'}\right)\left(\frac{\sigma_y}{Y}\right) + \left(\frac{\sigma_y}{Y}\right)^2 + \left(\frac{\sigma_s}{S}\right)^2 = 1$$

$$\text{Quadrant 3: } \left(\frac{\sigma_x}{X'}\right)^2 - \frac{1}{r_3}\left(\frac{\sigma_x}{X'}\right)\left(\frac{\sigma_y}{Y'}\right) + \left(\frac{\sigma_y}{Y'}\right)^2 + \left(\frac{\sigma_s}{S}\right)^2 = 1$$

$$\text{Quadrant 4: } \left(\frac{\sigma_x}{X}\right)^2 - \frac{1}{r_4}\left(\frac{\sigma_x}{X}\right)\left(\frac{\sigma_y}{Y'}\right) + \left(\frac{\sigma_y}{Y'}\right)^2 + \left(\frac{\sigma_s}{S}\right)^2 = 1$$

where $r_1 = \frac{X}{Y}$, $r_2 = \frac{X'}{Y}$, $r_3 = \frac{X'}{Y'}$, $r_4 = \frac{X}{Y'}$ and X, Y, X', Y' and S are defined respectively as XA(K), YA(K), XP(K), YP(K) and S(K). But since $\sigma_x^{(k)}$ and $\sigma_y^{(k)}$ are expressions in 2 variables, their signs cannot be determined, and hence $\sigma_x^{(k)}$, $\sigma_y^{(k)}$ and $\sigma_s^{(k)}$ are substituted into the yield condition for each quadrant, thus obtaining 4 quadratic equations of the form

$$EA_i^{(k)^2} + FA_i^{(k)} + GT^2 - 1 = 0$$

where E, F and G are constants and $A_i^{(k)} = N_1, N_2, N_6, M_1, M_2$ or M_6

For each input value of temperature, the four quadratic equations are solved by the quadratic formula and the solutions are used to compute $\sigma_x^{(k)}$ and $\sigma_y^{(k)}$. From the signs of $\sigma_x^{(k)}$ and $\sigma_y^{(k)}$, it is determined which yield

condition should have been used and the corresponding solutions are assigned to the quadrant associated with this yield condition.

Thus, a solution which represents a computed value of N_1, N_2, N_6 M_1, M_2, or M_6 is valid if the quadrant to which it has been assigned is the same quadrant as that of the yield condition which it satisfies.

A. 3. 1 INPUT PARAMETER DEFINITIONS

Parameter	Definitions
KQR	KQR defines a data control card. KQR = 1 implies return to the main program. KQR = 0 implies that Subroutine PARTWO is to continue reading data. Note: KQR = 1 permits using the main program without entry into Subroutine PARTWO.
LL	LL defines the particular case under consideration. For a Plate: LL = 1 implies $N_1 \neq 0$ LL = 2 implies $N_2 \neq 0$ LL = 3 implies $N_6 \neq 0$ LL = 4 implies $M_1 \neq 0$ LL = 5 implies $M_2 \neq 0$ LL = 6 implies $M_6 \neq 0$ For a Cylinder or Pressure Vessel: LL = 1 implies $N_1 \neq 0$ LL = 2 implies $N_6 \neq 0$ LL = 3 implies $2N_1 = N_2$

Parameter	Definition
JK	JK is a format control that defines which quadratic equations are to be printed. JK = 1 implies cases N_1 or M_1 JK = 2 implies cases N_2 or M_2 JK = 3 implies cases N_6 or M_6 Note: For case $2N_1 = N_2$, choose JK = 2
NM	NM is the number of input values of temperature.
MATRIX T	T(K) is temperature (Degrees F)
MATRIX XA	XA(K) is the axial tensile strength (psi) of the kth layer.
MATRIX YA	YA(K) is the transverse tensile strength (psi) of the kth layer.
MATRIX XP	YP(K) is the axial compressive strength (psi) of the kth layer.
MATRIX YP	YP(K) is the transverse compressive strength (psi) of the kth layer.
MATRIX S	S(K) is the shear strength (psi) of the kth layer.
TITLE	TITLE is an alphanumeric description of the case under consideration.

A.3.2 INPUT DATA CARD LISTING

Card No.	Parameter	Data Field	Format
1	KQR, LL, JK	1-3	I1
	NM	4-5	I2
2 to P	T	1-72	F12.6

Note: Card No. $P = 2 + \left[\frac{NM-1}{6}\right]$ where NM is the number of input values of temperature and [ ] represents the greatest integer function.

Card No.	Parameter	Data Field	Format
P + 1 to Q	XA	1-72	E12.6

Note: Card No. $Q = (P + 1) + \left[\frac{N-1}{6}\right]$

Card No.	Parameter	Data Field	Format
Q + 1 to R	YA	1-72	E12.6

Note: Card No. $R = (Q + 1) + \left[\frac{N-1}{6}\right]$

Card No.	Parameter	Data Field	Format
R + 1 to S	XP	1-72	E12.6

Note: Card No. $S = (R + 1) + \left[\frac{N-1}{6}\right]$

Card No.	Parameter	Data Field	Format
S + 1 to T	YP	1-72	E12.6

Note: Card No. $T = (S + 1) + \left[\frac{N-1}{6}\right]$

Card No.	Parameter	Data Field	Format
T + 1 to U	S	1-72	E12.6

Note: Card No. $U = (T + 1) + \left[\frac{N-1}{6}\right]$

Card No.	Parameter	Data Field	Format
U + 1	TITLE	1-72	12A6

A.3.3 OUTPUT OF PROGRAM

(1) Repeated input data.

(2) For a cylinder or pressure vessel:
For each layer the quadratic equation obtained from the appropriate yield condition for each quadrant in unknowns T and N_i or M_i, i = 1, 2 or 6.

Solutions of each quadratic equation for input values of temperature and the appropriate quadrant to which these solutions belong.

(3) For a plate output as given in (2) for each layer surface.

Note:

(1) A solution is valid if the quadrant to which it belongs agrees with the quadrant of the quadratic equation which it satisfies.

(2) A complex solution is represented by -.77777777 E-77. A complex solution implies that no real values of N_i or M_i will satisfy the yield condition, i.e., the temperature stresses have already resulted in failure of the laminate.

A.3.4 PROGRAM LISTING

At the end of this appendix is a listing of the Fortran statements which make up the program MN CM, its supporting Subroutine RW MATS and Subroutine PARTWO.

A.3.5 SAMPLE PROBLEM

The sample output presented at the end of this appendix is that obtained for a two-layer, angle-ply cylinder, all layers intact, where θ = 15 degrees. Subroutine PARTWO considers the case $N_1 \neq 0$, $N_2 = N_6 = 0$.

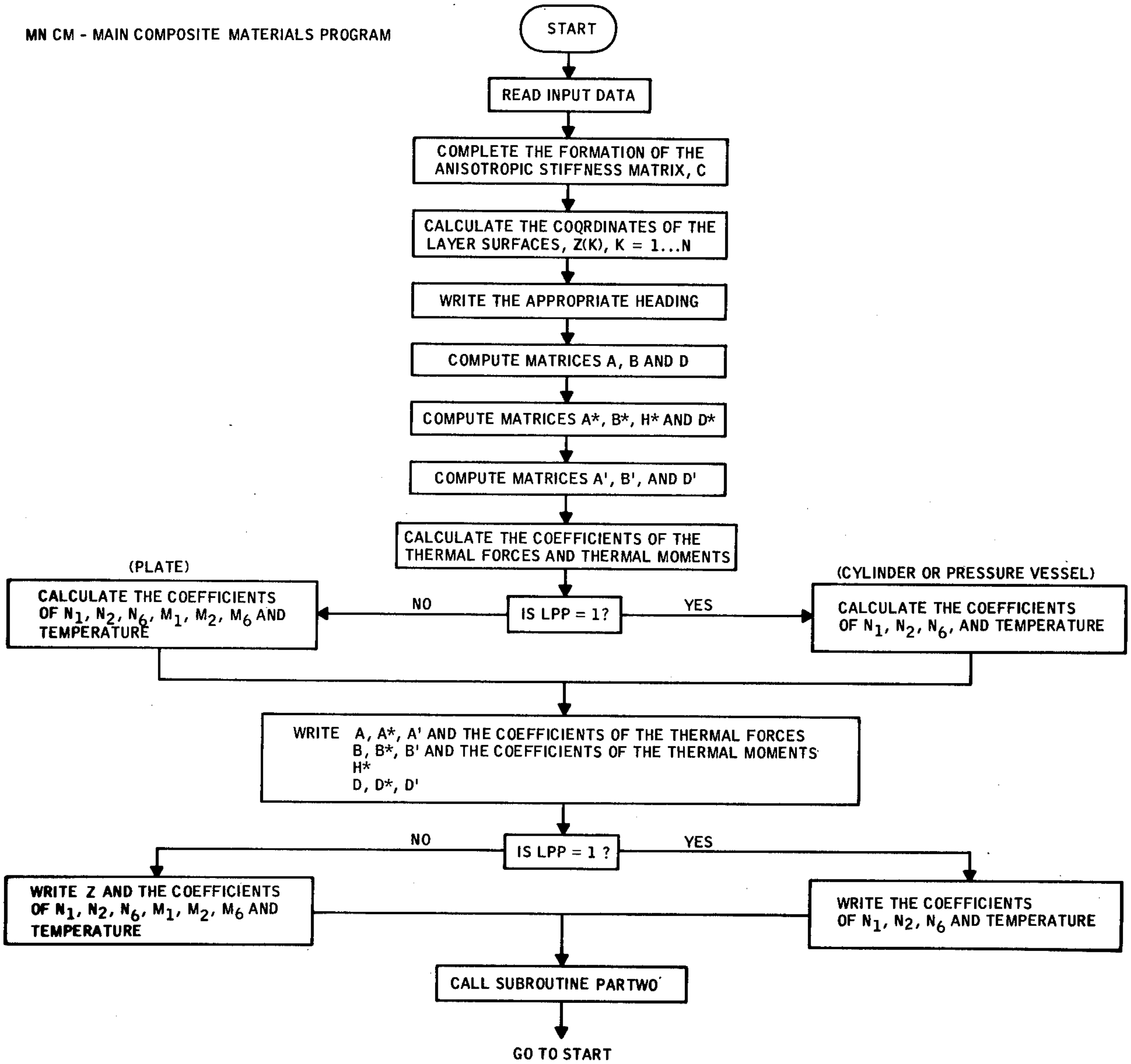
Since the anisotropic stiffness matrix C is symmetric, only six of its coefficients need be printed. Also, since the stress components of a cylinder are not a function of M_i, only the coefficients of N_i and temperature are printed. Typical output format for a flat laminate plate is as shown in a previous report, NASA CR-224. For a cylinder, the coefficients of the stress components are given per layer since, within each layer, the stresses are uniform. For a plate, the coefficients of the stress components are given for each layer surface, as illustrated in NASA CR-224.

Using the method outlined in Paragraph A. 3, those solutions which represent the correct values of N_1 in the sample problem for the given values of temperature are as follows:

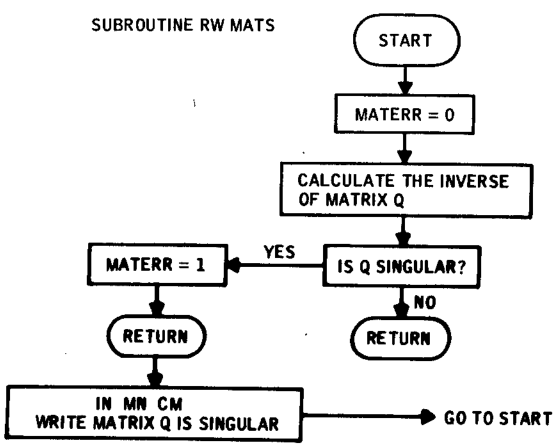
(1) For Compression - solution 2 of the quadratic equation given for Quadrant 2.

(2) For Tension - solution 1 of the quadratic equation given for Quadrant 4.

MN CM - MAIN COMPOSITE MATERIALS PROGRAM

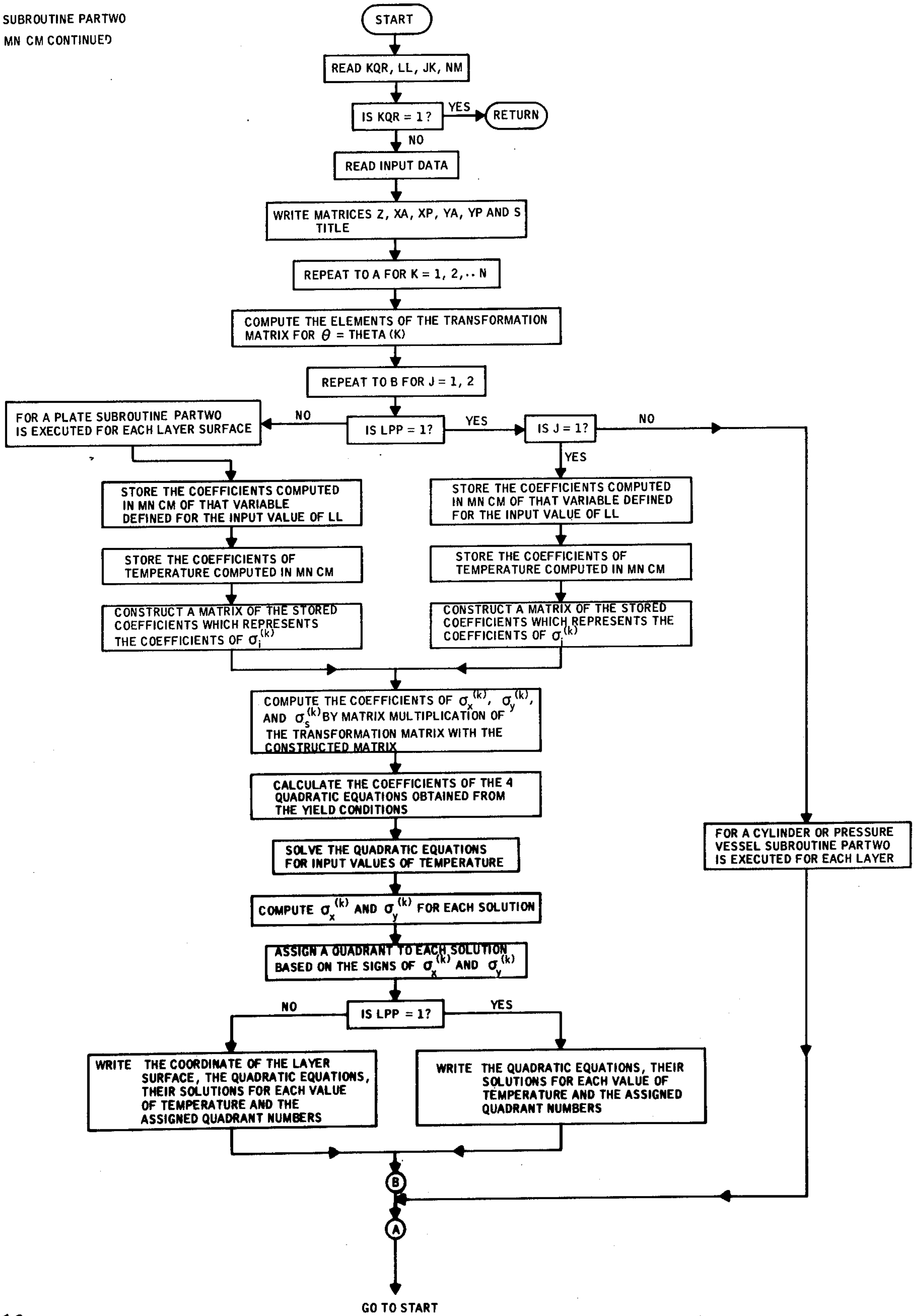


SUBROUTINE RW MATS



SUBROUTINE PARTWO

MN CM CONTINUED



FORTRAN IV COMPUTER LISTING

```
      CMN CM
        COMMON      THETA(50),N,TH(3,3),LPP,LL,PCNO(3,50,2),RB(3,50,2),
       X      PCNT(3,50,2),PCNTR(3,50,2),PCMO(3,50,2),PCMT(3,50,2),
       X      PCMTR(3,50,2),RC(3,50,2),PCT(3,50,2),RS(3,2),RD(3,2),XA(50)
       X      ,S(50),XP(50),YA(50),YP(50),CVS(4),CVP(4),CTS(4),NM,
       X      SOL(4,50,2),T(50),SIGMX(2),SIGMY(2),IQUAD(4,50,2),PRB(3,50),
       X      CNO(3,50),CNTR(3,50),CNT(3,50),PRC(3,50),CT(3,50),TITLE(10)
       X      ,JK,Z(55)
        DIMENSION   ALPHA(3,50),H(50),A(3,3),B(3,3),D(3,3),C(3,3,50),
       X      HS(50),HC(50),AN(3,6),X(3,3),ASTAR(3,3),BSTAR(3,3),
       X      HSTAR(3,3),DSTAR(3,3),DPRI(3,3),BPRI(3,3),APRI(3,3),
       X      SUM(3,50),TSUM(3),TADD(3),RNT(3),RMT(3),SASR(3),
       X      DSUM(3,55),CSUM(3,50,2)
      1 READ (8,2)  N,THTA,LPP,J,RM,LKL
      2 FORMAT (I2,F5.2,2I1,F12.6,I1)
C       N = NO. OF LAYERS
C       MAXIMUM VALUE OF N IS N = 50
C       THTA IMPLIES ANGLE - PLY
C       LPP = 1 IMPLIES PRESSURE VESSEL OR CYLINDER
C       LPP = 2 IMPLIES PLATE
C       J = 1 IMPLIES CROSS-PLY
C       J = 2 IMPLIES ANGLE-PLY
C       J = 3 IMPLIES GENERAL LAMINATE
C       RM = CROSS-PLY RATIO
C       LKL = 0 IMPLIES ALL LAYERS INTACT
C       LKL = 1 IMPLIES ALL LAYERS DEGRADED
        READ(8,6) (H(K), K = 1,N)
      6 FORMAT(6F12.6)
        READ(8,7)  (C(1,1,K),C(1,2,K),C(2,2,K),C(3,1,K),C(3,2,K),C(3,3,K)
       X      ,K=1,N)
      7 FORMAT (6E12.6)
        READ(8,7)  ((ALPHA(I,K),I=1,3),K=1,N)
        READ (8,7) (THETA(K), K=1,N)
        TOTAL = 0.0
        DO 11 K = 1,N
        C(2,1,K) = C(1,2,K)
        C(1,3,K) = C(3,1,K)
        C(2,3,K) = C(3,2,K)
     11 TOTAL = TOTAL + H(K)
        Z(1) = -TOTAL/2.
        MM = N + 1
        DO 12 K = 2,MM
     12 Z(K) = Z(K-1) + H(K-1)
        IF (J .EQ. 2) GO TO 300
        IF (J .EQ. 3) GO TO 600
        WRITE(5,200) RM,N,N
    200 FORMAT(1H1,37X,9HCROSS-PLY,4X,3HM =F5.3,5X,17HALL LAYERS INTACT/
       X      50X,I2,1X,12HLAYERS (N = I2,1H))
        GO TO 215
    600 WRITE(5,625) N,N
    625 FORMAT(1H1,41X,16HGENERAL LAMINATE,4X,17HALL LAYERS INTACT/
       X      51X,I2,1X,12HLAYERS (N = I2,1H))
        GO TO 215
    300 IF (LKL .EQ. 1)  GO TO 212
        WRITE (5,210)  THTA,N,N
    210 FORMAT(1H1,33X,9HANGLE-PLY,4X,8HTHETA = F5.2,1X,7HDEGREES,4X,
       X      17HALL LAYERS INTACT/
```



```
       X      52X,I2,1X,12HLAYERS (N = I2,1H))
        GO TO 215
    212 WRITE (5,214)   THTA,N,N
    214 FORMAT(1H1,33X,9HANGLE-PLY,4X,8HTHETA = F5.2,1X,7HDEGREES,4X,
       X      19HALL LAYERS DEGRADED/
       X      52X,I2,1X,12HLAYERS (N = I2,1H))
    215 WRITE (5,220)
    220 FORMAT(/1H0,1X,5HLAYER,2X,9HTHICKNESS,2X,14HCOORDINATES OF/
       X      3X,3HNO.,3X,9HOF LAYERS,2X,14HLAYER SURFACES,15X,
       X      26HCOEFS. OF STIFFNESS MATRIX,14X,27HCOEFS. OF THERMAL EXPAN
       XSION/
       X      9X,8H(INCHES),6X,8H(INCHES),22X,17H(10+6 LB./IN.SQ.),22X,
       X      21H(10-6 IN./IN./DEG.F.)//
       X      4X,1HK,6X,4HH(K),5X,4HZ(K),4X,6HZ(K+1),3X,6HC(1,1),3X,
       X      6HC(1,2),3X,6HC(2,2),3X,6HC(6,1),3X,6HC(6,2),3X,6HC(6,6),2X,
       X      8HALPHA(1),1X,8HALPHA(2),1X,8HALPHA(6)//)
        WRITE (5,225)   (K,H(K),Z(K),Z(K+1),C(1,1,K),C(1,2,K),C(2,2,K),
       X      C(3,1,K),C(3,2,K),C(3,3,K),ALPHA(1,K),ALPHA(2,K),ALPHA(3,K)
       X      ,K=1,N)
    225 FORMAT(3X,I2,3X,0PF9.4,0PF9.4,0PF9.4,-6PF9.4,-6PF9.4,-6PF9.4,
       X      -6PF9.4,-6PF9.4,-6PF9.4,
       X             6PF9.4,6PF9.4,6PF9.4)
        DO 10 K = 1,N
        HS(K) = Z(K+1)**2 - Z(K)**2
     10 HC(K) = Z(K+1)**3 - Z(K)**3
        DO 20 I = 1,3
        DO 20 J = 1,3
        A(I,J) = 0.0
        B(I,J) = 0.0
        D(I,J) = 0.0
        DO 30 K = 1,N
        A(I,J) = A(I,J) + C(I,J,K) * H(K)
        B(I,J) = B(I,J) + C(I,J,K) * HS(K)
     30 D(I,J) = D(I,J) + C(I,J,K) * HC(K)
        B(I,J) = B(I,J)/2.
        D(I,J) = D(I,J)/3.
     20 CONTINUE
        L = 0
        DO 35 I = 1,3
        DO 35 J = 1,3
     35 AN(I,J) = A(I,J)
     33 DO 38 I = 1,3
        DO 38 J = 4,6
     38 AN(I,J) = 0.0
        DO 39 I = 1,3
     39 AN(I,I+3) = 1.0
        IF (L .EQ. 1) GO TO 34
        CALL MATS (AN,X,3,3,MATERR)
        IF (MATERR) 32,32,31
     31 WRITE (5,3)  ((A(I,J), I = 1,3), J = 1,3)
      3 FORMAT (1H0,20HMATRIX A IS SINGULAR//(3(-6PF8.4)))
        GO TO 1
     32 CALL F4MAMU (3,3,3,X,B,BSTAR)
        DO 40 I = 1,3
        DO 40 J = 1,3
        ASTAR(I,J) = X(I,J)
     40 BSTAR(I,J) = -BSTAR(I,J)
```

```
      CALL F4MAMU (3,3,3,B,X,HSTAR)
      CALL F4MAMU (3,3,3,HSTAR,B,DSTAR)
      CALL F4MSB  (3,3,D,DSTAR)
      DO 45  I = 1,3
      DO 45  J = 1,3
   45 AN(I,J) = DSTAR(I,J)
      L = 1
      GO TO 33
   34 CALL MATS (AN,DPRI,3,3,MATERR)
      IF (MATERR) 36,36,13
   13 WRITE (5,5)  ((DSTAR(I,J), I = 1,3),  J = 1,3)
    5 FORMAT  (1H0,24HMATRIX DSTAR IS SINGULAR//(3(-6PF8.4)))
      GO TO 1
   36 CALL F4MAMU (3,3,3,BSTAR,DPRI,BPRI)
      CALL F4MAMU (3,3,3,BPRI,HSTAR,APRI)
      CALL F4MSB (3,3,ASTAR,APRI)
      DO 50  I = 1,3
      DO 50  K = 1,N
      SUM(I,K) = 0.0
      DO 50  J = 1,3
   50 SUM(I,K) = SUM(I,K) + C(I,J,K)*ALPHA(J,K)
      DO 60 I = 1,3
      TSUM(I) = 0.0
      TADD(I) = 0.0
      DO 55 K = 1,N
      TSUM(I) = TSUM(I) + SUM(I,K)*H(K)
   55 TADD(I) = TADD(I) + SUM(I,K)*HS(K)
      RNT(I) = TSUM(I)
   60 RMT(I) = TADD(I)/2.
      IF (LPP .EQ. 2) GO TO 100
      DO 70  K = 1,N
      DO 70  I = 1,3
      CNO(I,K) = 0.0
      CNT(I,K) = 0.0
      CNTR(I,K)= 0.0
      DO 70 J = 1,3
      CNO(I,K) = CNO(I,K) + C(I,J,K)*ASTAR(J,1)
      CNT(I,K) = CNT(I,K) + C(I,J,K)*ASTAR(J,2)
   70 CNTR(I,K)= CNTR(I,K) + C(I,J,K)*ASTAR(J,3)
      DO 90 I = 1,3
      SASR(I) = 0.0
      DO 90 J = 1,3
   90 SASR(I) = SASR(I) + ASTAR(I,J)*RNT(J)
      DO 115 K = 1,N
      DO 115 I = 1,3
      CT(I,K) = 0.0
      DO 110 J = 1,3
  110 CT(I,K) = CT(I,K) + C(I,J,K)*SASR(J)
  115 CT(I,K) = CT(I,K) - SUM(I,K)
      GO TO 700
  100 DO 75 K = 1,N
      DO 75 I = 1,3
      DO 75 LR = 1,2
      PCNO(I,K,LR) = 0.0
      PCNT(I,K,LR) = 0.0
      PCNTR(I,K,LR) = 0.0
      PCMO(I,K,LR) = 0.0
```



```
      PCMT(I,K,LR) = 0.0
   75 PCMTR(I,K,LR) =0.0
      DO 80 K = 1,N
      DO 80 I = 1,3
      DO 80 J = 1,3
      PCNO(I,K,1) = PCNO(I,K,1)+C(I,J,K)*(APRI(J,1)+Z(K)*BPRI(J,1))
      PCNT(I,K,1) = PCNT(I,K,1)+C(I,J,K)*(APRI(J,2)+Z(K)*BPRI(J,2))
      PCNTR(I,K,1) = PCNTR(I,K,1)+C(I,J,K)*(APRI(J,3)+Z(K)*BPRI(J,3))
      PCNO(I,K,2) = PCNO(I,K,2)+C(I,J,K)*(APRI(J,1)+Z(K+1)*BPRI(J,1))
      PCNT(I,K,2) = PCNT(I,K,2)+C(I,J,K)*(APRI(J,2)+Z(K+1)*BPRI(J,2))
      PCNTR(I,K,2) = PCNTR(I,K,2)+C(I,J,K)*(APRI(J,3)+Z(K+1)*BPRI(J,3))
      PCMO(I,K,1) = PCMO(I,K,1)+C(I,J,K)*(BPRI(J,1)+Z(K)*DPRI(J,1))
      PCMT(I,K,1) = PCMT(I,K,1)+C(I,J,K)*(BPRI(J,2)+Z(K)*DPRI(J,2))
      PCMTR(I,K,1) = PCMTR(I,K,1)+C(I,J,K)*(BPRI(J,3)+Z(K)*DPRI(J,3))
      PCMO(I,K,2) = PCMO(I,K,2)+C(I,J,K)*(BPRI(J,1)+Z(K+1)*DPRI(J,1))
      PCMT(I,K,2) = PCMT(I,K,2)+C(I,J,K)*(BPRI(J,2)+Z(K+1)*DPRI(J,2))
   80 PCMTR(I,K,2) = PCMTR(I,K,2)+C(I,J,K)*(BPRI(J,3)+Z(K+1)*DPRI(J,3))
      MM = N + 1
      DO 120 K = 1,MM
      DO 120 I = 1,3
      DSUM(I,K) = 0.0
      DO 120 J = 1,3
  120 DSUM(I,K) = DSUM(I,K) + (APRI(I,J) + Z(K)*BPRI(I,J))*RNT(J) +
     X (BPRI(I,J) + Z(K)*DPRI(I,J))*RMT(J)
      DO 140  K = 1,N
      DO 140  I = 1,3
      CSUM(I,K,1) = 0.0
      CSUM(I,K,2) = 0.0
      DO 130  J = 1,3
      CSUM(I,K,1) = CSUM(I,K,1) + C(I,J,K)*DSUM(J,K)
  130 CSUM(I,K,2) = CSUM(I,K,2) + C(I,J,K)*DSUM(J,K+1)
      PCT(I,K,1) = CSUM(I,K,1) - SUM(I,K)
  140 PCT(I,K,2) = CSUM(I,K,2) - SUM(I,K)
  700 WRITE(5,230)
  230 FORMAT(///1H0,15X,1HA,31X,2HA*,27X,7HA PRIME,12X,22HCOEF. OF THERM
     XAL FORCE/
     X      10X,14H(10+6 LB./IN.),18X,14H(10-6 IN./LB.),18X,
     X      14H(10-6 IN./LB.),11X,16H(LB./IN./DEG.F.)//)
      WRITE(5,235)   (A(I,1),A(I,2),A(I,3),ASTAR(I,1),ASTAR(I,2),
     X      ASTAR(I,3),APRI(I,1),APRI(I,2),APRI(I,3),I,RNT(I) ,I=1,3)
  235 FORMAT(1X,-6PF10.4,-6PF10.4,-6PF10.4,2X,6PF10.4,6PF10.4,
     X      6PF10.4,2X,6PF10.4,6PF10.4,6PF10.4,5X,1HN,
     X      I1,3H-T ,1X,0PF8.4)
      WRITE(5,240)
  240 FORMAT(/1H0,15X,1HB,31X,2HB*,27X,7HB PRIME,12X,23HCOEF. OF THERMAL
     X MOMENT/
     X      12X,10H(10+6 IN.),22X,10H(10+0 IN.),21X,12H(10-6 1/LB.),14X,
     X      12H(LB./DEG.F.)//)
      WRITE(5,245)   (B(I,1),B(I,2),B(I,3),BSTAR(I,1),BSTAR(I,2),
     X      BSTAR(I,3),BPRI(I,1),BPRI(I,2),BPRI(I,3),I,RMT(I) ,I=1,3)
  245 FORMAT(1X,-6PF10.4,-6PF10.4,-6PF10.4,2X,0PF10.4,0PF10.4,
     X      0PF10.4,2X,6PF10.4,6PF10.4,6PF10.4,5X,1HM,
     X      I1,3H-T ,1X,0PF8.4)
      WRITE(5,250)
  250 FORMAT(/1H0,47X,2HH*/44X,10H(10+0 IN.)//)
      WRITE(5,255)   (HSTAR(I,1),HSTAR(I,2),HSTAR(I,3) ,I=1,3)
  255 FORMAT(33X,3F10.4)
```

FORTRAN 4 PROGRAM MN CM

```
      WRITE(5,260)
  260 FORMAT(/1H0,15X,1HD,31X,2HD*,27X,7HD PRIME/
     X       10X,13H(10+6 LB.IN.),19X,13H(10+6 LB.IN.),18X,
     X       15H(10-6 1/LB.IN.)//)
      WRITE(5,265)   (D(I,1),D(I,2),D(I,3),DSTAR(I,1),DSTAR(I,2),
     X       DSTAR(I,3),DPRI(I,1),DPRI(I,2),DPRI(I,3) ,I=1,3)
  265 FORMAT(1X,-6PF10.4,-6PF10.4,-6PF10.4,2X,-6PF10.4,-6PF10.4,
     X       -6PF10.4,2X,6PF10.4,6PF10.4,6PF10.4)
      IF (LPP .EQ. 1) GO TO 400
      WRITE (5,270)
  270 FORMAT(/1H0,6X,1HZ,8X,6HSTRESS,3X,11HCOEF. OF N1,2X,11HCOEF. OF N2
     X       ,2X,11HCOEF. OF N6,2X,11HCOEF. OF M1,2X,11HCOEF. OF M2,2X,
     X       11HCOEF. OF M6,2X,14HCOEF. OF TEMP./
     X       5X,5H(IN.),4X,9HCOMPONENT,4X,7H(1/IN.),6X,7H(1/IN.),6X,
     X       7H(1/IN.),4X,10H(1/IN.SQ.),3X,10H(1/IN.SQ.),3X,
     X       10H(1/IN.SQ.),3X,15H(LB./IN.SQ./F.)//)
      DO 500  K = 1,N
      WRITE(5,275) K
  275 FORMAT(50X,9H-- LAYER ,I2,3H --//)
      WRITE(5,280)  Z(K), (PCNO(I,K,1),PCNT(I,K,1),PCNTR(I,K,1),
     X       PCMO(I,K,1),PCMT(I,K,1),PCMTR(I,K,1),PCT(I,K,1) ,I=1,3),
     X       Z(K+1), (PCNO(I,K,2),PCNT(I,K,2),PCNTR(I,K,2),
     X       PCMO(I,K,2),PCMT(I,K,2),PCMTR(I,K,2),PCT(I,K,2), I=1,3)
  280 FORMAT(3X,F8.4,4X,7HSIGMA 1,4X,F8.4,5F13.4,6X,F8.4/
     X       21X,1H2,4X,F8.4,5F13.4,6X,F8.4/
     X       21X,1H6,4X,F8.4,5F13.4,6X,F8.4/)
  500 CONTINUE
  308 CALL PARTWO
      GO TO 1
  400 WRITE(5,285)
  285 FORMAT(/1H0,30X,6HSTRESS,3X,11HCOEF. OF N1,2X,11HCOEF. OF N2,2X,
     X       11HCOEF. OF N6,2X,14HCOEF. OF TEMP./
     X       29X,9HCOMPONENT,4X,7H(1/IN.),6X,7H(1/IN.),6X,7H(1/IN.),4X,
     X       15H(LB./IN.SQ./F.)//)
      DO 550 K = 1,N
      WRITE (5,290) K
  290 FORMAT(55X,9H-- LAYER ,I2,3H --//)
      WRITE (5,295)   (CNO(I,K),CNT(I,K),CNTR(I,K),CT(I,K) ,I=1,3)
  295 FORMAT(30X,7HSIGMA 1,4X,F8.4,2F13.4,6X,F8.4/
     X       36X,1H2,4X,F8.4,2F13.4,6X,F8.4/
     X       36X,1H6,4X,F8.4,2F13.4,6X,F8.4/)
  550 CONTINUE
      GO TO 308
      END
```

FORTRAN 4 PROGRAM SUB RW MATS

```
CSUB RW MATS
      SUBROUTINE MATS(A,X,N,M,MATERR)
      DIMENSION A(3,6),X(3,3)
      MATERR=0
      MM=N+M                                                            MATS0003
      DO 15 I=2,N                                                       MATS0004
   70 II=I-1                                                            MATS0005
   7  DO 15 J=1,II                                                      MATS0006
    8 IF (A(I,J).EQ.0.0) GO TO 15
    9 IF ((ABS(A(J,J))-ABS(A(I,J))).LT.0.0) GO TO 11
   10 R=A(I,J)/A(J,J)                                                   MATS0009
      GO TO 130                                                         MATS0010
   11 R=A(J,J)/A(I,J)                                                   MATS0011
      DO 12 K=1,MM                                                      MATS0012
      B=A(J,K)                                                          MATS0013
      A(J,K)=A(I,K)                                                     MATS0014
   12 A(I,K)=B                                                          MATS0015
  130 JJ=J+1                                                            MATS0016
   13 DO 14 K=JJ,MM                                                     MATS0017
   14 A(I,K)=A(I,K)-R*A(J,K)                                            MATS0018
   15 CONTINUE                                                          MATS0019
      IF ((ABS(A(N,N))-1.0E-10).GT.0.0) GO TO 17
   16 CONTINUE
  100 FORMAT(26H0                    ELEMENT(I2,1H,I2,1H),
     X       38H VERY SMALL. CASE DELETED BY MATS      )
      WRITE (5,100) N,N
      MATERR=1
      GO TO 500
   17 DO28J=1,M                                                         MATS0022
      KK=N+J                                                            MATS0023
      X(N,J)=A(N,KK)/A(N,N)                                             MATS0024
      DO28I=2,N                                                         MATS0025
      JJ=N-I+1                                                          MATS0026
      B=0.                                                              MATS0027
      II=N-I+2                                                          MATS0028
      DO 25 K=II,N                                                      MATS0029
   25 B=B+A(JJ,K)*X(K,J)                                                MATS0030
      IF ((ABS(A(JJ,JJ))-1.0E-10).LE.0.0) GO TO 16
   28 X(JJ,J)=(A(JJ,KK)-B)/A(JJ,JJ)                                     MATS0032
  500 RETURN
      END
```

```
CPARTWO
      SUBROUTINE PARTWO
      COMMON      THETA(50),N,TM(3,3),LPP,LL,PCNO(3,50,2),RB(3,50,2),
     X      PCNT(3,50,2),PCNTR(3,50,2),PCMO(3,50,2),PCMT(3,50,2),
     X      PCMTR(3,50,2),RC(3,50,2),PCT(3,50,2),RS(3,2),RD(3,2),XA(50)
     X      ,S(50),XP(50),YA(50),YP(50),CVS(4),CVP(4),CTS(4),NM,
     X      SOL(4,50,2),T(50),SIGMX(2),SIGMY(2),IQUAD(4,50,2),PRB(3,50),
     X      CNO(3,50),CNTR(3,50),CNT(3,50),PRC(3,50),CT(3,50),TITLE(10)
     X      ,JK,Z(55)
    1 READ (8,2) KQR,LL,JK,NM
    2 FORMAT (3I1,I2)
C     KQR = 0 IMPLIES SUBROUTINE IS TO CONTINUE READING
C     KQR = 1 IMPLIES RETURN TO THE MAIN PROGRAM
C     LL IMPLIES CASE UNDER CONSIDERATION
C     FOR PLATE
C       LL = 1 IMPLIES  N1 NOT EQUAL TO 0.0
C       LL = 2 IMPLIES  N2 NOT EQUAL TO 0.0
C       LL = 3 IMPLIES  N6 NOT EQUAL TO 0.0
C       LL = 4 IMPLIES M1 NOT EQUAL TO 0.0
C       LL = 5 IMPLIES M2 NOT EQUAL TO 0.0
C       LL = 6 IMPLIES M6 NOT EQUAL TO 0.0
C     FOR CYLINDER
C       LL = 1 IMPLIES N1 NOT EQUAL TO 0.0
C       LL = 2 IMPLIES N6 NOT EQUAL TO 0.0
C       LL = 3 IMPLIES 2N1 = N2
C     JK = 1 IMPLIES CASES N1 OR M1
C     JK = 2 IMPLIES CASES N2 OR M2
C     JK = 6 IMPLIES CASES N6 OR M6
C     NM = NO. OF INPUT VALUES OF TEMPERATURE
C     MAXIMUM VALUE OF NM = 50
      IF (KQR .EQ. 1) GO TO 10
      READ (8,6) (T(K),  K=1,NM)
      READ (8,7) (XA(K), K=1,N)
      READ (8,7) (YA(K), K=1,N)
      READ (8,7) (XP(K), K=1,N)
      READ (8,7) (YP(K), K=1,N)
      READ (8,7) (S(K), K=1,N)
    6 FORMAT (6F12.6)
    7 FORMAT (6E12.6)
      READ (8,4) TITLE
    4 FORMAT(12A6)
  308 WRITE(5,303)
  303 FORMAT(1H1,   1X,1HZ,3X,22HAXIAL TENSILE STRENGTH,2X,26HAXIAL COMP
     XRESSIVE STRENGTH,3X,27HTRANSVERSE TENSILE STRENGTH,2X,31HTRANSVERS
     XE COMPRESSIVE STRENGTH/1X,4H(IN),9X,5H(PSI),22X,5H(PSI),23X,5H(PSI
     X),26X,5H(PSI)//)
      DO 306 K=1,N
      WRITE (5,307) Z(K),XA(K),XP(K),YA(K),YP(K)
  307 FORMAT(F8.4,3X,E13.6,12X,E13.6,16X,E13.6,18X,E13.6)
  306 CONTINUE
      WRITE (5,309)  (S(K),  K = 1,N)
  309 FORMAT (1H0,52X,14HSHEAR STRENGTH/57X,5H(PSI)//(52X,E13.6))
      WRITE (5,703) TITLE
  703 FORMAT(1H1,47X,7HCASE   ,10A6)
      TEMP = -.77777777E-77
      DO 599 K = 1,N
      RM = COS(THETA(K))
```

FORTRAN 4 PROGRAM PARTWO

```
      RN = SIN(THETA(K))
      TM(1,1) = RM*RM
      TM(1,2) = RN*RN
      RPMN = RM*RN
      TM(1,3) = 2.*RPMN
      TM(2,1) = TM(1,2)
      TM(2,2) = TM(1,1)
      TM(2,3) = -TM(1,3)
      TM(3,1) = -RPMN
      TM(3,2) = RPMN
      TM(3,3) = TM(1,1) - TM(1,2)
      IF (K .EQ. 1) GO TO 731
      WRITE (5,733)
  733 FORMAT(1H1)
  731 WRITE(5,710) K
  710 FORMAT(/1H0,52X,9H-- LAYER ,I2,3H --/)
      DO 598 J = 1,2
      IF (LPP .EQ. 1) GO TO 801
      GO TO (601,602,603,604,605,606) , LL
  601 DO 610 I = 1,3
  610 RB(I,K,J) = PCNO(I,K,J)
      GO TO 622
  602 DO 612 I = 1,3
  612 RB(I,K,J) = PCNT(I,K,J)
      GO TO 622
  603 DO 614 I = 1,3
  614 RB(I,K,J) = PCNTR(I,K,J)
      GO TO 622
  604 DO 616 I = 1,3
  616 RB(I,K,J) = PCMO(I,K,J)
      GO TO 622
  605 DO 618 I = 1,3
  618 RB(I,K,J) = PCMT(I,K,J)
      GO TO 622
  606 DO 620 I = 1,3
  620 RB(I,K,J) = PCMTR(I,K,J)
  622 DO 624 I = 1,3
  624 RC(I,K,J) = PCT(I,K,J)
      DO 626 I = 1,3
      RS(I,1) = RB(I,K,J)
  626 RS(I,2) = RC(I,K,J)
      GO TO 627
  801 IF (J .EQ. 2) GO TO 598
      GO TO (802,804,806), LL
  802 DO 810 I = 1,3
  810 PRB(I,K) = CNO(I,K)
      GO TO 817
  804 DO 812 I = 1,3
  812 PRB(I,K) = CNTR(I,K)
      GO TO 817
  806 DO 814 I = 1,3
  814 PRB(I,K) = .5*CNO(I,K) + CNT(I,K)
  817 DO 819 I = 1,3
  819 PRC(I,K) = CT(I,K)
      DO 821 I = 1,3
      RS(I,1) = PRB(I,K)
  821 RS(I,2) = PRC(I,K)
```

```
FORTRAN 4 PROGRAM        PARTWO

      627 CALL F4MAMU(3,3,2,TH,RS,RD)
          S1 = RD(1,1)**2
          S2 = RD(1,1)*RD(2,1)
          S3 = RD(2,1)**2
          S4 = RD(3,1)**2
          S5 = 2.*RD(1,1)*RD(1,2)
          S6 = RD(1,2)*RD(2,1) + RD(1,1)*RD(2,2)
          S7 = 2.*RD(2,1)*RD(2,2)
          S8 = 2.*RD(3,1)*RD(3,2)
          S9 = RD(1,2)**2
          S10 = RD(1,2)*RD(2,2)
          S11 = RD(2,2)**2
          S12 = RD(3,2)**2
          R1 = XA(K)/YA(K)
          R2 = XP(K)/YA(K)
          R3 = XP(K)/YP(K)
          R4 = XA(K)/YP(K)
          SQ = S(K)**2
          YAS = YA(K)**2
          XAS = XA(K)**2
          YPS = YP(K)**2
          XPS = XP(K)**2
          XY = XA(K)*YA(K)
          XPYP = XP(K)*YP(K)
          XYP = XA(K)*YP(K)
          XPY = XP(K)*YA(K)
          CVS(1) = S1/XAS -S2/(R1*XY)+ S3/YAS + S4/SQ
          CVS(2) = S1/XPS -S2/(R2*XPY)+ S3/YAS + S4/SQ
          CVS(3) = S1/XPS -S2/(R3*XPYP)+ S3/YPS + S4/SQ
          CVS(4) = S1/XAS -S2/(R4*XYP)+ S3/YPS + S4/SQ
          CVP(1) = S5/XAS -S6/(R1*XY)+ S7/YAS + S8/SQ
          CVP(2) = S5/XPS -S6/(R2*XPY)+ S7/YAS + S8/SQ
          CVP(3) = S5/XPS -S6/(R3*XPYP)+ S7/YPS + S8/SQ
          CVP(4) = S5/XAS -S6/(R4*XYP) + S7/YPS + S8/SQ
          CTS(1) = S9/XAS -S10/(R1*XY)+ S11/YAS + S12/SQ
          CTS(2) = S9/XPS -S10/(R2*XPY)+ S11/YAS + S12/SQ
          CTS(3) = S9/XPS -S10/(R3*XPYP)+ S11/YPS + S12/SQ
          CTS(4) = S9/XAS -S10/(R4*XYP)+ S11/YPS + S12/SQ
          DO 640 I = 1,4
          DO 640 JL = 1,NM
          DISC =(CVP(I)*T(JL))**2 - 4.*CVS(I)*(CTS(I)*T(JL)**2 - 1.)
      634 IF (DISC .LT. 0.0)  GO TO 636
          SOL(I,JL,1) = (-CVP(I)*T(JL) + SQRT(DISC))/(2.*CVS(I))
          SOL(I,JL,2) = (-CVP(I)*T(JL) - SQRT(DISC))/(2.*CVS(I))
          GO TO 639
      636 SOL(I,JL,1) = TEMP
          SOL(I,JL,2) = TEMP
      639 DO 640  IL = 1,2
          SIGMX(IL) = RD(1,1)*SOL(I,JL,IL) + RD(1,2)*T(JL)
          SIGMY(IL) = RD(2,1)*SOL(I,JL,IL) + RD(2,2)*T(JL)
          IF (SIGMX(IL) .GE. 0.0  .AND.  SIGMY(IL) .GE. 0.0) GO TO 642
          IF (SIGMX(IL) .LT. 0.0  .AND.  SIGMY(IL) .GT. 0.0) GO TO 644
          IF (SIGMX(IL) .LT. 0.0  .AND.  SIGMY(IL) .LT. 0.0) GO TO 646
          IQUAD(I,JL,IL) = 4
          GO  TO  640
      642 IQUAD(I,JL,IL) = 1
          GO TO 640
```

```
FORTRAN 4 PROGRAM        PARTWO

      644 IQUAD(I,JL,IL) = 2
          GO TO 640
      646 IQUAD(I,JL,IL) = 3
      640 CONTINUE
          IF (J .EQ. 2) GO TO 711
          IF (LPP  .EQ. 1)  GO TO 715
          WRITE (5,712) Z(K)
      712 FORMAT(4X,4HZ = ,F8.4)
          GO TO 715
      711 WRITE (5,713) Z(K+1)
      713 FORMAT(1H1,3X,4HZ = ,F8.4)
      715 DO 717 I = 1,4
          IF (LPP .EQ. 1) GO TO 719
          IF (LL .GT. 3) GO TO 721
      719 WRITE (5,720) I,CVS(I),JK,CVP(I),JK,CTS(I)
      720 FORMAT(1H0,54X,9HQUADRANT ,I1//
         X       26X,E13.6,2H*N,I1,4H**2 ,  E13.6,2H*N,I1,3H*T ,  E13.6,
         X       13H*T**2 - 1 = 0//)
          GO TO 723
      721 WRITE (5,725) I,CVS(I),JK,CVP(I),JK,CTS(I)
      725 FORMAT(1H0,54X,9HQUADRANT ,I1//
         X       26X,E13.6,2H*M,I1,4H**2 ,  E13.6,2H*M,I1,3H*T ,  E13.6,
         X       13H*T**2 - 1 = C//)
      723 WRITE(5,727)
      727 FORMAT(9X,11HTEMPERATURE,13X,10HSOLUTION 1,8X,8HQUADRANT,7X,
         X       10HSOLUTION 2,8X,8HQUADRANT/
         X       10X,8H(DEG. F)//)
          DO 718  JL = 1,NM
          WRITE(5,729)T(JL),SOL(I,JL,1),IQUAD(I,JL,1),SOL(I,JL,2),
         X       IQUAD(I,JL,2)
      729 FORMAT(11X,F7.1,13X,E13.6,10X,I1, 9X,E13.6,10X,I1)
      718 CONTINUE
      717 CONTINUE
      598 CONTINUE
      599 CONTINUE
          GO TO 1
       10 RETURN
          END
```

COMPUTER OUTPUT SAMPLE PROBLEM

ANGLE-PLY THETA = 15.00 DEGREES ALL LAYERS INTACT
2 LAYERS (N = 2)

LAYER NO.	THICKNESS OF LAYERS (INCHES)	COORDINATES OF LAYER SURFACES (INCHES)		COEFS. OF STIFFNESS MATRIX (10+6 LB./IN.SQ.)						COEFS. OF THERMAL EXPANSION (10-6 IN./IN./DEG.F.)		
K	H(K)	Z(K)	Z(K+1)	C(1,1)	C(1,2)	C(2,2)	C(6,1)	C(6,2)	C(6,6)	ALPHA(1)	ALPHA(2)	ALPHA(6)
1	0.5000	-0.5000	-0.0000	7.3420	0.9320	2.7430	-1.1290	-0.1993	1.5190	4.0292	10.8700	1.9750
2	0.5000	-0.0000	0.5000	7.3420	0.9320	2.7430	1.1290	0.1993	1.5190	4.0292	10.8700	-1.9750

A (10+6 LB./IN.)			A* (10-6 IN./LB.)			A PRIME (10-6 IN./LB.)			COEF. OF THERMAL FORCE (LB./IN./DEG.F.)	
7.3420	0.9320	0.	0.1423	-0.0484	0.	0.1547	-0.0466	-0.0000	N1-T	37.4835
0.9320	2.7430	0.	-0.0484	0.3810	0.	-0.0466	0.3812	-0.0000	N2-T	33.1780
0.	0.	1.5190	0.	0.	0.6583	-0.0000	-0.0000	0.7205	N3-T	0.

B (10+6 IN.)			B* (10+0 IN.)			B PRIME (10-6 1/LB.)			COEF. OF THERMAL MOMENT (LB./DEG.F.)	
-0.0000	-0.0000	0.2822	0.0000	0.0000	-0.0378	0.0000	-0.0000	-0.3265	M1-T	-0.0000
-0.0000	-0.0000	0.0498	-0.0000	0.0000	-0.0053	-0.0000	0.0000	-0.0461	M2-T	-0.0000
0.2822	0.0498	-0.0000	-0.1858	-0.0328	0.0000	-0.3265	-0.0461	0.0000	M3-T	0.9288

H* (10+0 IN.)		
-0.0000	0.0000	0.1858
-0.0000	-0.0000	0.0328
0.0378	0.0053	-0.0000

D (10+6 LB.IN.)			D* (10+6 LB.IN.)			D PRIME (10-6 1/LB.IN.)		
0.6118	0.0777	-0.0000	0.5594	0.0684	-0.0000	1.8561	-0.5595	0.0000
0.0777	0.2286	-0.0000	0.0684	0.2269	-0.0000	-0.5595	4.5749	0.0000
-0.0000	-0.0000	0.1266	-0.0000	-0.0000	0.1157	0.0000	0.0000	8.6462

STRESS COMPONENT	COEF. OF N1 (1/IN.)	COEF. OF N2 (1/IN.)	COEF. OF N6 (1/IN.)	COEF. OF TEMP. (LB./IN.SQ./F.)
		-- LAYER 1 --		
SIGMA 1	1.0000	-0.0000	-0.7433	0.
2	-0.0000	1.0000	-0.1312	0.
6	-0.1511	-0.0213	1.0000	-2.6548
		-- LAYER 2 --		
SIGMA 1	1.0000	-0.0000	0.7433	0.
2	-0.0000	1.0000	0.1312	0.
6	0.1511	0.0213	1.0000	2.6548

Z (IN)	AXIAL TENSILE STRENGTH (PSI)	AXIAL COMPRESSIVE STRENGTH (PSI)	TRANSVERSE TENSILE STRENGTH (PSI)	TRANSVERSE COMPRESSIVE STRENGTH (PSI)
-0.5000	0.150000+006	0.150000+006	0.120000+005	0.200000+005
-0.0000	0.150000+006	0.150000+006	0.120000+005	0.200000+005

SHEAR STRENGTH (PSI)
0.100000+005
0.100000+005

CASE N1 NOT EQUAL TO 0.0

-- LAYER 1 --

QUADRANT 1

0.188120-009*N1**2 -0.514332-008*N1*T 0.652518-007*T**2 - 1 = 0

TEMPERATURE (DEG. F)	SOLUTION 1	QUADRANT	SOLUTION 2	QUADRANT
-400.0	0.672653+005	4	-0.782016+005	2
-200.0	0.701312+005	4	-0.755994+005	2
-100.0	0.715312+005	4	-0.742653+005	2
0.	0.729092+005	4	-0.729092+005	2
200.0	0.755994+005	4	-0.701312+005	2
400.0	0.782016+005	4	-0.672653+005	2

QUADRANT 2

0.188120-009*N1**2 -0.514332-008*N1*T 0.652518-007*T**2 - 1 = 0

TEMPERATURE (DEG. F)	SOLUTION 1	QUADRANT	SOLUTION 2	QUADRANT
-400.0	0.672653+005	4	-0.782016+005	2
-200.0	0.701312+005	4	-0.755994+005	2
-100.0	0.715312+005	4	-0.742653+005	2
0.	0.729092+005	4	-0.729092+005	2
200.0	0.755994+005	4	-0.701312+005	2
400.0	0.782016+005	4	-0.672653+005	2

QUADRANT 3

0.187796-009*N1**2 -0.524414-008*N1*T 0.574208-007*T**2 - 1 = 0

TEMPERATURE (DEG. F)	SOLUTION 1	QUADRANT	SOLUTION 2	QUADRANT
-400.0	0.672656+005	4	-0.784355+005	2
-200.0	0.701493+005	4	-0.757343+005	2
-100.0	0.715683+005	4	-0.743608+005	2
0.	0.729722+005	4	-0.729722+005	2
200.0	0.757343+005	4	-0.701493+005	2
400.0	0.784355+005	4	-0.672656+005	2

QUADRANT 4

0.187796-009*N1**2 -0.524414-008*N1*T 0.574208-007*T**2 - 1 = 0

TEMPERATURE (DEG. F)	SOLUTION 1	QUADRANT	SOLUTION 2	QUADRANT
-400.0	0.672656+005	4	-0.784355+005	2
-200.0	0.701493+005	4	-0.757343+005	2
-100.0	0.715683+005	4	-0.743608+005	2
0.	0.729722+005	4	-0.729722+005	2
200.0	0.757343+005	4	-0.701493+005	2
400.0	0.784355+005	4	-0.672656+005	2

-- LAYER 2 --

QUADRANT 1

0.188120-009*N1**2 -0.514332-008*N1*T 0.652518-007*T**2 - 1 = 0

TEMPERATURE (DEG. F)	SOLUTION 1	QUADRANT	SOLUTION 2	QUADRANT
-400.0	0.672653+005	4	-0.782016+005	2
-200.0	0.701312+005	4	-0.755994+005	2
-100.0	0.715312+005	4	-0.742653+005	2
0.	0.729092+005	4	-0.729092+005	2
200.0	0.755994+005	4	-0.701312+005	2
400.0	0.782016+005	4	-0.672653+005	2

QUADRANT 2

0.188120-009*N1**2 -0.514332-008*N1*T 0.652518-007*T**2 - 1 = 0

TEMPERATURE (DEG. F)	SOLUTION 1	QUADRANT	SOLUTION 2	QUADRANT
-400.0	0.672653+005	4	-0.782016+005	2
-200.0	0.701312+005	4	-0.755994+005	2
-100.0	0.715312+005	4	-0.742653+005	2
0.	0.729092+005	4	-0.729092+005	2
200.0	0.755994+005	4	-0.701312+005	2
400.0	0.782016+005	4	-0.672653+005	2

QUADRANT 3

0.187796-009*N1**2 -0.524414-008*N1*T 0.574208-007*T**2 - 1 = 0

TEMPERATURE (DEG. F)	SOLUTION 1	QUADRANT	SOLUTION 2	QUADRANT
-400.0	0.672656+005	4	-0.784355+005	2
-200.0	0.701493+005	4	-0.757343+005	2
-100.0	0.715683+005	4	-0.743608+005	2
0.	0.729722+005	4	-0.729722+005	2
200.0	0.757343+005	4	-0.701493+005	2
400.0	0.784355+005	4	-0.672656+005	2

QUADRANT 4

0.187796-009*N1**2 -0.524414-008*N1*T 0.574208-007*T**2 - 1 = 0

TEMPERATURE (DEG. F)	SOLUTION 1	QUADRANT	SOLUTION 2	QUADRANT
-400.0	0.672656+005	4	-0.784355+005	2
-200.0	0.701493+005	4	-0.757343+005	2
-100.0	0.715683+005	4	-0.743608+005	2
0.	0.729722+005	4	-0.729722+005	2
200.0	0.757343+005	4	-0.701493+005	2
400.0	0.784355+005	4	-0.672656+005	2

APPENDIX B

A RELAXATION METHOD OF SOLUTION OF THE LONGITUDINAL SHEAR PROBLEM FOR A DOUBLY PERIODIC RECTANGULAR ARRAY OF ELASTIC INCLUSIONS IN AN INFINITE ELASTIC BODY

B.1 INTRODUCTION

The solution of the problem outlined in Section 3 has been formulated using a finite difference representation and a numerical relaxation procedure designed for high-speed digital computer operation. The finite difference approximations of the partial derivatives contained in Equations (55) and (56) make use of irregular grid spacings in both coordinate directions, as indicated in Figure B-1. This is an important feature of the solution in that it permits the use of close grid spacings in regions where it is desired to determine stresses very accurately, e.g., in areas of high stress concentration where stress gradients are very high, while permitting a coarser spacing in less critical regions. This permits a given degree of accuracy with a minimum amount of numerical computation and computer storage capacity.

The matrix-inclusion interface is located in the grid array in the following manner. If a grid line in the y-direction intersects the matrix-inclusion interface at a given point, then there must be a corresponding grid line in the x-direction which also intersects the interface at the same point, i.e., the intersection point is a grid node lying on the interface.

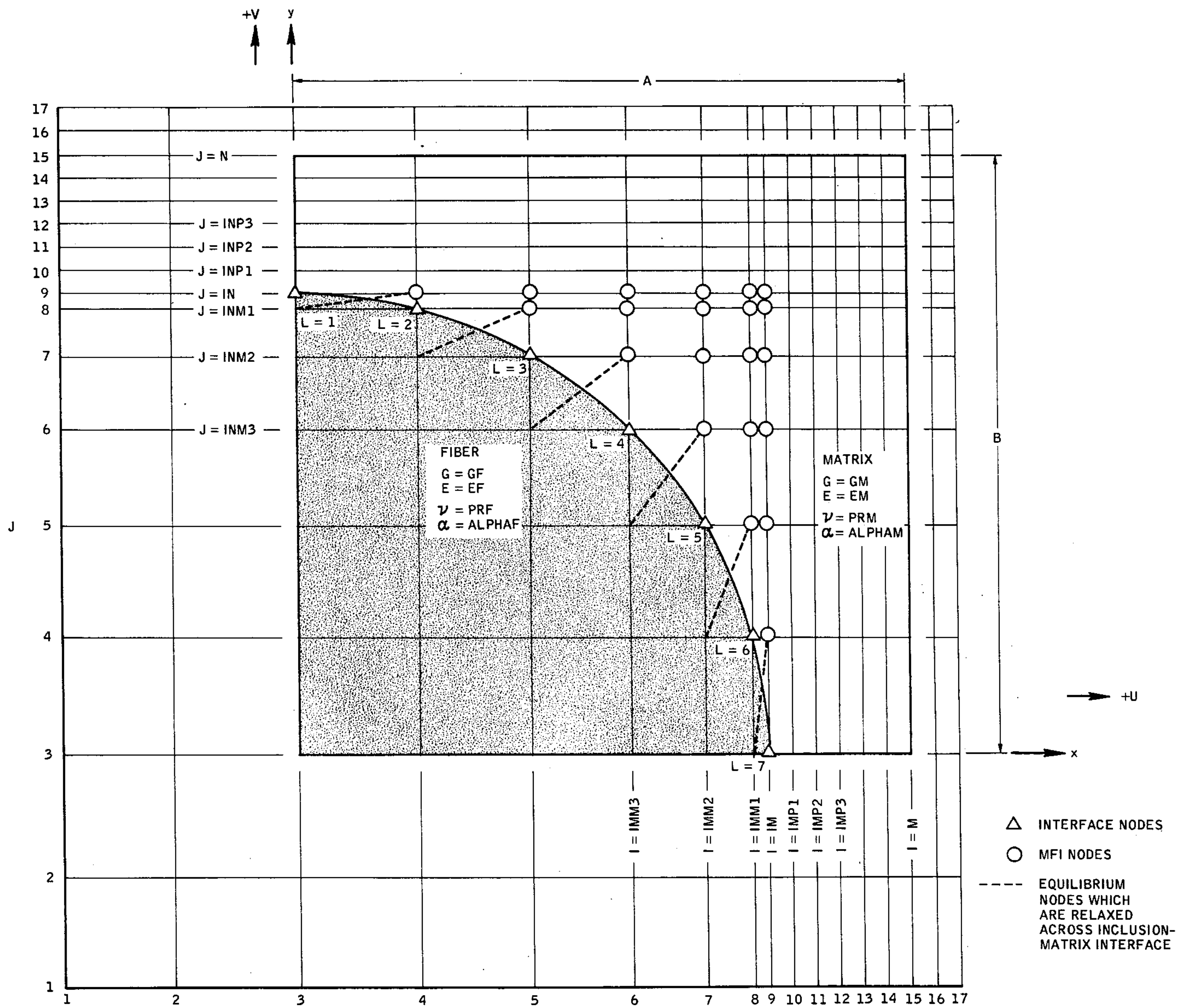


Figure B-1. First Quadrant of the Fundamental Region Showing Typical Grid Lines and Notation Used

B. 2 FINITE DIFFERENCE REPRESENTATIONS

The finite difference representations of the partial derivatives are of the following forms:

(1) First Irregular Central Differences.

$$\left.\frac{\partial w}{\partial x}\right|_{i,j} = \frac{1}{a_1 a_3 (a_1 + a_3)} \left[a_3^2 w_{i+1,j} + (a_1^2 - a_3^2) w_{i,j} - a_1^2 w_{i-1,j}\right]$$

$$\left.\frac{\partial w}{\partial y}\right|_{i,j} = \frac{1}{a_2 a_4 (a_2 + a_4)} \left[a_4^2 w_{i,j+1} + (a_2^2 - a_4^2) w_{i,j} - a_2^2 w_{i,j-1}\right]$$

(2) Second Irregular Central Differences.

$$\left.\frac{\partial^2 w}{\partial x^2}\right|_{i,j} = \frac{2}{a_1 a_3 (a_1 + a_3)} \left[a_3 w_{i+1,j} - (a_1 + a_3) w_{i,j} + a_1 w_{i-1,j}\right]$$

$$\left.\frac{\partial^2 w}{\partial y^2}\right|_{i,j} = \frac{2}{a_2 a_4 (a_2 + a_4)} \left[a_4 w_{i,j+1} - (a_2 + a_4) w_{i,j} + a_2 w_{i,j-1}\right]$$

(3) First Irregular Forward Differences.

$$\left.\frac{\partial w}{\partial x}\right|_{i,j} = \frac{1}{a_1 a_9 (a_9 - a_1)} \left[- (a_9^2 - a_1^2) w_{i,j} + a_9^2 w_{i+i,j} - a_1^2 w_{i+2,j}\right]$$

$$\left.\frac{\partial w}{\partial y}\right|_{i,j} = \frac{1}{a_2 a_{10} (a_{10} - a_2)} \left[- (a_{10}^2 - a_2^2) w_{i,j} + a_{10}^2 w_{i,j+1} - a_2^2 w_{i,j+2}\right]$$

(4) First Irregular Backward Differences.

(Continued on next page)

$$\left.\frac{\partial w}{\partial x}\right|_{i,j} = \frac{1}{a_3\, a_{11}\,(a_{11} - a_3)}\left[(a_{11}^2 - a_3^2)\, w_{i,j} - a_{11}^2\, w_{i-1,j} + a_3^2\, w_{i-2,j}\right]$$

$$\left.\frac{\partial w}{\partial y}\right|_{i,j} = \frac{1}{a_4\, a_{12}\,(a_{12} - a_4)}\left[(a_{12}^2 - a_4^2)\, w_{i,j} - a_{12}^2\, w_{i,j-1} + a_4^2\, w_{i,j-2}\right]$$

The terms a_1 through a_{12} represent distances measured from the point (i, j) at which the difference form is being expressed (point 0 in Figure B-2 to surrounding points (numbered 1 through 12 in Figure B-2). Node points 5 through 8 are not actually used in the longitudinal shear problem, since they are associated with partial derivatives of the form $\partial^2/\partial x \partial y$ which do not appear in the formulation. The subscripts on each displacement term, w, identify the grid coordinates of that displacement in terms of the point (i, j).

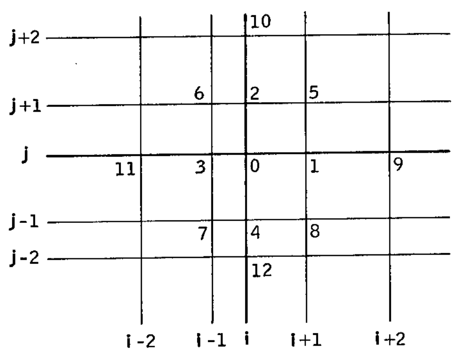


Figure B-2. Node Identification Numbering System

B.3 NUMERICAL PROCEDURE

Central differences are used in representing the equilibrium equation, Equation (56). In representing the boundary condition equations, Equations (58) and (60), and the interface continuity equation, Equation (63), it becomes necessary to use either forward or backward differences in order to remain within the first quadrant of the fundamental region.

The fundamental region is bounded by the grid lines $3 \leq i \leq m$, $3 \leq j \leq n$ (see Figure B-1). The computer storage array is bounded by the grid lines $1 \leq i \leq m + 2$, $1 \leq j \leq n + 2$, the two additional grid lines exterior to each side of the fundamental region being used only for indexing purposes in the program.

The maximum total grid array size has been established as 33 x 33 and the minimum total grid array size must be 9 x 9. Thus, if the total grid array size is (M + 2) x (N + 2), i.e., an array with M + 2 grid lines parallel to the y-axis and N + 2 grid lines parallel to the x-axis, where $9 \leq (M + 2) \leq 33$, $9 \leq (N + 2) \leq 33$, then the usable grid node array size is (M-2) x (N-2) because of the indexing grid lines exterior to the fundamental region.

For a maximum total grid array size of 33 x 33, the usable grid array size is therefore 29 x 29, and for a minimum total grid array size of 9 x 9, the usable grid array size is 5 x 5.

The main control program LONGSHEAR begins by reading the input data from the punched data cards. The program first reads and stores the physical aspects of the problem including grid node array spacing, location of nodes which lie on the inclusion interface, the sine and cosine of the angle which the normal to the interface at each interface node makes with the x axis and the material properties of the inclusion and matrix. Next a code number (MFI) is given to each node which identifies it as being located either in the matrix (MFI=1), in the fiber (MFI=2) or on the interface (MFI=3). Another code (KNT) is assigned to each node indicating the type of equation to be satisfied at that node, i.e. (equilibrium, interface continuity, or boundary) and also the difference representation used for that equation, i.e., forward, central, or backward. There are a total of 17 different node types.

With this information, the program generates the coefficients of the difference representations of the equilibrium, interface, and boundary equations. The coefficients for the interior equilibrium nodes are stored in the two-dimensional (33, 33) arrays E1 through E5. The interface coefficients are stored in the single subscript (70) arrays C1 through C29 and the boundary coefficients are stored in the single subscript (35) arrays D1 through D12.

All of the coefficients for each node equation are stored in the computer core, thus eliminating time consuming recalculation or tape access during the solution process.

The remainder of the main program logic controls the flow between subroutines to affect the desired solution.

B. 4 SUPPORTING SUBROUTINES

B. 4. 1 SUBROUTINE RSDLS

This subroutine calculates a residual at each grid node using the existing displacement field and the difference representation of the appropriate equation at each grid node.

RSDLS will be entered NRD times, calculating a new residual at each grid node, using the displacement field obtained from subroutine RLXLS (or the specified input displacements when RSDLS is entered the first time). The displacements existing at each grid node and its surrounding nodes are put into the appropriate equation for that node and a residual is computed which represents the extent to which the existing displacements do not satisfy the equation. In the first entry to RSDLS at the beginning of the problem, the only displacements existing are the unit displacements along one boundary, all other displacements being set equal to zero. The result is that the equations are trivially satisfied at each grid node except the first row in from the displaced boundary where residuals are calculated. These residuals create residuals at surrounding nodes during the solution process and thus propagate the boundary displacement throughout the array.

B. 4. 2 SUBROUTINE RLXLS

Subroutine RLXLS employs a systematic relaxation procedure (successive overrelaxation) on the residuals in the grid node array to arrive at a set of displacements which are a solution of the difference equations.

This subroutine is the portion of the program which solves the set of equations representing the problem, and as such is the key element in the relaxation technique.

Indexing from node to node begins in the row adjacent to the displaced boundary and progresses toward the interior of the fundamental region. This is done to transmit the boundary displacement most rapidly to the other nodes. At each node, the KNT code is tested to determine the type of equation to be satisfied at that node. The coefficient in the difference equation for the node multiplying the displacement at that node is placed in CAT.

The residual existing at each node represents the extent to which the difference equation is not yet satisfied at that node and this error is arbitrarily assumed to be entirely caused by an error in displacement at that node. A change in displacement can be calculated which will cause the residual at the grid node to be reduced to zero, thus satisfying the equation at that node. Actually, the change in displacement is further increased by multiplying it by a factor OMB, in effect "overrelaxing" the residual. In theory,* the value of OMB can vary from $0 < \text{OMB} < 2$. The case of $\text{OMB} < 1$ is termed underrelaxation and $\text{OMB} > 1$ is overrelaxation.

An optimum value of the relaxation factor OMB has been found to be about 1.75 for the present solution.

After computing the desired displacement change at the node and actually changing the displacement value, the program indexes to the eight surrounding nodes (see Figure B-2). The residual at each of these nodes is changed in proportion to the influence of the changed displacement on the equation at the node point. This amount is the ratio of the coefficient of the changed displacement to the coefficient stored in CAT. This process is

*Young, David, "Iterative Methods for Solving Partial Difference Equations of Elliptic Type," Transactions of the American Mathematical Society, Vol 76, pp 92-111, January-June 1954.

repeated many times throughout the array until the residual at each node is reduced to a value small enough such that subsequent relaxations would no longer induce a significant change in displacement at any grid node.

At the grid nodes interior to the inclusion and lying on the $x = 0$ or $y = 0$ boundaries, (IMM1, 3) and (3, INM1), a forward difference cannot be taken which will always have all three points interior to the inclusion. For this reason, the usual relaxation procedure has been replaced with an interpolation-relaxation scheme at these points. At the end of each relaxation cycle, the displacement at these two points is calculated using a Fortran Function Subroutine AINTPL. This library subroutine uses all of the displacements along the boundary interior to the inclusion and by the method of Lagrangian interpolation, which can accommodate the irregular grid spacing, computes a new value for the displacement. The difference between this new displacement and the previous one is then used to relax the residuals at all affected surrounding grid nodes. Using this method, the final displacement value is the result of interaction with surrounding nodes and not the result of interpolation alone. This library subroutine can be easily replaced with any Lagrangian interpolation scheme desired if AINTPL is not available.

Two exits are possible from Subroutine RLXLS. At the beginning of each relax cycle, the total number of cycles already executed is compared to the input value of NRX. When these are equal, control returns to the main program. At the end of each relaxation cycle, the total number of cycles already executed is compared to the input value of NRXBT, which is the number of relaxation cycles to be executed before testing the stresses at selected test points. When the number of relaxation cycles exceeds NRXBT, the stresses TZX and TZY are calculated at the specified test points and compared with the stresses existing at the end of the previous relaxation cycle. If the sum of the squares of these stresses at all test points has changed by an amount less than a specified percentage, read in as PCGPRX, then control returns to the main program.

Printed output from Subroutine RLXLS consists of an I and J node index, displacement and residual for each node point in the array. Printout occurs for the first (NCPRLX) number of consecutive relaxation cycles following an exit from Subroutine RSDLS and every (NPRLX) multiple cycle thereafter. Printout will also occur for the last relaxation cycle executed when exit from RLXLS is a result of satisfying the condition of minimum change in stress at the test points. At the end of each printout, a record of the number of test points which have not yet satisfied the percentage change in stress condition, since testing began, is given.

B. 4. 3 SUBROUTINE STRLS

Subroutine STRLS is entered after Subroutines RSDLS and RLXLS have been executed the specified number of times. STRLS then calculates the average shear stress existing along the boundary having the specified unit displacement. An effective composite shear modulus is calculated by multiplying the average shear stress by the proper quadrant dimension and dividing this product by the unit displacement. Each displacement in the array is then multiplied by the ratio of the average shear stress desired to the average shear stress developed. This yields the desired displacement field.

Using this displacement field, Subroutine STRLS then calculates the shear stresses τ_{zx} and τ_{zy} and the shear stress resultant $\tau_{zxy} = (\tau_{zx}^2 + \tau_{zy}^2)^{1/2}$ at each node of the grid array. These are printed along with the identifying I and J indices and the displacements.

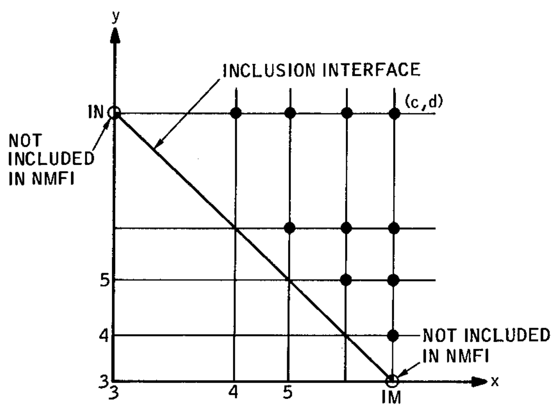
At each interface node, where stresses can be calculated both in the inclusion and in the matrix, a zero is printed. The interface stresses are then printed on a separate page along with the effective composite shear modulus. The inclusion shear stresses, τ_{zx} at L = 1 and τ_{zy} at L = NL, cannot be calculated and are printed as zero.

B.5 INPUT PARAMETER DEFINITION

Parameter	Definition
TITLE	TITLE is an alphanumeric description of the particular problem under consideration (up to 72 characters).
M N	M and N identify the boundaries of the fundamental region (see Figure B-1).
NRX	NRX is the maximum number of times the program will execute Subroutine RLXLS between successive returns to Subroutine RSDLS.
NRD	NRD is the number of times the program will enter Subroutine RSDLS.
IM	IM is the number of the I coordinate grid line at which the inclusion crosses the x-axis, i.e., at grid node (IM, 3). Grid nodes are indexed in the program as (I, J).
IN	IN is the number of the J coordinate grid line at which the inclusion crosses the y-axis, i.e., at grid node (3, IN).
NPRLX	NPRLX is an integer indicating that subroutine RLXLS will be printed at every integral multiple of NPRLX.

Parameter	Definition
NCPRLX	NCPRLX is an integer which indicates the number of consecutive outputs of the results of Subroutine RLXLS to be printed, beginning with the first entry to RLXLS, i.e., the first NCPRLX outputs of Subroutine RLXTS will be printed.
NL	NL is the number of grid nodes lying on the inclusion interface and includes the grid nodes referenced in the definitions of IM and IN (see Figure B-1).
NMFI	Construct a line perpendicular to the y-axis and passing through the grid node referenced in the definition of IN and another line perpendicular to the x-axis and passing through the grid node referenced in the definition of IM. These lines will intersect at some grid node (c, d). NMFI is the number of grid nodes contained in the region exterior to the inclusion and its interface node points, but lying on or within the lines constructed through point (c, d). Note: The grid nodes referenced in the definitions of IM and IN are not included in the above sum.

Parameter	Definition
	Example: NMFI = 10



NKPROB

NKPROB = 1 indicates that Problem 1 only is to be solved.

NKPROB = 2 indicates that Problem 2 only is to be solved.

NKPROB = 3 indicates that both Problems 1 and 2 are to be solved (combined loading).

NTP

NTP is the number of test points ($1 \leq NTP \leq 10$).

Note: Choose as test points only those grid nodes which are interior to the matrix.

NRXBT

NRXBT is the number of times the program will execute the Subroutine RLXLS before testing the selected test points.

Parameter	Definition
KSYM	KSYM = 0 indicates an unsymmetrical inclusion or inclusion spacing. An inclusion is unsymmetrical if, when rotated 90 degrees about its longitudinal axis, the transformed inclusion does not occupy the same space as the original inclusion. KSYM = 1 indicates that both inclusion and spacing are symmetrical.
MATRIX IJTP	Matrix IJTP contains the coordinates of the test points used in testing the percent change of stress per relax. IJTP (2N-1) = I coordinate and IJTP (2N) = J coordinate of the Nth test point.
PCGPRX	PCGPRX is the maximum percent change in stress allowed at any of the test points, per relax, before exiting from Subroutine RLXLS.
MATRIX HX	HX(I) is the absolute value of the distance between grid lines I and I+1.
MATRIX HY	HY(J) is the absolute value of the distance between grid lines J and J+1.
GF	GF is the shear modulus, G_f, of the fiber (lb/in.^{2}).
GM	GM is the shear modulus, G_m, of the matrix (lb/in.^{2}).

Parameter	Definition
OMB	OMB is the relaxation factor to be used. $0 < OMB < 2$, with optimum convergence usually being obtained for OMB near 1.7.
VF	VF is the percent fiber content by volume of the composite. Note: VF is input for printout purposes only and is not used in the calculations.
MATRICES LI, LJ	Associated with each grid node on the interface of the inclusion is an L number. The grid node referenced in the definition of IN has an L number equal to 1, i.e., L = 1. Proceeding clockwise along the interface the next grid node has an L number equal to 2, i.e., L = 2. Continuing as described above implies that the grid node referenced in the definition of IM has an L number equal to NL, i.e., L = NL. Matrices LI and LJ contain the I and J coordinates respectively, of the grid nodes on the interface of the inclusion where LI(N) is the I coordinate and LJ(N) is the J coordinate of that grid node whose L number is equal to N, i.e., L = N.

Parameter	Definition
MATRICES COST, SINT	Matrices COST and SINT contain $\text{Cos}\theta_n$ and $\text{Sin}\theta_n$, respectively, where θ_n is defined as follows:

For an arbitrary grid node (I, J) on the interface of the inclusion whose L number is some value such that $1 < L < NL$, θ_n is defined as the angle between the normal to the inclusion surface at (I, J) and the positive x-axis.

Thus

$$\text{COST (L)} = \text{Cos}\theta_n$$
$$\text{SINT (L)} = \text{Sin}\theta_n$$

For L = 1, i.e., the grid node referenced in the definition of IN, θ_n is defined to be 90 degrees which implies

$$\text{COST (1)} = \text{Cos } 90^\circ = 0.0$$
$$\text{SINT (1)} = \text{Sin } 90^\circ = 1.0$$

For L = NL, i.e., the grid node referenced in the definition of IM, θ_n is defined to be 0 degrees which implies

$$\text{COST (NL)} = \text{Cos } 0^\circ = 1.0$$
$$\text{SINT (NL)} = \text{Sin } 0^\circ = 0.0$$

TZXB	TZXB is the desired average shear stress (lb/in.^{2}) at infinity in the x-direction.
TZYB	TZYB is the desired average shear stress (lb/in.^{2}) at infinity in the y-direction.

Parameter	Definition
MATRICES MFII, MFIJ	Matrices MFII and MFIJ contain the I and J coordinates respectively of those grid nodes referenced in the definition of NMFI. No particular input order is required.

B.6 INPUT DATA CARD LISTING

Card No.	Parameter	Data Field	Format
1	TITLE	1-72	12A6
2	M, N, NRX	1-3, 4-6, 7-9	I3
	NRD, IM, IN	10-12, 13-15, 16-18	I3
	NPRLX, NCPRLX	19-21, 22-24	I3
	NL, NMFI	25-27, 28-30	I3
	NKPROB, NTP	31-33, 34-36	I3
	NRXBT	37-39	I3
	KSYM	40-42	I3
3	IJTP	1-60	I3
4	PCGPRX	1-12	E12.6
5 to L	HX(I) where I = 3...M-1	1-72	E12.6

NOTE: Card No. K $= \left[\frac{M-3}{6}\right] + (L + 1)$ where [ ] represents the greatest integer function. The maximum allowable value of K is L + 5.

Card No.	Parameter	Data Field	Format
L+1 to K	HY(J) where J = 3...N-1	1-72	E12.6

NOTE: Card No. K $= \left[\frac{N-3}{6}\right] + (L + 1)$ where [ ]represents the greatest integer function. The maximum allowable value of K is L + 5.

Card No.	Parameter	Data Field	Format
K+1	GF, GM	1-24	E12.6
	OMB, VF	25-48	E12.6
K+2 to J	LI(L), LJ(L)	1-72	I3
	where L = 1...NL		
J+1 to I	COST(L), SINT(L)	1-72	E12.6
	where L = 1...NL		
I+1	TZXB, TZYB	1-24	E12.6
I+2 to LC	MFII(K), MFIJ(K)	1-72	I3
	where K = 1...NMFI		

B.7 OUTPUT OF PROGRAM

(1) Repeated input data.

(2) Dimensions of first quadrant of the fundamental region, A and B, where:

$$A = \sum_{I=3}^{M-1} HX(I)$$

and

$$B = \sum_{J=3}^{N-1} HY(J)$$

(3) If NKPROB = 1 or 2:

(a) Results of the kth entry into Subroutine RSDLS

(b) Results of Subroutine RLXLS, NCPRLX consecutive times, every integral multiple of NPRLX, and the last execution.

NOTE: (a) and (b) are printed consecutively for each value of k where k = 1. . . NRD. Output includes the I and J coordinate of each node of the grid array and the corresponding displacements and residuals at each grid node.

If NKPROB = 1 and k = 1, the residuals computed in Subroutine RSDLS will be zero everywhere except at those grid nodes in the M-1 column at J = 4. . . N-1.

If NKPROB = 2 and k = 1, the residuals computed in Subroutine RSDLS will be zero everywhere except at those grid nodes in the N-1 row at I = 4. . . M-1.

(c) Results of Subroutine STRLS for the particular problem solved, i.e., Problem 1 or Problem 2.

(4) If NKPROB = 3:

Results of Subroutine STRLS for Problems 1 and 2 combined.

Output will include:

(a) The I and J coordinates of each grid node and its corresponding displacement w.

(b) The shear stress components TZX and TZY and the resultant shear stress TZXY at each interior and boundary node.

(c) The shear stress components and the resultant shear stress at each interface node for both filament and matrix.

(d) GX and GY, which are defined as the effective composite shear moduli in the x and y coordinate directions, respectively.

B.8 SAMPLE PROBLEM

The sample solution presented at the end of this appendix is that of the elliptical inclusion array shown in the upper left of Figure 26.

On the first page of output is printed the title ELLIPTICAL INCLUSION and the other input data. The grid node array size of 15 by 15 is the number of grid lines in the fundamental area. The computer solution uses two grid lines outside this area and so M and N are input as 17. The quadrant dimensions A and B are merely the sum of the distances between grid lines in the x and y directions respectively. The ellipse represented has a major to minor axes ratio of 2:1 and a fiber volume of 70 percent. The input values of matrix and inclusion shear modulus, relaxation factor, imposed loads, and fiber volume are also listed.

Following this are the I and J coordinates of the ten test points at which the change in stress per relaxation cycle is to be calculated. The spacing between each grid line is listed under GRID SPACING. First, the horizontal spacing HX (I) is given. The distance shown for I = 3 is the horizontal distance from grid line 3 to grid line 4. Similarly, HY (J) is the vertical grid spacing.

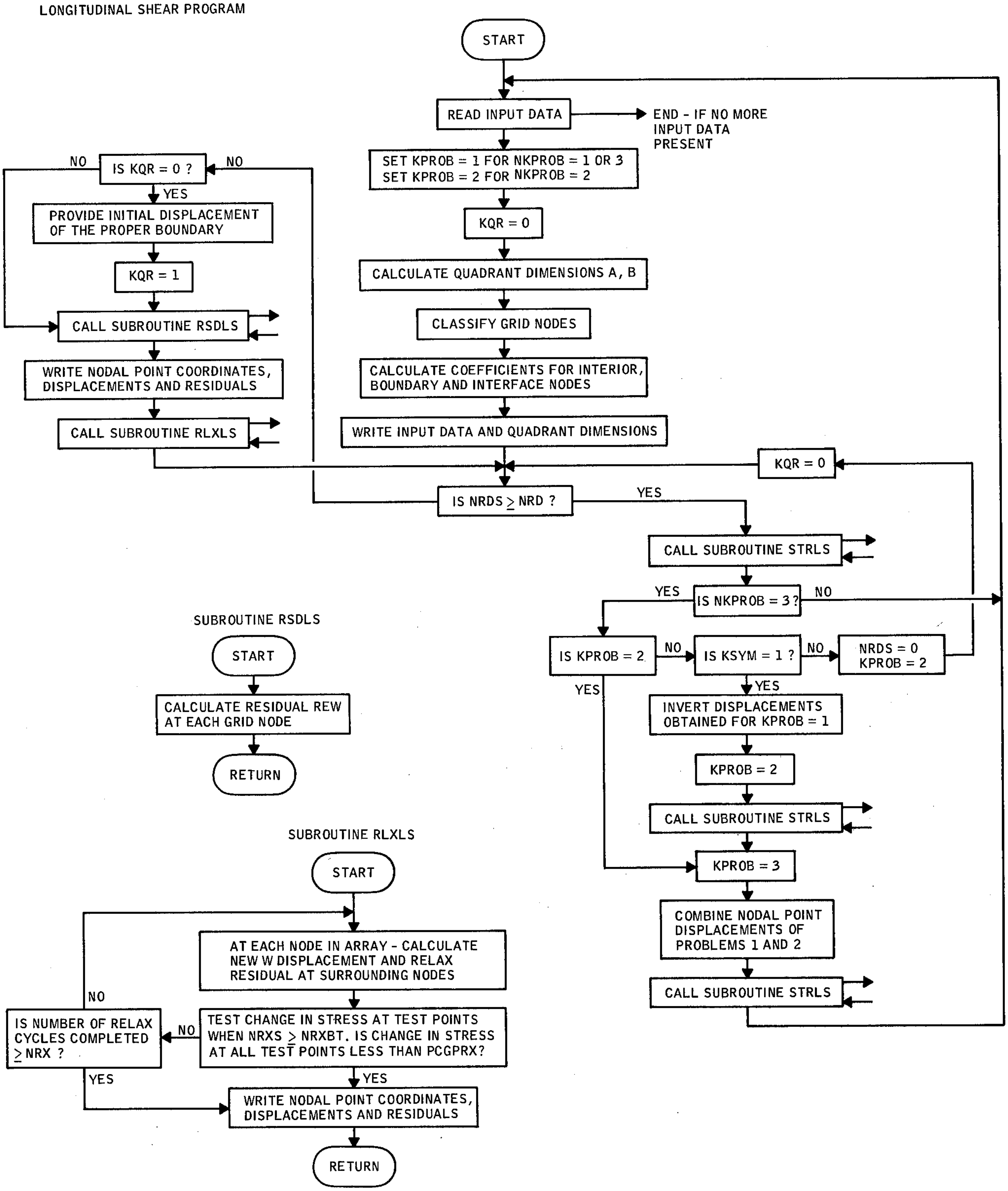
The first entry into Subroutine RSDLS results in zero residuals at all grid nodes except those adjacent to the right boundary which is given a unit displacement. In this row, the residuals are equal to 0.4958×10^{10}. As the effect of these residuals spreads throughout the array during the relaxation process, they become progressively smaller.

The relaxation process was halted after 110 relaxation cycles when all 10 test points recorded a change in stress of less than 0.05 percent per relaxation cycle. At this point, the largest residual in the entire array had an exponent of 10^5. This represents a decrease of 5 orders of magnitude.

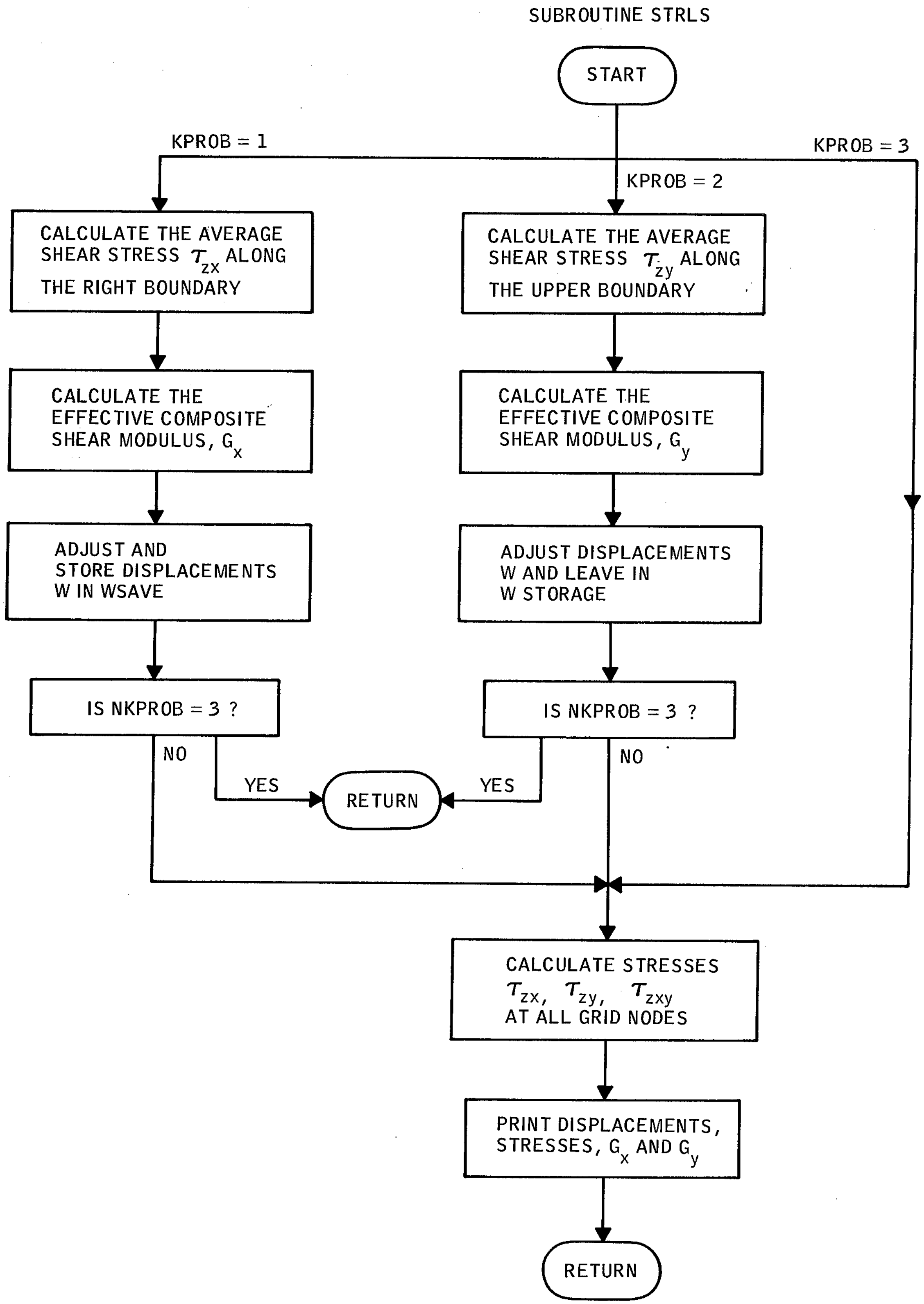
The interior and boundary stresses are printed, followed by the interface stresses. The stress concentration factor (as shown in Figure 26)

is determined by the matrix interface stress at I = 11, J = 3, i.e., 3921.1 psi, divided by the imposed shear stress of 1000 psi, i.e., SCF = 3.921. Next is printed the effective composite shear modulus in the x direction of 0.869×10^6. The shear modulus in the y direction was not calculated since the example problem shown involved an imposed shear stress along the x = a boundary only; Problem 2, i.e., an imposed shear stress along the y = b boundary only, was not solved for in this example.

LONGITUDINAL SHEAR PROGRAM



LONGITUDINAL SHEAR PROGRAM CONTINUED



FORTRAN IV COMPUTER LISTING

FORTRAN 4 PROGRAM   LONGSHEAR

```
      CLONGSHEAR
       COMMON W,WL,WSAVE,W1,W1S,W2,W2S,TZX,TZY,TZXB,TZYB,TZXBS,TZYBS,
      1TZXM,TZYM,TZXF,TZYF,REW,HX,HY,OMB,GF,GM,GX,GY,F1,F2,COST,SINT,
      2CAT,C1,C2,C3,C4,C5,C6,C7,C8,C9,C10,C11,C12,C13,C14,C15,C16,C17,
      3C18,C19,C20,C21,C22,C23,C24,C25,C26,C27,C28,C29,D1,D2,D3,D4,D5,
      4D6,D7,D8,D9,D10,D11,D12,E1,E2,E3,E4,E5,
      5M,MM1,MM2,MM3,MP1,MP2,N,NM1,NM2,NM3,NP1,NP2,IN,INM1,INM2,INM3,
      6INP1,INP2,INP3,IM,IMM1,IMM2,IMM3,IMP1,IMP2,IMP3,NL,NLM1,NLM2,
      7LN,LI,LJ,LAT,KNAT,NMFI,MFIJ,MFI,KNT,KPROB,IJTP,MFII,
      8NRX,NRD,NRXS,NRDS,NPRLX,NCPRLX,NTP,NPT,TZXY1,TZXY2,PCGPRX,TZXY
      9,NRXBT,NKPROB,A,B,FP1
       DIMENSION W(33,33),REW(33,33),TZX(33,33),TZY(33,33),F1(33,33),
      1E2(33,33),E3(33,33),E4(33,33),E5(33,33),KNT(33,33),LN(33,33),
      2C29(33,33),MFI(33,33),WSAVE(33,33),TZXY(33,33),
      3C6(70),C7(70),C8(70),C9(70),C10(70),C11(70),C12(70),C13(70),C14(70
      4),C15(70),C16(70),C17(70),C18(70),C19(70),C20(70),C21(70),C22(70),
      5C23(70),C24(70),C25(70),C26(70),C27(70),C28(70),COST(7 ),SINT(70)
      6,D1(35),D2(35),D3(35),D4(35),D5(35),D6(35),D7(35),D8(35),D9(35),
      7HX(35),HY(35),WL(70),LT(70),LJ(70),C1(70),C2(70),C3(70),C4(70),
      8C5(70),D10(35),D11(35),D12(35),TZXM(70),TZYM(70),TZXF(70),TZYF(70)
      9,IJTP(20),TZXY1(10),TZXY2(10),MFIT(90),MFIJ(90)
       DIMENSION TITLE(12)
      C
      C    A RELAXATION SOLUTION OF THE LONGITUDINAL SHEAR PROBLEM FOR A
      C    DOUBLY PERIODIC RECTANGULAR ARRAY OF ELASTIC INCLUSIONS IN AN
      C    INFINITE ELASTIC BODY
      C
     1 DO 102 I=1,33
       DO 102 J=1,33
       W(I,J)=0.0
       REW(I,J)=0.0
       TZX(I,J)=0.0
       TZY(I,J)=0.0
       WSAVE(I,J) = 0.0
   102 CONTINUE
       GX = 0.0
       GY = 0.0
       KQR = 0
   101 READ (8,208) TITLE
       READ (8,201) M,N,NRX,NRD,IM,IN,NPRLX,NCPRLX,NL,NMFI,NKPROB,NTP
      1,NRXBT,KSYM
       IF (NKPROB .EQ. 2) GO TO 62
       KPROB = 1
       GO TO 61
    62 KPROB = 2
    61 NTP2=NTP+2
       READ (8,201) (IJTP(IJ),IJ=1,NTP2 )
       DO 44 IJ=1,10
       TZXY1(IJ)=0.
    44 TZXY2(IJ)=0.
       READ(8,202) PCGPRX
       MM1=M-1
       MM2=M-2
       MM3=M-3
       NM1=N-1
       NM2=N-2
       NM3=N-3
```

FORTRAN 4 PROGRAM   LONGSHEAR

```
       MP1=M+1
       MP2=M+2
       NP1=N+1
       NP2=N+2
       NLM1=NL-1
       NLM2=NL-2
       IMP3=IM+3
       IMP2=IM+2
       IMP1=IM+1
       IMM1=IM-1
       IMM2=IM-2
       IMM3=IM-3
       INP3=IN+3
       INP2=IN+2
       INP1=IN+1
       INM1=IN-1
       INM2=IN-2
       INM3=IN-3
       READ (8,202) (HX(I),I=3,MM1 )
       READ (8,202) (HY(J),J=3,NM1)
       A=0.0
       B=0.0
       DO 42 I=3,MM1
    42 A=A+HX(I)
       DO 43 J=3,NM1
    43 B=B+HY(J)
       HX(M)=HX(MM1)
       HX(MP1)=HX(MM2)
       HY(N)=HY(NM1)
       HY(NP1)=HY(NM2)
       HX(2)=HX(3)
       HX(1)=HX(4)
       HY(2)=HY(3)
       HY(1)=HY(4)
       READ (8,202) GF,GM,OMB,VF
       READ (8,201) ((LI(L),LJ(L)),L=1,NL)
       READ (8,202) ((COST(L),SINT(L)),L=1,NL)
       READ (8,202) TZXB,TZYB
       DO 33 I=3,M
       DO 33 J=INP1,N
    33 MFI(I,J)=1
       DO 34 I=IMP1,M
       DO 34 J=3,IN
    34 MFI(I,J)=1
       DO 35 I=3,IM
       DO 35 J=3,IN
    35 MFI(I,J)=2
       DO 37 L=1,NL
       I=LI(L)
       J=LJ(L)
    37 MFI(I,J)=3
       DO 12 L=1,NL
       I=LI(L)
       J=LJ(L)
       LN(I,J)=L
    12 CONTINUE
       DO 20 I=4,MM1
```

```
      DO 20 J=4,NM1
      KNT(I,J)=2
   20 CONTINUE
      DO 21 J=1,NP2
      KNT(1,J)=1
      KNT(2,J)=1
      KNT(MP1,J)=1
      KNT(MP2,J)=1
   21 CONTINUE
      DO 22 I=3,M
      KNT(I,1)=1
      KNT(I,2)=1
      KNT(I,NP1)=1
      KNT(I,NP2)=1
   22 CONTINUE
      DO 23 J=4,NM1
      KNT(3,J)=8
      KNT(M,J)=9
   23 CONTINUE
      DO 24 I=4,MM1
      KNT(I,3)=10
      KNT(I,N)=11
   24 CONTINUE
      KNT(3,3)=12
      KNT(3,N)=13
      KNT(M,N)=14
      KNT(M,3)=15
      KNT(IMM1,3)=16
      KNT(3,INM1)=17
      DO 25 L=3,NLM2
      I=LI(L)
      J=LJ(L)
      KNT(I,J)=3
   25 CONTINUE
      I=LI(1)
      J=LJ(1)
      KNT(I,J)=6
      I=LI(2)
      J=LJ(2)
      KNT(I,J)=4
      I=LI(NLM1)
      J=LJ(NLM1)
      KNT(I,J)=5
      I=LI(NL)
      J=LJ(NL)
      KNT(I,J)=7
      DO 4 I=4,MM1
      DO 4 J=4,NM1
      A1=HX(I)
      A2=HY(J)
      A3=HX(I-1)
      A4=HY(J-1)
      E1(I,J)=((-2.0/(A1*A3))+(-2.0/(A2*A4)))*GM
      E2(I,J)=(2.0/(A1*(A1+A3)))*GM
      E3(I,J)=(2.0/(A2*(A2+A4)))*GM
      E4(I,J)=(2.0/(A3*(A1+A3)))*GM
      E5(I,J)=(2.0/(A4*(A2+A4)))*GM
```

FORTRAN 4 PROGRAM LONGSHEAR

```
    4 CONTINUE
      DO 41 I=4,IM
      DO 41 J=4,IN
      A1=HX(I)
      A2=HY(J)
      A3=HX(I-1)
      A4=HY(J-1)
      E1(I,J)=((-2.0/(A1*A3))+(-2.0/(A2*A4)))*GF
      E2(I,J)=(2.0/(A1*(A1+A3)))*GF
      E3(I,J)=(2.0/(A2*(A2+A4)))*GF
      E4(I,J)=(2.0/(A3*(A1+A3)))*GF
   41 E5(I,J)=(2.0/(A4*(A2+A4)))*GF
      READ (8,201) ((MFII(K),MFIJ(K)),K=1,NMFI)
      DO 36 K=1,NMFT
      I = MFII(K)
      J = MFIJ(K)
      A1=HX(I)
      A2=HY(J)
      A3=HX(I-1)
      A4=HY(J-1)
      E1(I,J)=((-2.0/(A1*A3))+(-2.0/(A2*A4)))*GM
      E2(I,J)=(2.0/(A1*(A1+A3)))*GM
      E3(I,J)=(2.0/(A2*(A2+A4)))*GM
      E4(I,J)=(2.0/(A3*(A1+A3)))*GM
      E5(I,J)=(2.0/(A4*(A2+A4)))*GM
   36 MFI(I,J)=1
      DO 7 L=1,NL
      I=LI(L)
      J=LJ(L)
      A1=HX(I)
      A2=HY(J)
      A3=HX(I-1)
      A4=HY(J-1)
      A9=A1+HX(I+1)
      A10=A2+HY(J+1)
      A11=A3+HX(I-2)
      A12=A4+HY(J-2)
      B1=COST(L)/(A9-A1)
      B2=SINT(L)/(A10-A2)
      B3=COST(L)/(A11-A3)
      B4=SINT(L)/(A12-A4)
      C1(L)=(GM*(A9**2-A1**2)*B1)/(A1*A9)
      C2(L)=(GM*(A10**2-A2**2)*B2)/(A2*A10)
      C3(L)=(GF*(A11**2-A3**2)*B3)/(A3*A11)
      C4(L)=(GF*(A12**2-A4**2)*B4)/(A4*A12)
      C7(L)=-(GM*A9*B1)/A1
      C8(L)=-(GM*A10*B2)/A2
      C9(L)=-(GF*A11*B3)/A3
      C10(L)=-(GF*A12*B4)/A4
      C11(L)=(GM*A1*B1)/A9
      C12(L)=(GM*A2*B2)/A10
      C13(L)=(GF*A3*B3)/A11
      C14(L)=(GF*A4*B4)/A12
      C5(L)=(2.0*GF*A3*B3)/A11
      C6(L)=2.0*C5(L)
      C15(L)=(4.0*GF*A4*B4)/A12
      C16(L)=C15(L)/2.0
```

```
      C17(L)=(GF*(A12**2-A4**2))/(A4*A12*(A12-A4))
      C18(L)=(GM*(A10**2-A2**2))/(A2*A10*(A10-A2))
      C19(L)=(-GM*A10)/(A2*(A10-A2))
      C20(L)=(-GF*A12)/(A4*(A12-A4))
      C21(L)=(GM*A2)/(A10*(A10-A2))
      C22(L)=(GF*A4)/(A12*(A12-A4))
      C23(L)=(GF*(A11**2-A3**2))/(A3*A11*(A11-A3))
      C24(L)=(GM*(A9**2-A1**2))/(A1*A9*(A9-A1))
      C25(L)=(-GM*A9)/(A1*(A9-A1))
      C26(L)=(-GF*A11)/(A3*(A11-A3))
      C27(L)=(GM*A1)/(A9*(A9-A1))
      C28(L)=(GF*A3)/(A11*(A11-A3))
      C29(L)=-4.0*C13(L)
    7 CONTINUE
C POINTS 16 AND 17
      A1=HX(IMM1)
      A3=HX(IMM2)
      A9=HX(IM)+A1
      A11=HX(IMM3)+A3
      A2=HY(INM1)
      A4=HY(INM2)
      A10=HY(IN)+A2
      A12=HY(INM3)+A4
      L=NL+1
      LI(L)=IMM1
      LJ(L)=3
      C23(L)=((GF*(A11**2-A3**2))/(A3*A11*(A11-A3)))*(-1.0)
      C24(L)=((GM*(A9**2-A1**2))/(A1*A9*(A9-A1)))*(-1.0)
      C25(L)=( GM*A9)/(A1*(A9-A1))
      C26(L)=( GF*A11)/(A3*(A11-A3))
      C27(L)=((GM*A1)/(A9*(A9-A1)))*(-1.0)
      C28(L)=((GF*A3)/(A11*(A11-A3)))*(-1.0)
      L=NL+2
      LI(L)=3
      LJ(L)=INM1
      C17(L)=((GF*(A12**2-A4**2))/(A4*A12*(A12-A4)))*(-1.0)
      C18(L)=((GM*(A10**2-A2**2))/(A2*A10*(A10-A2)))*(-1.0)
      C19(L)=( GM*A10)/(A2*(A10-A2))
      C20(L)=( GF*A12)/(A4*(A12-A4))
      C21(L)=((GM*A2)/(A10*(A10-A2)))*(-1.0)
      C22(L)=((GF*A4)/(A12*(A12-A4)))*(-1.0)
      A2=HY(3)
      A10=HY(4)+A2
      A4=HY(NM1)
      A12=HY(NM2)+A4
      DO 8 I=4,IM
      D1(I)=(-(A10**2-A2**2)/(A2*A10*(A10-A2)))*GF
      D2(I)=(A10/(A2*(A10-A2)))*GF
      D3(I)=(-A2/(A10*(A10-A2)))*GF
      D4(I)=((A12**2-A4**2)/(A4*A12*(A12-A4)))*GM
      D5(I)=(-A12/(A4*(A12-A4)))*GM
      D6(I)=(A4/(A12*(A12-A4)))*GM
    8 CONTINUE
      DO 81 I=IMP1,MM1
      D1(I)=(-(A10**2-A2**2)/(A2*A10*(A10-A2)))*GM
      D2(I)=(A10/(A2*(A10-A2)))*GM
      D3(I)=(-A2/(A10*(A10-A2)))*GM
```

FORTRAN 4 PROGRAM LONGSHEAR

```
      D4(I)=((A12**2-A4**2)/(A4*A12*(A12-A4)))*GM
      D5(I)=(-A12/(A4*(A12-A4)))*GM
      D6(I)=(A4/(A12*(A12-A4)))*GM
   81 CONTINUE
      A1=HX(3)
      A9=HX(4)+A1
      A3=HX(MM1)
      A11=HX(MM2)+A3
      DO 9 J=4,IN
      D7(J)=(-(A9**2-A1**2)/(A1*A9*(A9-A1)))*GF
      D8(J)=(A9/(A1*(A9-A1)))*GF
      D9(J)=(-A1/(A9*(A9-A1)))*GF
      D10(J)=((A11**2-A3**2)/(A3*A11*(A11-A3)))*GM
      D11(J)=(-A11/(A3*(A11-A3)))*GM
      D12(J)=(A3/(A11*(A11-A3)))*GM
    9 CONTINUE
      DO 91 J=INP1,NM1
      D7(J)=(-(A9**2-A1**2)/(A1*A9*(A9-A1)))*GM
      D8(J)=(A9/(A1*(A9-A1)))*GM
      D9(J)=(-A1/(A9*(A9-A1)))*GM
      D10(J)=((A11**2-A3**2)/(A3*A11*(A11-A3)))*GM
      D11(J)=(-A11/(A3*(A11-A3)))*GM
      D12(J)=(A3/(A11*(A11-A3)))*GM
   91 CONTINUE
      WRITE (5,206) TITLE,MM2,NM2,A,B,GM,GF,OMB,TZXR,TZYR,VF
      WRITE (5,207) (IJTP(IJ),IJ=1,NTP2)
      WRITE (5,209) ((I,HX(I)),I=3,MM1)
      WRITE (5,210) ((J,HY(J)),J=3,NM1)
    3 NRXS=0
      NRDS=0
   10 IF (NRDS-NRD) 5,6,6
    5 IF (KPROB .EQ. 2)  GO TO 52
      IF (KQR .NE. 0) GO TO 63
      W1S=1.0
      DO 30 J=3,N
   30 W(M,J)=W1S
      KQR = 1
      GO TO 63
   52 IF (KQR .NE. 0) GO TO 63
      W2S=1.0
      DO 104 I=1,M
      DO 104 J=1,N
      W(I,J)=0.0
      REW(I,J)=0.0
  104 CONTINUE
      DO 31 I=3,M
   31 W(I,N)=W2S
      KQR = 1
   63 CALL RSDLS
      NRDS=NRDS+1
      WRITE(5,203) NRDS,KPROB
      WRITE(5,204)
      WRITE (5,205) (((I,J,W(I,J),REW(I,J)),J=3,N),I=3,M)
      DO 46 IJ=1,10
   46 TZXY1(IJ)=0.
      CALL RLXLS
      GO TO 10
```

FORTRAN 4 PROGRAM LONGSHEAR

```
      6 CALL STRLS
        IF (NKPROB .NE. 3) GO TO 1
        IF (KPROB .EQ. 2) GO TO 64
        IF (KSYM .EQ. 1) GO TO 65
        KPROB = 2
        KQR = 0
        NRDS = 0
        NRXS = 0
        GO TO 10
     65 DO 66 I = 3,M
        DO 66 J = 3,N
     66 W(I,J) = WSAVE(J,I)/FP1
        KPROB = 2
        CALL STRLS
     64 KPROB = 3
        DO 67 I = 3,M
        DO 67 J = 3,N
     67 W(I,J) = W(I,J) + WSAVE(I,J)
        CALL STRLS
        GO TO 1
    201 FORMAT (24I3)
    202 FORMAT (6E12.6)
    203 FORMAT (1H1,49X,21HRESULTS OF RESID NO. ,I2,5X,11HPROBLEM NO.,I3/)
    204 FORMAT (1H ,//,6X,1HI,3X,1HJ,18X,1HW,18X, 8HRESIDUAL,///)
    205 FORMAT (1H ,3X,2I4,6X,2E20.8)
    206 FORMAT (1H1,30X,55HL O N G I T U D I N A L   S H E A R   A N A L
       1 Y S I S ,///,12A6,///,55X,10HINPUT DATA,///,
       245H GRID NODE ARRAY SIZE                         =,I2,4H BY ,I2,//,
       321H QUADRANT DIMENSIONS ,6X,3HA =,1F6.3,6X,3HB =,1F6.3,      //,
       445H MATRIX SHEAR MODULUS       PSI               =,1E12.4,     //,
       545H INCLUSION SHEAR MODULUS    PSI               =,1E12.4,     //,
       645H RELAXATION FACTOR    (OMEGA BAR)             =,1F6.3,      //,
       745H AVERAGE ZX SHEAR LOADING AT INFINITY (PSI) =,1F9.2,        //,
       845H AVERAGE ZY SHEAR LOADING AT INFINITY (PSI) =,1F9.2,        //,
       945H PERCENT FIBER BY VOLUME                      =,1F9.2  )
    207 FORMAT (1H ,//,24H TEST POINT COORDINATES ,//,6X,1HI,3X,1HJ,//,
       1(3X,2I4))
    208 FORMAT (12A6)
    209 FORMAT (1H1,////,14H  GRID SPACING ,//,6X,1HI,8X,5HHX(I),//,
       1(3X,I4,3X,F12.8))
    210 FORMAT (1H ,///,6X,1HJ,8X,5HHY(J),//,
       1(3X,I4,3X,F12.8))
        END
```

FORTRAN 4 PROGRAM RSDLS

```
CRSDLS
      SUBROUTINE RSDLS
      COMMON W,WL,WSAVE,W1,W1S,W2,W2S,TZX,TZY,TZXB,TZYB,TZXBS,TZYBS,
     1TZXM,TZYM,TZXF,TZYF,REW,HX,HY,OMB,GF,GM,GX,GY,F1,F2,COST,SINT,
     2CAT,C1,C2,C3,C4,C5,C6,C7,C8,C9,C10,C11,C12,C13,C14,C15,C16,C17,
     3C18,C19,C20,C21,C22,C23,C24,C25,C26,C27,C28,C29,D1,D2,D3,D4,D5,
     4D6,D7,D8,D9,D10,D11,D12,E1,E2,E3,E4,E5,
     5M,MM1,MM2,MM3,MP1,MP2,N,NM1,NM2,NM3,NP1,NP2,IN,INM1,INM2,INM3,
     6INP1,INP2,INP3,IM,IMM1,IMM2,IMM3,IMP1,IMP2,IMP3,NL,NLM1,NLM2,
     7LN,LI,LJ,LAT,KNAT,NMFI,MFIJ,MFI,KNT,KPROB,IJTP,MFII,
     8NRX,NRD,NRXS,NRDS,NPRLX,NCPRLX,NTP,NPT,TZXY1,TZXY2,PCGPRX,TZXY
      DIMENSION W(33,33),REW(33,33),TZX(33,33),TZY(33,33),E1(33,33),
     1E2(33,33),E3(33,33),E4(33,33),E5(33,33),KNT(33,33),LN(33,33),
     2C29(33,33),MFI(33,33),WSAVE(33,33),TZXY(33,33),
     3C6(70),C7(70),C8(70),C9(70),C10(70),C11(70),C12(70),C13(70),C14(70
     4),C15(70),C16(70),C17(70),C18(70),C19(70),C20(70),C21(70),C22(70),
     5C23(70),C24(70),C25(70),C26(70),C27(70),C28(70),COST(70),SINT(70)
     6,D1(35),D2(35),D3(35),D4(35),D5(35),D6(35),D7(35),D8(35),D9(35),
     7HX(35),HY(35),WL(70),LI(70),LJ(70),C1(70),C2(70),C3(70),C4(70),
     8C5(70),D10(35),D11(35),D12(35),TZXM(70),TZYM(70),TZXF(70),TZYF(70)
     9,IJTP(20),TZXY1(10),TZXY2(10),MFII(90),MFIJ(90)
      DO 3 I=4,MM1
      DO 3 J=4,NM1
      REW(I,J)=E1(I,J)*W(I,J)+E2(I,J)*W(I+1,J)+E3(I,J)*W(I,J+1)+E4(I,J)*
     1        W(I-1,J)+E5(I,J)*W(I,J-1)
    3 CONTINUE
      NLM2=NL-2
      DO 4 L=3,NLM2
      I=LI(L)
      J=LJ(L)
      REW(I,J)=(C3(L)+C4(L)+C1(L)+C2(L))*W(I,J)+C7(L)*W(I+1,J)+C8(L)*W
     1        (I,J+1)+C9(L)*W(I-1,J)+C10(L)*W(I,J-1)+C11(L)*W(I+2,J)+C12
     2        (L)*W(I,J+2)+C13(L)*W(I-2,J)+C14(L)*W(I,J-2)
    4 CONTINUE
      GO TO (1,2),KPROB
    1 DO 8 J=3,N
      REW(3,J)=0.0
    8 REW(M,J)=0.0
      DO 5 I=4,MM1
      REW(I,3)=D1(I)*W(I,3)+D2(I)*W(I,4)+D3(I)*W(I,5)
      REW(I,N)=D4(I)*W(I,N)+D5(I)*W(I,NM1)+D6(I)*W(I,NM2)
    5 CONTINUE
      J=LJ(2)
      REW(4,J)=(C1(2)+C2(2)+C5(2)+C4(2))*W(4,J)+C7(2)*W(5,J)+C8(2)*W(4,
     1        J+1)+C29(2)*W(3,J)+C10(2)*W(4,J-1)+C11(2)*W(6,J)+C12(2)*W(4,
     2        J+2)+C14(2)*W(4,J-2)
      L=NL-1
      I=LI(L)
      REW(I,4)=(C1(L)+C2(L)+C3(L)+C15(L))*W(I,4)+C7(L)*W(I+1,4)+C8(L)*W
     1        (I,5)+C9(L)*W(I-1,4)-C15(L)*W(I,3)+C11(L)*W(I+2,4)+C12(L)*W
     2        (I,6)+C13(L)*W(I-2,4)
      REW(IM,3)=(C23(NL)+C24(NL))*W(IM,3)+C25(NL)*W(IM+1,3)+C26(NL)*W
     1        (IM-1,3)+C27(NL)*W(IM+2,3)+C28(NL)*W(IM-2,3)
      I=IMM1
      L=NL+1
      REW(I,3)=C26(L)*W(I,3)+(C23(L)+C24(L))*W(I+1,3)+C28(L)*W(I-1,3)
     1        +C25(L)*W(I+2,3)+C27(L)*W(I+3,3)
```

FORTRAN 4 PROGRAM RSDLS

```
              GO TO 6
            2 DO 9 I=3,M
              REW(I,3)=0.0
            9 REW(I,N)=0.0
              DO 7 J=4,NM1
              REW(3,J)=D7(J)*W(3,J)+D8(J)*W(4,J)+D9(J)*W(5,J)
              REW(M,J)=D10(J)*W(M,J)+D11(J)*W(MM1,J)+D12(J)*W(MM2,J)
            7 CONTINUE
              J=LJ(2)
              REW(4,J)=(C1(2)+C2(2)+C6(2)+C4(2))*W(4,J)+C7(2)*W(5,J)+C8(2)*W(4,
             1      J+1)+C29(2)*W(3,J)+C10(2)*W(4,J-1)+C11(2)*W(6,J)+C12(2)*W(4,
             2      J+2)+C14(2)*W(4,J-2)
              L=NL-1
              I=LI(L)
              REW(I,4)=(C1(L)+C2(L)+C3(L)+C16(L))*W(I,4)+C7(L)*W(I+1,4)+C8(L)*W
             1      (I,5)+C9(L)*W(I-1,4)-C15(L)*W(I,3)+C11(L)*W(I+2,4)+C12(L)*W
             2      (I,6)+C13(L)*W(I-2,4)
              REW(3,IN)=(C17(1)+C18(1))*W(3,IN)+C19(1)*W(3,IN+1)+C20(1)*W(3,IN-1
             1      )+C21(1)*W(3,IN+2)+C22(1)*W(3,IN-2)
              J=INM1
              L=NL+2
              REW(3,J)=C20(L)*W(3,J)+(C17(L)+C18(L))*W(3,J+1)+C22(L)*W(3,J-1)
             1      +C19(L)*W(3,J+2)+C21(L)*W(3,J+3)
            6 RETURN
              END
```

FORTRAN 4 PROGRAM RLXLS

```
      CRLXLS
              SUBROUTINE RLXLS
              COMMON W,WL,WSAVE,W1,W1S,W2,W2S,TZX,TZY,TZXB,TZYB,TZXBS,TZYBS,
             1TZXM,TZYM,TZXF,TZYF,REW,HX,HY,OMB,GF,GM,GX,GY,F1,F2,COST,SINT,
             2CAT,C1,C2,C3,C4,C5,C6,C7,C8,C9,C10,C11,C12,C13,C14,C15,C16,C17,
             3C18,C19,C20,C21,C22,C23,C24,C25,C26,C27,C28,C29,D1,D2,D3,D4,D5,
             4D6,D7,D8,D9,D10,D11,D12,E1,E2,E3,E4,E5,
             5M,MM1,MM2,MM3,MP1,MP2,N,NM1,NM2,NM3,NP1,NP2,IN,INM1,INM2,INM3,
             6INP1,INP2,INP3,IM,IMM1,IMM2,IMM3,IMP1,IMP2,IMP3,NL,NLM1,NLM2,
             7LN,LI,LJ,LAT,KNAT,NMFI,MFIJ,MFI,KNT,KPROB,IJTP,MFII,
             8NRX,NRD,NRXS,NRDS,NPRLX,NCPRLX,NTP,NPT,TZXY1,TZXY2,PCGPRX,TZXY
             9,NRXBT
              DIMENSION W(33,33),REW(33,33),TZX(33,33),TZY(33,33),E1(33,33),
             1E2(33,33),E3(33,33),E4(33,33),E5(33,33),KNT(33,33),LN(33,33),
             2C29(33,33),MFI(33,33),WSAVE(33,33),TZXY(33,33),
             3C6(70),C7(70),C8(70),C9(70),C10(70),C11(70),C12(70),C13(70),C14(70
             4),C15(70),C16(70),C17(70),C18(70),C19(70),C20(70),C21(70),C22(70),
             5C23(70),C24(70),C25(70),C26(70),C27(70),C28(70),COST(70),SINT(70)
             6,D1(35),D2(35),D3(35),D4(35),D5(35),D6(35),D7(35),D8(35),D9(35),
             7HX(35),HY(35),WL(70),LI(70),LJ(70),C1(70),C2(70),C3(70),C4(70),
             8C5(70),D10(35),D11(35),D12(35),TZXM(70),TZYM(70),TZXF(70),TZYF(70)
             9,IJTP(20),TZXY1(10),TZXY2(10),MFII(90),MFIJ(90)
              DIMENSION XX(35),YY(35),AA(70)
              NRXS=0
              NPRXS=0
         1001 IF(NRXS-NRX) 1002,1003,1003
         1002 NRXS=NRXS+1
              NPRXS=NPRXS+1
              GO TO (4201,4202),KPROB
         4201 KMM1 = MM1
              KMP2 = MP2
              KNM1 = NM1
              KNP2 = NP2
              GO TO 4203
         4202 KMM1 = NM1
              KMP2 = NP2
              KNM1 = MM1
              KNP2 = MP2
         4203 DO 50 III = 2,KMM1
              II = KMP2 - III
              DO 50 JJJ = 2,KNM1
              JJ = KNP2 - JJJ
              GO TO (1111,1112),KPROB
         1111 I=II
              J=JJ
              GO TO 1113
         1112 I=JJ
              J=II
         1113 KNAT=KNT(I,J)
              GO TO (50,202,203,204,205,206,207,208,209,210,211,50,50,50,50,
             150,50),KNAT
          202 CAT=E1(I,J)
              GO TO 1
          203 LAT=LN(I,J)
              CAT=C1(LAT)+C2(LAT)+C3(LAT)+C4(LAT)
              GO TO 1
          204 LAT=LN(I,J)
```

FORTRAN 4 PROGRAM   RLXLS

```
           GO TO (2041,2042),KPROB
      2041 CAT=C1(LAT)+C2(LAT)+C4(LAT)+C5(LAT)
           GO TO 1
      2042 CAT=C1(LAT)+C2(LAT)+C4(LAT)+C6(LAT)
           GO TO 1
       205 LAT=LN(I,J)
           GO TO (2051,2052),KPROB
      2051 CAT=C1(LAT)+C2(LAT)+C3(LAT)+C15(LAT)
           GO TO 1
      2052 CAT=C1(LAT)+C2(LAT)+C3(LAT)+C16(LAT)
           GO TO 1
       206 LAT=LN(I,J)
           CAT=C17(LAT)+C18(LAT)
           GO TO (50,1),KPROB
       207 LAT=LN(I,J)
           CAT=C23(LAT)+C24(LAT)
           GO TO (1,50),KPROB
       208 GO TO (50,2082),KPROB
      2082 CAT=D7(J)
           GO TO 1
       209 GO TO (50,2092),KPROB
      2092 CAT=D10(J)
           GO TO 1
       210 GO TO (2101,50),KPROB
      2101 CAT=D1(I)
           GO TO 1
       211 GO TO (2111,50),KPROB
      2111 CAT=D4(I)
           GO TO 1
         1 DO 51 KIJ=1,9
           GO TO (9022,9023,9024,9025,9026,9027,9028,9029,9021),KIJ
      9021 KI=I
           KJ=J
           GO TO 30
      9022 KI=I+1
           KJ=J
           GO TO 30
      9023 KI=I
           KJ=J+1
           GO TO 30
      9024 KI=I-1
           KJ=J
           GO TO 30
      9025 KI=I
           KJ=J-1
           GO TO 30
      9026 KI=I+2
           KJ=J
           GO TO 30
      9027 KI=I
           KJ=J+2
           GO TO 30
      9028 KI=I-2
           KJ=J
           GO TO 30
      9029 KI=I
           KJ=J-2
```

FORTRAN 4 PROGRAM   RLXLS

```
        30 KN=KNT(KI,KJ)
           GO TO (301,302),KPROB
       301 GO TO (51,2,3,4,5,51,7,51,51,10,11,51,51,51,51,51,51),KN
       302 GO TO (51,2,3,4,5,6,51,8,9,51,51,51,51,51,51,-1,51),KN
         2 GO TO (22,23,24,25,51,51,51,51,21),KIJ
        21 W(I,J)=W(I,J)-REW(I,J)*OMB/CAT
           REW(I,J)     =REW(I,J)     *(1.0-OMB)
           GO TO 51
        22 REW(KI,KJ)  =REW(KI,KJ)  -REW(I,J)*OMB*(E4 (KI,KJ)/CAT)
           GO TO 51
        23 REW(KI,KJ)  =REW(KI,KJ)  -REW(I,J)*OMB*(E5 (KI,KJ)/CAT)
           GO TO 51
        24 REW(KI,KJ)  =REW(KI,KJ)  -REW(I,J)*OMB*(E2 (KI,KJ)/CAT)
           GO TO 51
        25 REW(KI,KJ)  =REW(KI,KJ)  -REW(I,J)*OMB*(E3 (KI,KJ)/CAT)
           GO TO 51
         3 L=LN(KI,KJ)
           GO TO (32,33,34,35,36,37,38,39,31),KIJ
        31 W(I,J)=W(I,J)-REW(I,J)*OMB/CAT
           REW(I,J)     =REW(I,J)     *(1.0-OMB)
           GO TO 51
        32 REW(KI,KJ)  =REW(KI,KJ)  -REW(I,J)*OMB*(C9 (L)/CAT)
           GO TO 51
        33 REW(KI,KJ)  =REW(KI,KJ)  -REW(I,J)*OMB*(C10(L)/CAT)
           GO TO 51
        34 REW(KI,KJ)  =REW(KI,KJ)  -REW(I,J)*OMB*(C7 (L)/CAT)
           GO TO 51
        35 REW(KI,KJ)  =REW(KI,KJ)  -REW(I,J)*OMB*(C8 (L)/CAT)
           GO TO 51
        36 REW(KI,KJ)  =REW(KI,KJ)  -REW(I,J)*OMB*(C13(L)/CAT)
           GO TO 51
        37 REW(KI,KJ)  =REW(KI,KJ)  -REW(I,J)*OMB*(C14(L)/CAT)
           GO TO 51
        38 REW(KI,KJ)  =REW(KI,KJ)  -REW(I,J)*OMB*(C11(L)/CAT)
           GO TO 51
        39 REW(KI,KJ)  =REW(KI,KJ)  -REW(I,J)*OMB*(C12(L)/CAT)
           GO TO 51
         4 L=LN(KI,KJ)
           GO TO (42,43,44,45,51,47,48,49,41),KIJ
        41 W(I,J)=W(I,J)-REW(I,J)*OMB/CAT
           REW(I,J)     =REW(I,J)     *(1.0-OMB)
           GO TO 51
        42 REW(KI,KJ)  =REW(KI,KJ)  -REW(I,J)*OMB*(C29(L)/CAT)
           GO TO 51
        43 REW(KI,KJ)  =REW(KI,KJ)  -REW(I,J)*OMB*(C10(L)/CAT)
           GO TO 51
        44 REW(KI,KJ)  =REW(KI,KJ)  -REW(I,J)*OMB*(C7 (L)/CAT)
           GO TO 51
        45 REW(KI,KJ)  =REW(KI,KJ)  -REW(I,J)*OMB*(C8 (L)/CAT)
           GO TO 51
        47 REW(KI,KJ)  =REW(KI,KJ)  -REW(I,J)*OMB*(C14(L)/CAT)
           GO TO 51
        48 REW(KI,KJ)  =REW(KI,KJ)  -REW(I,J)*OMB*(C11(L)/CAT)
           GO TO 51
        49 REW(KI,KJ)  =REW(KI,KJ)  -REW(I,J)*OMB*(C12(L)/CAT)
           GO TO 51
         5 L=LN(KI,KJ)
```

```
      GO TO (52,53,54,55,56,51,58,59,46),KIJ
   46 W(I,J)=W(I,J)-REW(I,J)*OMB/CAT
      REW(I,J)=REW(I,J)*(1.0-OMB)
      GO TO 51
   52 REW(KI,KJ)=REW(KI,KJ)-REW(I,J)*OMB*( C9 (L)/CAT)
      GO TO 51
   53 REW(KI,KJ)=REW(KI,KJ)-REW(I,J)*OMB*(-C15(L)/CAT)
      GO TO 51
   54 REW(KI,KJ)=REW(KI,KJ)-REW(I,J)*OMB*( C7 (L)/CAT)
      GO TO 51
   55 REW(KI,KJ)=REW(KI,KJ)-REW(I,J)*OMB*( C8 (L)/CAT)
      GO TO 51
   56 REW(KI,KJ)=REW(KI,KJ)-REW(I,J)*OMB*( C13(L)/CAT)
      GO TO 51
   58 REW(KI,KJ)=REW(KI,KJ)-REW(I,J)*OMB*( C11(L)/CAT)
      GO TO 51
   59 REW(KI,KJ)=REW(KI,KJ)-REW(I,J)*OMB*( C12(L)/CAT)
      GO TO 51
    6 L=LN(KI,KJ)
      GO TO (51,63,51,65,51,67,51,69,61),KIJ
   61 W(I,J)=W(I,J)-REW(I,J)*OMB/CAT
      REW(I,J)=REW(I,J)*(1.0-OMB)
      GO TO 51
   63 REW(KI,KJ)=REW(KI,KJ)-REW(I,J)*OMB*( C20(L)/CAT)
      GO TO 51
   65 REW(KI,KJ)=REW(KI,KJ)-REW(I,J)*OMB*( C19(L)/CAT)
      GO TO 51
   67 REW(KI,KJ)=REW(KI,KJ)-REW(I,J)*OMB*( C22(L)/CAT)
      GO TO 51
   69 REW(KI,KJ)=REW(KI,KJ)-REW(I,J)*OMB*( C21(L)/CAT)
      GO TO 51
    7 L=LN(KI,KJ)
      GO TO (72,51,74,51,76,51,78,51,71),KIJ
   71 W(I,J)=W(I,J)-REW(I,J)*OMB/CAT
      REW(I,J)=REW(I,J)*(1.0-OMB)
      GO TO 51
   72 REW(KI,KJ)=REW(KI,KJ)-REW(I,J)*OMB*( C26(L)/CAT)
      GO TO 51
   74 REW(KI,KJ)=REW(KI,KJ)-REW(I,J)*OMB*( C25(L)/CAT)
      GO TO 51
   76 REW(KI,KJ)=REW(KI,KJ)-REW(I,J)*OMB*( C28(L)/CAT)
      GO TO 51
   78 REW(KI,KJ)=REW(KI,KJ)-REW(I,J)*OMB*( C27(L)/CAT)
      GO TO 51
    8 GO TO (51,51,84,51,51,51,88,51,81),KIJ
   81 W(I,J)=W(I,J)-REW(I,J)*OMB/CAT
      REW(I,J)=REW(I,J)*(1.0-OMB)
      GO TO 51
   84 REW(KI,KJ)=REW(KI,KJ)-REW(I,J)*OMB*( D8 (J)/CAT)
      GO TO 51
   88 REW(KI,KJ)=REW(KI,KJ)-REW(I,J)*OMB*( D9 (J)/CAT)
      GO TO 51
    9 GO TO (92,51,51,51,96,51,51,51,91),KIJ
   91 W(I,J)=W(I,J)-REW(I,J)*OMB/CAT
      REW(I,J)=REW(I,J)*(1.0-OMB)
      GO TO 51
   92 REW(KI,KJ)=REW(KI,KJ)-REW(I,J)*OMB*( D11(J)/CAT)
```

FORTRAN 4 PROGRAM RLXLS

```
      GO TO 51
   96 REW(KI,KJ)=REW(KI,KJ)-REW(I,J)*OMB*( D12(J)/CAT)
      GO TO 51
   10 GO TO (51,51,51,105,51,51,51,109,101),KIJ
  101 W(I,J)=W(I,J)-REW(I,J)*OMB/CAT
      REW(I,J)=REW(I,J)*(1.0-OMB)
      GO TO 51
  105 REW(KI,KJ)=REW(KI,KJ)-REW(I,J)*OMB*( D2 (I)/CAT)
      GO TO 51
  109 REW(KI,KJ)=REW(KI,KJ)-REW(I,J)*OMB*( D3 (I)/CAT)
      GO TO 51
   11 GO TO (51,113,51,51,51,117,51,51,111),KIJ
  111 W(I,J)=W(I,J)-REW(I,J)*OMB/CAT
      REW(I,J)=REW(I,J)*(1.0-OMB)
      GO TO 51
  113 REW(KI,KJ)=REW(KI,KJ)-REW(I,J)*OMB*( D5 (I)/CAT)
      GO TO 51
  117 REW(KI,KJ)=REW(KI,KJ)-REW(I,J)*OMB*( D6 (I)/CAT)
      GO TO 51
   51 CONTINUE
   50 CONTINUE
      GO TO (160,170),KPROB
  160 DO 161 I=3,IM
  161 YY(I-2)=W(I,3)
      XX(1)=0.0
      DO 162 I=4,IM
  162 XX(I-2)=XX(I-3)+HX(I-1)
      IMM4=IM-4
      DO 163 I=1,IMM4
      II=2*I-1
      AA(II)=XX(I)
  163 AA(II+1)=YY(I)
      II=2*IMM3-1
      AA(II)=XX(IMM2)
      AA(II+1)=YY(IMM2)
      XXX=XX(IMM3)
      NN=IMM3
      WNEW=AINTPL (XXX,NN,AA)
      DELTAW=WNEW-W(IMM1,3)
      REW(IM,3)=REW(IM,3)+C26(NL)*DELTAW
      REW(IMM1,4)=REW(IMM1,4)-C15(NL-1)*DELTAW
      W(IMM1,3)=WNEW
      GO TO 1617
  170 DO 171 J=3,IN
  171 YY(J-2)=W(3,J)
      XX(1)=0.0
      DO 172 J=4,IN
  172 XX(J-2)=XX(J-3)+HY(J-1)
      INM4=IN-4
      DO 173 J=1,INM4
      JJ=2*J-1
      AA(JJ)=XX(J)
  173 AA(JJ+1)=YY(J)
      JJ=2*INM3-1
      AA(JJ)=XX(INM2)
      AA(JJ+1)=YY(INM2)
      XXX=XX(INM3)
```

FORTRAN 4 PROGRAM RLXLS

```
            NN=INM3
            WNEW=AINTPL (XXX,NN,AA)
            DELTAW=WNEW-W(3,INM1)
            REW(3,IN)=REW(3,IN)+C20(1)*DELTAW
            REW(4,INM1)=REW(4,INM1)+C29(2)*DELTAW
            W(3,INM1)=WNEW
       1617 CONTINUE
            IF (NRXS.LE.NRXBT) GO TO 3005
            NPT=0
            DO 3001 IJ=1,NTP
            I=IJTP(2*IJ-1)
            J=IJTP(2*IJ)
            A1=HX(I)
            A2=HY(J)
            A3=HX(I-1)
            A4=HY(J-1)
            TZX(I,J)=  (GM/(A1*A3*(A1+A3)))*(A3**2*W(I+1,J)+(A1**2-A3**2)*W(I,
           1J)-A1**2*W(I-1,J))
            TZY(I,J)=(GM/(A2*A4*(A2+A4)))*(A4**2*W(I,J+1)+(A2**2-A4**2)*W(I,J)
           1-A2**2*W(I,J-1))
       3001 TZXY2(IJ)=SQRT(TZX(I,J)**2+TZY(I,J)**2)
            DO 3002 IJ=1,NTP
            I=IJTP(2*IJ-1)
            J=IJTP(2*IJ)
            PCG=((TZXY2(IJ)-TZXY1(IJ))/TZXY2(IJ))*100.0
            IF (PCG.LE .PCGPRX) GO TO 3002
            NPT=NPT+1
       3002 TZXY1(IJ)=TZXY2(IJ)
            IF (NPT.EQ.0) GO TO 1003
       3005 CONTINUE
            IF(NRXS-NCPRLX) 1005,1005,1004
       1004 CONTINUE
            IF(NPRXS-NPRLX) 1001,1006,1006
       1006 NPRXS=0
       1005 CONTINUE
            WRITE (5,1041) NRXS,KPROB
       1041 FORMAT(1H1,49X,21H RESULTS OF RELAX NO. ,I4,5X,11HPROBLEM NO.,I3/)
            WRITE (5,1042) (((I,J,W(I,J),REW(I,J)),J=3,N),I=3,M)
       1042 FORMAT(1H ,//,6X,1HI,3X,1HJ,18X,1HW,18X,8HRESIDUAL,///,
           1(3X,2I4,6X,2E20,8))
            WRITE (5,1043) NPT,PCGPRX
            LPRX = NRXS
            GO TO 1001
       1003 IF (NRXS .EQ. LPRX)  GO TO 4044
            WRITE (5,1041) NRXS,KPROB
            WRITE (5,1042) (((I,J,W(I,J),REW(I,J)),J=3,N),I=3,M)
            WRITE (5,1043) NPT,PCGPRX
       1043 FORMAT(1H ,///,I10,92H  TEST POINTS HAVE NOT YET CONVERGED TO THE
           1SPECIFIED MINIMUM CHANGE IN STRESS PER RELAX OF ,F8.3,7HPERCENT)
       4044 RETURN
            END
```

FORTRAN 4 PROGRAM STRLS

```
      CSTRLS
            SUBROUTINE STRLS
            COMMON W,WL,WSAVE,W1,W1S,W2,W2S,TZX,TZY,TZXB,TZYB,TZXBS,TZYBS,
           1TZXM,TZYM,TZXF,TZYF,REW,HX,HY,OMB,GF,GM,GX,GY,F1,F2,COST,SINT,
           2CAT,C1,C2,C3,C4,C5,C6,C7,C8,C9,C10,C11,C12,C13,C14,C15,C16,C17,
           3C18,C19,C20,C21,C22,C23,C24,C25,C26,C27,C28,C29,D1,D2,D3,D4,D5,
           4D6,D7,D8,D9,D10,D11,D12,E1,E2,E3,E4,E5,
           5M,MM1,MM2,MM3,MP1,MP2,N,NM1,NM2,NM3,NP1,NP2,IN,INM1,INM2,INM3,
           6INP1,INP2,INP3,IM,IMM1,IMM2,IMM3,IMP1,IMP2,IMP3,NL,NLM1,NLM2,
           7LN,LI,LJ,LAT,KNAT,NMFI,MFIJ,MFI,KNT,KPROB,IJTP,MFII,
           8NRX,NRD,NRXS,NRDS,NPRLX,NCPRLX,NTP,NPT,TZXY1,TZXY2,PCGPRX,TZXY
           9,NRXBT,NKPROB,A,B,FP1
            DIMENSION W(33,33),REW(33,33),TZX(33,33),TZY(33,33),E1(33,33),
           1E2(33,33),E3(33,33),E4(33,33),E5(33,33),KNT(33,33),LN(33,33),
           2C29(33,33),MFI(33,33),WSAVE(33,33),TZXY(33,33),
           3C6(70),C7(70),C8(70),C9(70),C10(70),C11(70),C12(70),C13(70),C14(70
           4),C15(70),C16(70),C17(70),C18(70),C19(70),C20(70),C21(70),C22(70),
           5C23(70),C24(70),C25(70),C26(70),C27(70),C28(70),COST(70),SINT(70)
           6,D1(35),D2(35),D3(35),D4(35),D5(35),D6(35),D7(35),D8(35),D9(35),
           7HX(35),HY(35),WL(70),LI(70),LJ(70),C1(70),C2(70),C3(70),C4(70),
           8C5(70),D10(35),D11(35),D12(35),TZXM(70),TZYM(70),TZXF(70),TZYF(70)
           9,IJTP(20),TZXY1(10),TZXY2(10),MFII(90),MFIJ(90)
           A,TZXYF(70),TZXYM(70)
            GO TO (1,2,10),KPROB
          1 A3=HX(MM1)
            A11=HX(MM2)+HX(MM1)
            DO 3 J=3,N
            TZX(M,J)=
           1     (GM*((A11**2-A3**2)*W(M,J)-A11**2*W(MM1,J)+A3**2*W(MM2,J)))/
           2        (A3*A11*(A11-A3))
          3 CONTINUE
            TZX(M,3)=TZX(M,3)*HY(3)/2.0
            DO 200 J=4,NM1
        200 TZX(M,J)=TZX(M,J)*((HY(J-1)/2.0)+(HY(J)/2.0))
            TZX(M,N)=TZX(M,N)*HY(NM1)/2.0
            TZXBS=0.0
            DO 4 J=3,N
          4 TZXBS=TZXBS+TZX(M,J)
            TZXBS=TZXBS/B
            FP1 = TZXB/TZXBS
            DO 7 I=3,M
            DO 7 J=3,N
            W(I,J) = FP1*W(I,J)
            WSAVE(I,J)=W(I,J)
          7 CONTINUE
            GX=(A*TZXBS)/W1S
            IF (NKPROB .EQ. 1) GO TO 10
            RETURN
          2 A4=HY(NM1)
            A12=HY(NM1)+HY(NM2)
            DO 5 I=3,M
            TZY(I,N)=(GM/(A4*A12*(A12-A4)))*((A12**2-A4**2)*W(I,N)-A12**2*W(
           1I,NM1)+A4**2*W(I,NM2))
          5 CONTINUE
            TZY(3,N)=TZY(3,N)*HX(3)/2.0
            DO 201 I=4,MM1
        201 TZY(I,N)=TZY(I,N)*((HX(I-1)/2.0)+(HX(I)/2.0))
```

```
      TZY(M,N)=TZY(M,N)+HX(MM1)/2.0
      TZYBS=0.0
      DO 6 I=3,M
    6 TZYBS=TZYBS+TZY(I,N)
      TZYBS=TZYBS/A
      F =TZYB/TZYBS
      DO 8 I=3,M
      DO 8 J=3,N
      W(I,J)=F*W(I,J)
    8 CONTINUE
      GY=(B*TZYBS)/W2S
      IF (NKPROB .EQ. 2) GO TO 10
      RETURN
   10 DO 11 I=4,MM1
      DO 11 J=INP1,NM1
      A1=HX(I)
      A2=HY(J)
      A3=HX(I-1)
      A4=HY(J-1)
      TZX(I,J)=  (GM/(A1*A3*(A1+A3)))*(A3**2*W(I+1,J)+(A1**2-A3**2)*W(I,
     1J)-A1**2*W(I-1,J))
      TZY(I,J)=(GM/(A2*A4*(A2+A4)))*(A4**2*W(I,J+1)+(A2**2-A4**2)*W(I,J)
     1-A2**2*W(I,J-1))
   11 CONTINUE
      DO 12 I=IMP1,MM1
      DO 12 J=4,IN
      A1=HX(I)
      A2=HY(J)
      A3=HX(I-1)
      A4=HY(J-1)
      TZX(I,J)=  (GM/(A1*A3*(A1+A3)))*(A3**2*W(I+1,J)+(A1**2-A3**2)*W(I,
     1J)-A1**2*W(I-1,J))
      TZY(I,J)=(GM/(A2*A4*(A2+A4)))*(A4**2*W(I,J+1)+(A2**2-A4**2)*W(I,J)
     1-A2**2*W(I,J-1))
   12 CONTINUE
      DO 13 I=4,IM
      DO 13 J=4,IN
      A1=HX(I)
      A2=HY(J)
      A3=HX(I-1)
      A4=HY(J-1)
      A9=HX(I+1)+A1
      A10=HY(J+1)+A2
      A11=HX(I-2)+A3
      A12=HY(J-2)+A4
      K=MFI(I,J)
      GO TO (14,15,16),K
   14 CONTINUE
      A1=HX(I)
      A2=HY(J)
      A3=HX(I-1)
      A4=HY(J-1)
      TZX(I,J)=  (GM/(A1*A3*(A1+A3)))*(A3**2*W(I+1,J)+(A1**2-A3**2)*W(I,
     1J)-A1**2*W(I-1,J))
      TZY(I,J)=(GM/(A2*A4*(A2+A4)))*(A4**2*W(I,J+1)+(A2**2-A4**2)*W(I,J)
     1-A2**2*W(I,J-1))
      GO TO 13
```

FORTRAN 4 PROGRAM STRLS

```
   15 CONTINUE
      A1=HX(I)
      A2=HY(J)
      A3=HX(I-1)
      A4=HY(J-1)
      TZX(I,J)=(GF/(A1*A3*(A1+A3)))*(A3**2*W(I+1,J)+(A1**2-A3**2)*W(I,
     1J)-A1**2*W(I-1,J))
      TZY(I,J)=(GF/(A2*A4*(A2+A4)))*(A4**2*W(I,J+1)+(A2**2-A4**2)*W(I,J)
     1-A2**2*W(I,J-1))
      GO TO 13
   16 L=LN(I,J)
      IF (L.GT.2.AND.L.LT.NLM1)GO TO 19
      IF (L.EQ.2) GO TO 18
      GO TO 20
   18 TZXF(L)=(GF/(A3*A11*(A11-A3)))*((A11**2-A3**2)*W(I,J)-A11**2*W(I-1
     1,J)+A3**2*(W(I,J)-2.0*WSAVE(I,J)))
      TZYF(L)=(GF/(A4*A12*(A12-A4)))*((A12**2-A4**2)*W(I,J)-A12**2*W(I,
     1J-1)+A4**2*W(I,J-2))
      TZXM(L)=(GM/(A1*A9*(A9-A1)))*((A1**2-A9**2)*W(I,J)+A9**2*W(I+1,J)
     1-A1**2*W(I+2,J))
      TZYM(L)=(GM/(A2*A10*(A10-A2)))*((A2**2-A10**2)*W(I,J)+A10**2*W(I,
     1J+1)-A2**2*W(I,J+2))
      GO TO 13
   19 CONTINUE
      TZXF(L)=(GF/(A3*A11*(A11-A3)))*((A11**2-A3**2)*W(I,J)-A11**2*W(I-1
     1,J)+A3**2*W(I-2,J))
      TZYF(L)=(GF/(A4*A12*(A12-A4)))*((A12**2-A4**2)*W(I,J)-A12**2*W(I,
     1J-1)+A4**2*W(I,J-2))
      TZXM(L)=(GM/(A1*A9*(A9-A1)))*((A1**2-A9**2)*W(I,J)+A9**2*W(I+1,J)
     1-A1**2*W(I+2,J))
      TZYM(L)=(GM/(A2*A10*(A10-A2)))*((A2**2-A10**2)*W(I,J)+A10**2*W(I,
     1J+1)-A2**2*W(I,J+2))
      GO TO 13
   20 TZYF(L)=(GF/(A4*A12*(A12-A4)))*((A12**2-A4**2)*W(I,J)-A12**2*W(I,
     1J-1)+A4**2*(2.0*WSAVE(I,J)-W(I,J)))
      TZXF(L)=(GF/(A3*A11*(A11-A3)))*((A11**2-A3**2)*W(I,J)-A11**2*W(I-1
     1,J)+A3**2*W(I-2,J))
      TZXM(L)=(GM/(A1*A9*(A9-A1)))*((A1**2-A9**2)*W(I,J)+A9**2*W(I+1,J)
     1-A1**2*W(I+2,J))
      TZYM(L)=(GM/(A2*A10*(A10-A2)))*((A2**2-A10**2)*W(I,J)+A10**2*W(I,
     1J+1)-A2**2*W(I,J+2))
      GO TO 13
   13 CONTINUE
      L=1
      I=3
      J=IN
      A1=HX(I)
      A2=HY(J)
      A3=HX(I-1)
      A4=HY(J-1)
      A9=HX(I)+HX(I+1)
      A10=HY(J)+HY(J+1)
      A11=HX(I-1)+HX(I-2)
      A12=HY(J-1)+HY(J-2)
   17 TZXF(L)=0.0
      TZYF(L)=(GF/(A4*A12*(A12-A4)))*((A12**2-A4**2)*W(I,J)-A12**2*W(I,
     1J-1)+A4**2*W(I,J-2))
```

FORTRAN 4 PROGRAM STRLS

```
      TZXM(L)=(GM/(A1*A9*(A9-A1)))*((A1**2-A9**2)*W(I,J)+A9**2*W(I+1,J)
     1-A1**2*W(I+2,J))
      TZYM(L)=(GM/(A2*A10*(A10-A2)))*((A2**2-A10**2)*W(I,J)+A10**2*W(I,
     1J+1)-A2**2*W(I,J+2))
      L=NL
      I=IM
      J=3
      A1=HX(I)
      A2=HY(J)
      A3=HX(I-1)
      A4=HY(J-1)
      A9=HX(I)+HX(I+1)
      A10=HY(J)+HY(J+1)
      A11=HX(I-1)+HX(I-2)
      A12=HY(J-1)+HY(J-2)
   21 TZYF(L)=0.0
      TZXF(L)=(GF/(A3*A11*(A11-A3)))*((A11**2-A3**2)*W(I,J)-A11**2*W(I-1
     1,J)+A3**2*W(I-2,J))
      TZXM(L)=(GM/(A1*A9*(A9-A1)))*((A1**2-A9**2)*W(I,J)+A9**2*W(I+1,J)
     1-A1**2*W(I+2,J))
      TZYM(L)=(GM/(A2*A10*(A10-A2)))*((A2**2-A10**2)*W(I,J)+A10**2*W(I,
     1J+1)-A2**2*W(I,J+2))
      DO 37 L=1,NL
      I=LI(L)
      J=LJ(L)
      TZXYF(L)=SQRT(TZXF(L)**2+TZYF(L)**2)
   37 TZXYM(L)=SQRT(TZXM(L)**2+TZYM(L)**2)
C
C     STRESSES AT RECTANGULAR BOUNDARIES
C
      A1=HX(3)
      A9=HX(4)+A1
      DO 35 J=3,INM2
   35 TZX(3,J)=(GF/(A1*A9*(A9-A1)))*((A1**2-A9**2)*W(3,J)+A9**2*W(4,J)
     1-A1**2*W(5,J))
      DO 23 J=INP1,N
   23 TZX(3,J)=(GM/(A1*A9*(A9-A1)))*((A1**2-A9**2)*W(3,J)+A9**2*W(4,J)
     1-A1**2*W(5,J))
      TZX(3,INM1)=(GF*WSAVE(4,INM1))/HX(3)
      DO 24 J=4,INM1
      A2=HY(J)
      A4=HY(J-1)
   24 TZY(3,J)=(GF/(A2*A4*(A2+A4)))*( A4**2*W(3,J+1)+(A2**2-A4**2)*W(3,
     1J)-A2**2*W(3,J-1))
      DO 25 J=INP1,NM1
      A2=HY(J)
      A4=HY(J-1)
   25 TZY(3,J)=(GM/(A2*A4*(A2+A4)))*( A4**2*W(3,J+1)+(A2**2-A4**2)*W(3,
     1J)-A2**2*W(3,J-1))
      A3=HX(MM1)
      A11=HX(MM2)+A3
      DO 26 J=3,N
   26 TZX(M,J)=(GM/(A3*A11*(A11-A3)))*((A11**2-A3**2)*W(M,J)-A11**2*W(
     1MM1,J)+A3**2*W(MM2,J))
      DO 27 J=4,NM1
      A2=HY(J)
      A4=HY(J-1)
```

FORTRAN 4 PROGRAM STRLS

```
   27 TZY(M,J)=(GM/(A2*A4*(A2+A4)))*(A4**2*W(M,J+1)+(A2**2-A4**2)*W(M,J)
     1-A2**2*W(M,J-1))
      DO 28 I=4,IMM1
      A1=HX(I)
      A3=HX(I-1)
   28 TZX(I,3)=(GF/(A1*A3*(A1+A3)))*(A3**2*W(I+1,3)+(A1**2-A3**2)*W(I,3)
     1-A1**2*W(I-1,3))
      DO 29 I=IMP1,MM1
      A1=HX(I)
      A3=HX(I-1)
   29 TZX(I,3)=(GM/(A1*A3*(A1+A3)))*(A3**2*W(I+1,3)+(A1**2-A3**2)*W(I,3)
     1-A1**2*W(I-1,3))
      A2=HY(3)
      A10=HY(4)+A2
      DO 30 I=3,IMM2
   30 TZY(I,3)=(GF/(A2*A10*(A10-A2)))*((A2**2-A10**2)*W(I,3)+A10**2*W(I,
     14)-A2**2*W(I,5))
      DO 31 I=IMP1,M
   31 TZY(I,3)=(GM/(A2*A10*(A10-A2)))*((A2**2-A10**2)*W(I,3)+A10**2*W(I,
     14)-A2**2*W(I,5))
      TZY(IMM1,3)=(GF*(W(IMM1,4)-WSAVE(IMM1,4)))/HY(3)
      DO 32 I=4,MM1
      A1=HX(I)
      A3=HX(I-1)
   32 TZX(I,N)=(GM/(A1*A3*(A1+A3)))*(A3**2*W(I+1,N)+(A1**2-A3**2)*W(I,N)
     1-A1**2*W(I-1,N))
      A4=HY(NM1)
      A12=HY(NM2)+A4
      DO 33 I=3,M
   33 TZY(I,N)=(GM/(A4*A12*(A12-A4)))*((A12**2-A4**2)*W(I,N)-A12**2*W(I,
     1NM1)+A4**2*W(I,NM2))
      DO 34 L=1,NL
      I=LI(L)
      J=LJ(L)
      WL(L)=W(I,J)
      TZX(I,J)=0.0
   34 TZY(I,J)=0.0
      DO 36 I=3,M
      DO 36 J=3,N
   36 TZXY(I,J)=SQRT(TZX(I,J)**2+TZY(I,J)**2)
      WRITE (5,100)(((I,J,W(I,J),TZX(I,J),TZY(I,J),TZXY(I,J)),J=3,N),I=3
     1,M)
  100 FORMAT (1H1,45X,30HINTERIOR AND BOUNDARY STRESSES,/////,
     16X,1HI,3X,1HJ,18X,1HW,22X,3HTZX,17X,3HTZY,10X,17HTZXY (RESULTANT),
     2////,(3X,2I4,6X,F20.8,3F20.3))
      WRITE (5,101) ((LI(L),LJ(L),TZXM(L),TZYM(L),TZXYM(L),TZXF(L),TZYF
     1(L),TZXYF(L)),L=1,NL)
  101 FORMAT (1H1,51X,18HINTERFACE STRESSES,/////,
     136X,9HIN MATRIX,40X,12HIN INCLUSION,//,
     26X,1HI,3X,1HJ,11X,3HTZX,14X,3HTZY,11X,9HRESULTANT,11X,3HTZX,14X,
     33HTZY,11X,9HRESULTANT,///,
     4(3X,2I4,6F17.3))
      WRITE (5,102) GX,GY
  102 FORMAT (1H ,//,34H EFFECTIVE COMPOSITE SHEAR MODULUS,//,4H GX=,
     11E20.5,//,4H GY=,1E20.5)
      RETURN
      END
```

COMPUTER OUTPUT SAMPLE PROBLEM

LONGITUDINAL SHEAR ANALYSIS

ELLIPTICAL INCLUSION

INPUT DATA

```
GRID NODE ARRAY SIZE                              =15 BY 15
QUADRANT DIMENSIONS        A = 0.519       B = 1.000
MATRIX SHEAR MODULUS       PSI                 =  0.2000+006
INCLUSION SHEAR MODULUS    PSI                 =  0.4000+007
RELAXATION FACTOR   (OMEGA BAR)                = 1.750
AVERAGE ZX SHEAR LOADING AT INFINITY (PSI) =  1000.00
AVERAGE ZY SHEAR LOADING AT INFINITY (PSI) =      0.
PERCENT FIBER BY VOLUME                        =    70.00
```

TEST POINT COORDINATES

I	J
4	13
4	12
4	11
7	11
8	8
9	9
11	7
11	4
12	4
13	4

GRID SPACING

I	HX(I)
3	0.05746400
4	0.04222610
5	0.04069630
6	0.06475590
7	0.13994220
8	0.07991550
9	0.04000000
10	0.01594530
11	0.00635140
12	0.00635160
13	0.00635160
14	0.00635160
15	0.00635160
16	0.00635160

J	HY(J)
3	0.24562860
4	0.20463070
5	0.21974070
6	0.20000000
7	0.05000000
8	0.02100000
9	0.01400000
10	0.00689060
11	0.00635140
12	0.00635160
13	0.00635160
14	0.00635160
15	0.00635160
16	0.00635160

RESULTS OF RESID NO. 1 PROBLEM NO. 1

I	J	W	RESIDUAL
3	3	0.	0.
3	4	0.	0.
3	5	0.	0.
3	6	0.	0.
3	7	0.	0.
3	8	0.	0.
3	9	0.	0.
3	10	0.	0.
3	11	0.	0.
3	12	0.	0.
3	13	0.	0.
3	14	0.	0.
3	15	0.	0.
3	16	0.	0.
3	17	0.	0.
4	3	0.	0.
4	4	0.	0.
4	5	0.	0.
4	6	0.	0.
4	7	0.	0.
4	8	0.	0.
4	9	0.	0.
4	10	0.	0.
4	11	0.	0.
4	12	0.	0.
4	13	0.	0.
4	14	0.	0.
4	15	0.	0.
4	16	0.	0.
4	17	0.	0.
5	3	0.	0.
5	4	0.	0.
5	5	0.	0.
5	6	0.	0.
5	7	0.	0.
5	8	0.	0.
5	9	0.	0.
5	10	0.	0.
5	11	0.	0.
5	12	0.	0.
5	13	0.	0.
5	14	0.	0.
5	15	0.	0.
5	16	0.	0.
5	17	0.	0.
6	3	0.	0.
6	4	0.	0.
6	5	0.	0.
6	6	0.	0.
6	7	0.	0.
6	8	0.	0.
6	9	0.	0.
6	10	0.	0.
6	11	0.	0.
6	12	0.	0.
6	13	0.	0.
6	14	0.	0.
6	15	0.	0.
6	16	0.	0.
6	17	0.	0.
7	3	0.	0.
7	4	0.	0.
7	5	0.	0.
7	6	0.	0.
7	7	0.	0.
7	8	0.	0.
7	9	0.	0.
7	10	0.	0.
7	11	0.	0.
7	12	0.	0.
7	13	0.	0.
7	14	0.	0.
7	15	0.	0.
7	16	0.	0.
7	17	0.	0.
8	3	0.	0.
8	4	0.	0.
8	5	0.	0.
8	6	0.	0.
8	7	0.	0.
8	8	0.	0.
8	9	0.	0.
8	10	0.	0.
8	11	0.	0.
8	12	0.	0.
8	13	0.	0.
8	14	0.	0.
8	15	0.	0.
8	16	0.	0.
8	17	0.	0.
9	3	0.	0.
9	4	0.	0.
9	5	0.	0.
9	6	0.	0.
9	7	0.	0.
9	8	0.	0.
9	9	0.	0.
9	10	0.	0.
9	11	0.	0.
9	12	0.	0.
9	13	0.	0.
9	14	0.	0.
9	15	0.	0.
9	16	0.	0.
9	17	0.	0.
10	3	0.	0.
10	4	0.	0.
10	5	0.	0.
10	6	0.	0.
10	7	0.	0.
10	8	0.	0.
10	9	0.	0.
10	10	0.	0.
10	11	0.	0.
10	12	0.	0.
10	13	0.	0.
10	14	0.	0.
10	15	0.	0.

10	16	0.	0.
10	17	0.	0.
11	3	0.	0.
11	4	0.	0.
11	5	0.	0.
11	6	0.	0.
11	7	0.	0.
11	8	0.	0.
11	9	0.	0.
11	10	0.	0.
11	11	0.	0.
11	12	0.	0.
11	13	0.	0.
11	14	0.	0.
11	15	0.	0.
11	16	0.	0.
11	17	0.	0.
12	3	0.	0.
12	4	0.	0.
12	5	0.	0.
12	6	0.	0.
12	7	0.	0.
12	8	0.	0.
12	9	0.	0.
12	10	0.	0.
12	11	0.	0.
12	12	0.	0.
12	13	0.	0.
12	14	0.	0.
12	15	0.	0.
12	16	0.	0.
12	17	0.	0.
13	3	0.	0.
13	4	0.	0.
13	5	0.	0.
13	6	0.	0.
13	7	0.	0.
13	8	0.	0.
13	9	0.	0.
13	10	0.	0.
13	11	0.	0.
13	12	0.	0.
13	13	0.	0.
13	14	0.	0.
13	15	0.	0.
13	16	0.	0.
13	17	0.	0.
14	3	0.	0.
14	4	0.	0.
14	5	0.	0.
14	6	0.	0.
14	7	0.	0.
14	8	0.	0.
14	9	0.	0.
14	10	0.	0.
14	11	0.	0.
14	12	0.	0.
14	13	0.	0.
14	14	0.	0.
14	15	0.	0.
14	16	0.	0.
14	17	0.	0.
15	3	0.	0.
15	4	0.	0.
15	5	0.	0.
15	6	0.	0.
15	7	0.	0.
15	8	0.	0.
15	9	0.	0.
15	10	0.	0.
15	11	0.	0.
15	12	0.	0.
15	13	0.	0.
15	14	0.	0.
15	15	0.	0.
15	16	0.	0.
15	17	0.	0.
16	3	0.	0.
16	4	0.	0.49575113+010
16	5	0.	0.49575113+010
16	6	0.	0.49575113+010
16	7	0.	0.49575113+010
16	8	0.	0.49575113+010
16	9	0.	0.49575113+010
16	10	0.	0.49575113+010
16	11	0.	0.49575113+010
16	12	0.	0.49575113+010
16	13	0.	0.49575113+010
16	14	0.	0.49575113+010
16	15	0.	0.49575113+010
16	16	0.	0.49575113+010
16	17	0.	0.
17	3	0.10000000+001	0.
17	4	0.10000000+001	0.
17	5	0.10000000+001	0.
17	6	0.10000000+001	0.
17	7	0.10000000+001	0.
17	8	0.10000000+001	0.
17	9	0.10000000+001	0.
17	10	0.10000000+001	0.
17	11	0.10000000+001	0.
17	12	0.10000000+001	0.
17	13	0.10000000+001	0.
17	14	0.10000000+001	0.
17	15	0.10000000+001	0.
17	16	0.10000000+001	0.
17	17	0.10000000+001	0.

RESULTS OF RELAX NO. 116 PROBLEM NO. 1

I	J	W	RESIDUAL
3	3	0.	0.
3	4	0.	0.
3	5	0.	0.
3	6	0.	0.
3	7	0.	0.
3	8	0.	0.
3	9	0.	0.
3	10	0.	0.
3	11	0.	0.
3	12	0.	0.
3	13	0.	0.
3	14	0.	0.
3	15	0.	0.
3	16	0.	0.
3	17	0.	0.
4	3	0.36208566-001	-0.19016103+000
4	4	0.34268606-001	-0.90714801+002
4	5	0.29689896-001	-0.24172082+003
4	6	0.23298788-001	-0.75809827+003
4	7	0.17037325-001	-0.25379690+004
4	8	0.15660726-001	-0.52682039+004
4	9	0.15110390-001	-0.62804017+004
4	10	0.14749971-001	-0.39981055+003
4	11	0.15627010-001	-0.35287063+004
4	12	0.16315535-001	-0.20371057+004
4	13	0.16883871-001	-0.10804659+004
4	14	0.17328051-001	-0.10698458+003
4	15	0.17645281-001	0.87293423+003
4	16	0.17833762-001	0.18491147+004
4	17	0.17892572-001	-0.18971826+003
5	3	0.62980087-001	-0.34008831+000
5	4	0.59585409-001	-0.76949124+002
5	5	0.51573259-001	-0.19885271+003
5	6	0.40441258-001	-0.63014525+003
5	7	0.29491906-001	-0.25790069+004
5	8	0.27087291-001	-0.71266446+004
5	9	0.26155005-001	-0.42247939+003
5	10	0.31142670-001	-0.10581513+005
5	11	0.33073285-001	-0.56047732+004
5	12	0.34554900-001	-0.15810211+004
5	13	0.35756738-001	0.37849887+003
5	14	0.36683213-001	0.23352735+004
5	15	0.37337808-001	0.42775108+004
5	16	0.37723113-001	0.61931793+004
5	17	0.37840873-001	-0.50419339+003
6	3	0.89013255-001	-0.49461173+000
6	4	0.84174856-001	0.28904450+000
6	5	0.72755223-001	0.10033647+002
6	6	0.56995270-001	0.20455013+002
6	7	0.41404360-001	-0.10129771+004
6	8	0.37921124-001	-0.23309355+003
6	9	0.52259369-001	-0.67899667+004
6	10	0.59142203-001	-0.18339092+005
6	11	0.61794159-001	-0.36941880+004
6	12	0.63823838-001	0.42182179+004
6	13	0.65465980-001	0.70371000+004
6	14	0.66728046-001	0.98747623+004
6	15	0.67615949-001	0.12708502+005
6	16	0.68134054-001	0.15516982+005
6	17	0.68285232-001	-0.10166314+004
7	3	0.13110833+000	-0.75593553+000
7	4	0.12384917+000	0.43176683+001
7	5	0.10671603+000	0.20715837+002
7	6	0.83437427-001	0.53993150+002
7	7	0.60132144-001	-0.13080863+003
7	8	0.11601337+000	-0.48456575+004
7	9	0.13097342+000	-0.88531383+004
7	10	0.13828109+000	-0.34359582+005
7	11	0.14110646+000	0.34842340+004
7	12	0.14327289+000	0.19786154+005
7	13	0.14502630+000	0.23871410+005
7	14	0.14637043+000	0.28023176+005
7	15	0.14730846+000	0.32209330+005
7	16	0.14784290+000	0.36401885+005
7	17	0.14797557+000	-0.21481407+004
8	3	0.22635050+000	-0.14155845+001
8	4	0.21303483+000	-0.33884774+002
8	5	0.18160707+000	-0.51851987+002
8	6	0.14130375+000	-0.50531254+002
8	7	0.41712722+000	-0.11586234+004
8	8	0.44792948+000	-0.76245722+004
8	9	0.45628547+000	-0.13857875+005
8	10	0.46034622+000	-0.61531541+005
8	11	0.46188455+000	0.12636901+005
8	12	0.46304913+000	0.42418034+005
8	13	0.46397664+000	0.48707805+005
8	14	0.46466821+000	0.55102844+005
8	15	0.46512498+000	0.61564311+005
8	16	0.46534808+000	0.68062171+005
8	17	0.46533857+000	-0.39616282+004
9	3	0.28573689+000	-0.17886113+001
9	4	0.26784290+000	-0.65933902+002
9	5	0.22560933+000	-0.21344821+002
9	6	0.52636084+000	-0.92813474+002
9	7	0.67457103+000	-0.44787023+003
9	8	0.69112120+000	-0.48521925+004
9	9	0.69559076+000	-0.91599713+004
9	10	0.69774688+000	-0.43712619+005
9	11	0.69855303+000	0.10486761+005
9	12	0.69915820+000	0.32184964+005
9	13	0.69963523+000	0.36760141+005
9	14	0.69998477+000	0.41408742+005
9	15	0.70020752+000	0.46103390+005
9	16	0.70030417+000	0.50823206+005
9	17	0.70027542+000	-0.28794598+004
10	3	0.32892397+000	0.
10	4	0.29669847+000	-0.17424400+002
10	5	0.55302546+000	-0.18090677+001
10	6	0.72626395+000	-0.57875513+002
10	7	0.81129105+000	-0.24640315+003
10	8	0.82077058+000	-0.30741485+004
10	9	0.82332221+000	-0.58488018+004
10	10	0.82454772+000	-0.28313580+005
10	11	0.82500257+000	0.69774787+004
10	12	0.82534239+000	0.21097596+005
10	13	0.82560867+000	0.24073387+005
10	14	0.82580182+000	0.27096554+005
10	15	0.82592229+000	0.30149420+005

10	16	0.82597050+000	0.33218534+005
10	17	0.82594692+000	-0.18731457+004
11	3	0.35310588+000	-0.28880537+002
11	4	0.50458249+000	-0.33450567+002
11	5	0.68452006+000	-0.52113784+002
11	6	0.80672369+000	-0.38039565+003
11	7	0.86663202+000	-0.32219081+004
11	8	0.87330162+000	-0.75538818+004
11	9	0.87509286+000	-0.10943353+005
11	10	0.87595073+000	-0.28945519+005
11	11	0.87626765+000	-0.29203647+004
11	12	0.87650368+000	0.71488064+004
11	13	0.87668793+000	0.88110331+004
11	14	0.87682070+000	0.10523028+005
11	15	0.87690232+000	0.12274960+005
11	16	0.87693312+000	0.14060280+005
11	17	0.87691343+000	-0.14147285+004
12	3	0.52387875+000	-0.74803505-001
12	4	0.58730513+000	0.23851251+003
12	5	0.73700539+000	0.29706352+003
12	6	0.83885919+000	0.18864766+004
12	7	0.88876865+000	0.18018257+005
12	8	0.89431926+000	0.29405694+005
12	9	0.89580768+000	0.35380183+005
12	10	0.89651917+000	0.26843885+005
12	11	0.89678117+000	0.53831523+005
12	12	0.89697590+000	0.66678171+005
12	13	0.89712751+000	0.72137424+005
12	14	0.89723626+000	0.77542827+005
12	15	0.89730245+000	0.82861786+005
12	16	0.89732635+000	0.88063934+005
12	17	0.89730826+000	-0.12307532+004
13	3	0.61927331+000	-0.61755891-001
13	4	0.66994560+000	0.23149017+003
13	5	0.78954340+000	0.29285538+003
13	6	0.87103721+000	0.18796730+004
13	7	0.91095432+000	0.17706670+005
13	8	0.91538910+000	0.29135852+005
13	9	0.91657654+000	0.35259817+005
13	10	0.91714308+000	0.29073410+005
13	11	0.91735107+000	0.52100454+005
13	12	0.91750534+000	0.63322388+005
13	13	0.91762513+000	0.68385388+005
13	14	0.91771068+000	0.73391171+005
13	15	0.91776221+000	0.78309523+005
13	16	0.91777997+000	0.83112004+005
13	17	0.91776420+000	-0.10245378+004
14	3	0.71455423+000	-0.47727077-001
14	4	0.75251835+000	0.22236501+003
14	5	0.84212207+000	0.28495386+003
14	6	0.90324854+000	0.18447315+004
14	7	0.93317936+000	0.17192987+005
14	8	0.93650136+000	0.28554227+005
14	9	0.93738956+000	0.34774865+005
14	10	0.93781258+000	0.31126281+005
14	11	0.93796741+000	0.49749999+005
14	12	0.93808202+000	0.59147953+005
14	13	0.93817080+000	0.63742845+005
14	14	0.93823392+000	0.68277502+005
14	15	0.93827155+000	0.72724510+005
14	16	0.93828390+000	0.77057724+005
14	17	0.93827114+000	-0.79702023+003
15	3	0.80975003+000	-0.32744989-001
15	4	0.83504037+000	0.21106688+003
15	5	0.89473116+000	0.27323444+003
15	6	0.93548494+000	0.17807532+004
15	7	0.95543472+000	0.16470702+005
15	8	0.95764681+000	0.27647971+005
15	9	0.95823745+000	0.33908025+005
15	10	0.95851826+000	0.32968616+005
15	11	0.95862076+000	0.46767701+005
15	12	0.95869648+000	0.54150192+005
15	13	0.95875498+000	0.58205757+005
15	14	0.95879640+000	0.62198473+005
15	15	0.95882085+000	0.66104137+005
15	16	0.95882847+000	0.69899287+005
15	17	0.95881937+000	-0.54939997+003
16	3	0.90488917+000	-0.16826053-001
16	4	0.91752862+000	0.19759641+003
16	5	0.94736051+000	0.25762646+003
16	6	0.96773818+000	0.16873119+004
16	7	0.97771131+000	0.15536588+005
16	8	0.97881616+000	0.26409377+005
16	9	0.97911078+000	0.32648111+005
16	10	0.97925062+000	0.34570496+005
16	11	0.97930153+000	0.43150812+005
16	12	0.97933906+000	0.48336808+005
16	13	0.97936800+000	0.51783499+005
16	14	0.97938840+000	0.55165178+005
16	15	0.97940033+000	0.58461263+005
16	16	0.97940385+000	0.61651336+005
16	17	0.97939902+000	-0.28314684+003
17	3	0.10000000+001	0.
17	4	0.10000000+001	0.
17	5	0.10000000+001	0.
17	6	0.10000000+001	0.
17	7	0.10000000+001	0.
17	8	0.10000000+001	0.
17	9	0.10000000+001	0.
17	10	0.10000000+001	0.
17	11	0.10000000+001	0.
17	12	0.10000000+001	0.
17	13	0.10000000+001	0.
17	14	0.10000000+001	0.
17	15	0.10000000+001	0.
17	16	0.10000000+001	0.
17	17	0.10000000+001	0.

0 TEST POINTS HAVE NOT YET CONVERGED TO THE SPECIFIED MINIMUM CHANGE IN STRESS PER RELAX OF 0.050PERCENT

INTERIOR AND BOUNDARY STRESSES

I	J	W	TZX	TZY	TZXY (RESULTANT),
3	3	0.	1500.195	0.	1500.195
3	4	0.	1420.483	0.	1420.483
3	5	0.	1232.346	0.	1232.346
3	6	0.	968.055	0.	968.055
3	7	0.	710.533	0.	710.533
3	8	0.	653.830	0.	653.830
3	9	0.	630.215	0.	630.215
3	10	0.	613.307	0.	613.307
3	11	0.	0.	0.	0.
3	12	0.	23.727	0.	23.727
3	13	0.	24.556	0.	24.556
3	14	0.	25.226	0.	25.226
3	15	0.	25.715	0.	25.715
3	16	0.	26.012	0.	26.012
3	17	0.	26.108	0.	26.108
4	3	0.21628875-004	1510.925	-0.000	1510.925
4	4	0.20470056-004	1429.309	-37.742	1429.807
4	5	0.17735003-004	1236.679	-61.193	1238.192
4	6	0.13917330-004	969.483	-72.274	972.173
4	7	0.10177099-004	706.299	-67.588	709.525
4	8	0.93547995-005	648.523	-63.554	651.629
4	9	0.90260610-005	626.372	-61.954	629.428
4	10	0.88107682-005	0.	0.	0.
4	11	0.93346596-005	42.214	14.033	44.485
4	12	0.97459441-005	44.113	11.820	45.670
4	13	0.10085435-004	45.647	9.522	46.630
4	14	0.10350762-004	46.825	7.161	47.349
4	15	0.10540256-004	47.654	4.756	47.891
4	16	0.10652844-004	48.141	2.326	48.197
4	17	0.10687974-004	48.289	-0.113	48.289
5	3	0.37620612-004	1521.790	-0.000	1521.790
5	4	0.35592830-004	1438.228	-66.044	1439.744
5	5	0.30806841-004	1241.004	-106.810	1245.592
5	6	0.24157236-004	970.982	-126.157	979.143
5	7	0.17616736-004	702.025	-118.090	711.888
5	8	0.16180359-004	641.228	-108.688	650.374
5	9	0.15623467-004	0.	0.	0.
5	10	0.18602806-004	64.618	36.471	74.199
5	11	0.19756041-004	67.159	30.557	73.794
5	12	0.20641072-004	69.079	25.237	73.545
5	13	0.21358979-004	70.617	20.016	73.399
5	14	0.21912401-004	71.789	14.869	73.312
5	15	0.22303418-004	72.606	9.780	73.262
5	16	0.22533576-004	73.078	4.731	73.231
5	17	0.22603919-004	73.209	-0.301	73.210
6	3	0.53171300-004	1538.019	-0.000	1538.019
6	4	0.50281124-004	1451.496	-94.131	1454.545
6	5	0.43459705-004	1247.284	-151.677	1256.473
6	6	0.34045633-004	973.365	-179.165	989.717
6	7	0.24732538-004	696.169	-170.416	716.723
6	8	0.22651856-004	0.	0.	0.
6	9	0.31216683-004	103.102	67.869	123.434
6	10	0.35328085-004	106.820	50.186	118.022
6	11	0.36912208-004	108.244	41.920	116.078
6	12	0.38124619-004	109.330	34.533	114.654
6	13	0.39105539-004	110.203	27.313	113.537
6	14	0.39859423-004	110.866	20.220	112.695
6	15	0.40389804-004	111.322	13.223	112.105
6	16	0.40699290-004	111.573	6.294	111.751
6	17	0.40789594-004	111.620	-0.607	111.622
7	3	0.78316430-004	1576.298	-0.000	1576.298
7	4	0.73980232-004	1482.520	-141.227	1489.232
7	5	0.63745901-004	1261.185	-225.643	1281.211
7	6	0.49840627-004	979.568	-246.368	1015.178
7	7	0.35919418-004	0.	0.	0.
7	8	0.69299588-004	188.135	99.427	212.792
7	9	0.78235842-004	187.135	71.459	200.315
7	10	0.82601014-004	186.794	53.397	194.276
7	11	0.84288725-004	186.665	44.700	191.943
7	12	0.85582826-004	186.567	36.865	190.175
7	13	0.86630208-004	186.485	29.131	188.746
7	14	0.87433114-004	186.412	21.463	187.643
7	15	0.87993440-004	186.345	13.848	186.859
7	16	0.88312682-004	186.282	6.274	186.387
7	17	0.88391928-004	186.220	-1.283	186.224
8	3	0.13520852-003	1721.263	-0.001	1721.263
8	4	0.12725452-003	1596.546	-259.057	1617.427
8	5	0.10848142-003	1302.187	-401.334	1362.630
8	6	0.84406579-004	0.	0.	0.
8	7	0.24916735-003	355.747	91.830	367.409
8	8	0.26756681-003	334.404	55.245	338.936
8	9	0.27255819-003	328.656	39.806	331.058
8	10	0.27498384-003	325.836	29.304	327.152
8	11	0.27590275-003	324.740	24.191	325.640
8	12	0.27659841-003	323.897	19.676	324.494
8	13	0.27715245-003	323.212	15.227	323.571
8	14	0.27756555-003	322.684	10.800	322.865
8	15	0.27783840-003	322.312	6.394	322.376
8	16	0.27797166-003	322.095	2.009	322.102
8	17	0.27796598-003	322.033	-2.366	322.042
9	3	0.17068247-003	2311.497	-0.001	2311.497
9	4	0.15999365-003	1695.317	-348.128	1730.692
9	5	0.13476579-003	0.	0.	0.
9	6	0.31441711-003	589.908	124.259	602.853
9	7	0.40294919-003	400.510	49.342	403.538
9	8	0.41283529-003	379.329	29.603	380.483
9	9	0.41550514-003	373.574	21.210	374.175
9	10	0.41679308-003	370.772	15.436	371.093
9	11	0.41727463-003	369.707	12.627	369.923
9	12	0.41763612-003	368.900	10.178	369.041
9	13	0.41792107-003	368.256	7.774	368.338
9	14	0.41812987-003	367.774	5.382	367.814
9	15	0.41826292-003	367.454	3.004	367.466
9	16	0.41832066-003	367.295	0.639	367.295
9	17	0.41830348-003	367.295	-1.720	367.299
10	3	0.19647990-003	3326.086	0.	3326.086
10	4	0.17723027-003	0.	0.	0.
10	5	0.33034499-003	983.124	122.905	990.777
10	6	0.43382751-003	601.187	71.468	605.420
10	7	0.48461771-003	412.842	28.278	413.809
10	8	0.49028023-003	391.770	16.922	392.136
10	9	0.49180442-003	386.064	12.081	386.253
10	10	0.49253647-003	383.303	8.734	383.402
10	11	0.49280817-003	382.265	7.109	382.331
10	12	0.49301116-003	381.483	5.700	381.526

10	13	0.49317022-003	380.864	4.321	380.889
10	14	0.49328560-003	380.408	2.949	380.419
10	15	0.49335756-003	380.112	1.586	380.116
10	16	0.49338636-003	379.978	0.232	379.978
10	17	0.49337227-003	380.003	-1.119	380.005
11	3	0.21092475-003	0.	0.	0.
11	4	0.30140800-003	1556.435	90.792	1559.081
11	5	0.40889216-003	986.658	86.433	990.437
11	6	0.48188944-003	603.998	50.392	606.096
11	7	0.51767516-003	415.886	19.906	416.363
11	8	0.52165919-003	394.837	11.890	395.016
11	9	0.52272917-003	389.141	8.469	389.233
11	10	0.52324162-003	386.387	6.097	386.435
11	11	0.52343092-003	385.354	4.946	385.386
11	12	0.52357191-003	384.577	3.953	384.597
11	13	0.52368197-003	383.963	2.981	383.974
11	14	0.52376128-003	383.511	2.016	383.516
11	15	0.52381004-003	383.220	1.057	383.222
11	16	0.52382843-003	383.090	0.104	383.090
11	17	0.52381667-003	383.120	-0.845	383.121
12	3	0.31293445-003	2503.255	-0.000	2503.255
12	4	0.35082165-003	1555.195	61.698	1556.419
12	5	0.44024381-003	987.716	71.957	990.334
12	6	0.50108530-003	604.851	41.993	606.307
12	7	0.53089829-003	416.839	16.573	417.169
12	8	0.53421390-003	395.821	9.886	395.945
12	9	0.53510299-003	390.143	7.030	390.206
12	10	0.53552799-003	387.403	5.047	387.436
12	11	0.53568450-003	386.378	4.085	386.400
12	12	0.53580082-003	385.609	3.257	385.623
12	13	0.53589138-003	385.003	2.449	385.011
12	14	0.53595634-003	384.559	1.645	384.563
12	15	0.53599588-003	384.276	0.847	384.277
12	16	0.53601015-003	384.154	0.055	384.154
12	17	0.53599935-003	384.190	-0.735	384.191
13	3	0.36991757-003	1793.223	-0.000	1793.223
13	4	0.40018623-003	1553.761	49.292	1554.543
13	5	0.47162694-003	988.578	57.520	990.250
13	6	0.52030657-003	605.555	33.594	606.486
13	7	0.54415071-003	417.664	13.246	417.874
13	8	0.54679978-003	396.705	7.891	396.783
13	9	0.54750909-003	391.060	5.603	391.100
13	10	0.54784751-003	388.347	4.011	388.368
13	11	0.54797175-003	387.339	3.240	387.353
13	12	0.54806390-003	386.586	2.577	386.594
13	13	0.54813546-003	385.995	1.931	386.000
13	14	0.54818656-003	385.566	1.289	385.568
13	15	0.54821734-003	385.297	0.652	385.298
13	16	0.54822795-003	385.189	0.019	385.189
13	17	0.54821853-003	385.239	-0.612	385.239
14	3	0.42683280-003	1791.354	-0.000	1791.354
14	4	0.44951035-003	1552.647	36.930	1553.086
14	5	0.50303436-003	989.247	43.113	990.186
14	6	0.53954772-003	606.104	25.195	606.627
14	7	0.55742664-003	418.319	9.926	418.437
14	8	0.55941101-003	397.416	5.906	397.460
14	9	0.55994157-003	391.803	4.187	391.826
14	10	0.56019425-003	389.116	2.990	389.128
14	11	0.56028675-003	388.124	2.439	388.132
14	12	0.56035521-003	387.385	1.913	387.390
14	13	0.56040824-003	386.809	1.428	386.812
14	14	0.56044594-003	386.394	0.948	386.395

14	15	0.56046842-003	386.139	0.470	386.140
14	16	0.56047579-003	386.044	-0.004	386.044
14	17	0.56046817-003	386.107	-0.476	386.107
15	3	0.48369720-003	1790.020	-0.000	1790.020
15	4	0.49880417-003	1551.852	24.601	1552.047
15	5	0.53445994-003	989.723	28.729	990.140
15	6	0.55880386-003	606.498	16.796	606.731
15	7	0.57072069-003	418.804	6.612	418.857
15	8	0.57204206-003	397.953	3.930	397.972
15	9	0.57239488-003	392.371	2.782	392.380
15	10	0.57256262-003	389.707	1.981	389.712
15	11	0.57262384-003	388.730	1.593	388.733
15	12	0.57266907-003	388.005	1.262	388.007
15	13	0.57270402-003	387.442	0.941	387.444
15	14	0.57272876-003	387.041	0.619	387.041
15	15	0.57274337-003	386.799	0.302	386.799
15	16	0.57274792-003	386.716	-0.014	386.716
15	17	0.57274248-003	386.791	-0.328	386.791
16	3	0.54052774-003	1789.221	-0.000	1789.221
16	4	0.54807781-003	1551.376	12.295	1551.425
16	5	0.56589763-003	990.009	14.361	990.113
16	6	0.57807006-003	606.737	8.398	606.795
16	7	0.58402742-003	419.118	3.303	419.131
16	8	0.58468739-003	398.314	1.961	398.319
16	9	0.58486338-003	392.759	1.386	392.762
16	10	0.58494691-003	390.118	0.985	390.120
16	11	0.58497732-003	389.154	0.791	389.155
16	12	0.58499974-003	388.442	0.625	388.443
16	13	0.58501703-003	387.892	0.464	387.892
16	14	0.58502922-003	387.503	0.304	387.503
16	15	0.58503634-003	387.273	0.145	387.273
16	16	0.58503844-003	387.201	-0.012	387.201
16	17	0.58503556-003	387.287	-0.169	387.287
17	3	0.59734138-003	1788.689	0.000	1788.689
17	4	0.59734138-003	1551.059	-0.000	1551.059
17	5	0.59734138-003	990.200	0.	990.200
17	6	0.59734138-003	606.898	0.000	606.898
17	7	0.59734138-003	419.345	-0.000	419.345
17	8	0.59734138-003	398.587	-0.000	398.587
17	9	0.59734138-003	393.058	0.	393.058
17	10	0.59734138-003	390.438	0.000	390.438
17	11	0.59734138-003	389.487	-0.000	389.487
17	12	0.59734138-003	388.787	0.000	388.787
17	13	0.59734138-003	388.249	0.	388.249
17	14	0.59734138-003	387.871	0.	387.871
17	15	0.59734138-003	387.653	0.	387.653
17	16	0.59734138-003	387.592	0.	387.592
17	17	0.59734138-003	387.688	0.000	387.688

INTERFACE STRESSES

		IN MATRIX			IN INCLUSION		
I	J	TZX	TZY	RESULTANT	TZX	TZY	RESULTANT
3	11	22.764	0.	22.764	0.	0.	0.
4	10	28.140	15.379	32.560	613.307	-61.070	616.340
5	9	50.163	48.653	69.882	623.548	-103.462	632.073
6	8	100.010	95.271	138.126	631.926	-162.493	651.515
7	7	253.784	167.615	304.140	685.868	-290.481	744.845
8	6	561.361	237.691	609.609	996.445	-475.147	1103.933
9	5	972.668	202.765	993.578	1329.229	-638.150	1474.298
10	4	1558.658	175.394	1568.607	1752.007	-626.951	1860.806
11	3	3921.136	55.558	3921.544	3921.119	0.	3921.119

EFFECTIVE COMPOSITE SHEAR MODULUS

GX= 0.86894+006

GY= 0.

APPENDIX C

A RELAXATION METHOD OF SOLUTION OF THE TRANSVERSE NORMAL STRESS PROBLEM FOR A DOUBLY PERIODIC RECTANGULAR ARRAY OF ELASTIC INCLUSIONS IN AN INFINITE ELASTIC BODY

C.1 INTRODUCTION

The solution of the problem outlined in Section 4 has been formulated using a finite difference representation and a numerical relaxation procedure designed for high speed digital computer operation. The finite difference approximations of the partial derivatives contained in Equations (66), (67), and (68) make use of irregular grid spacings in both coordinate directions, as indicated in Figure C-1. This is an important feature of the solution in that it permits the use of close grid spacings in regions where it is desired to determine stresses very accurately, e.g., in areas of high stress concentration where stress gradients are high, while allowing a coarser spacing in less critical regions. This permits a given degree of accuracy with a minimum amount of numerical computation and computer storage capacity.

C.2 FINITE DIFFERENCE FORMS

The finite difference representations of the partial derivatives are of the following forms (where f represents either a u or a v displacement depending upon which derivative is being evaluated).

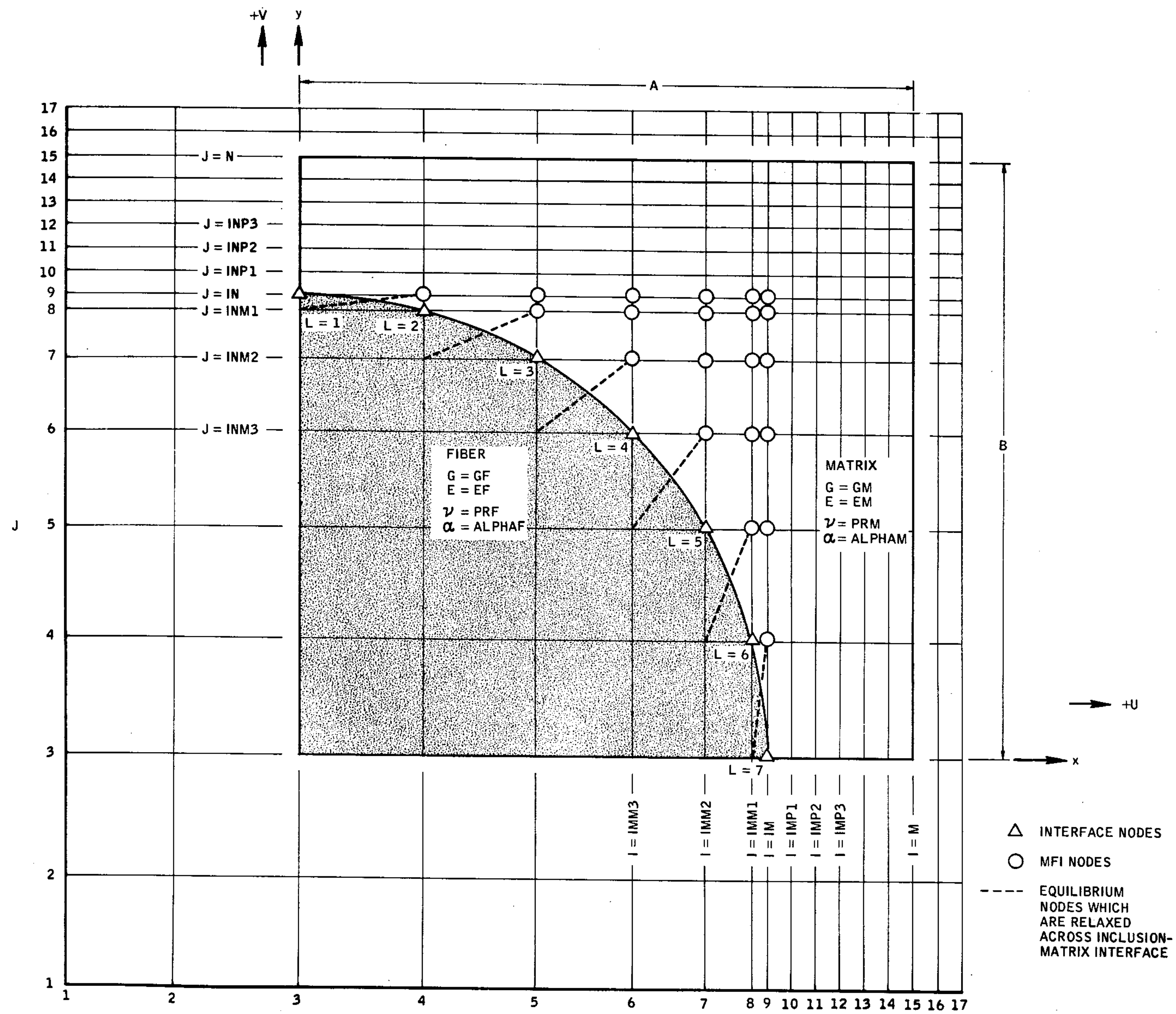


Figure C-1. First Quadrant of the Fundamental Region Showing Typical Grid Lines and Notation Used

(1) First irregular central differences

$$\left.\frac{\partial f}{\partial x}\right|_{i,j} = \frac{1}{a_1\, a_3\, (a_1 + a_3)} \left[a_3^2\, f_{i+1,j} + (a_1^2 - a_3^2)\, f_{i,j} - a_1^2\, f_{i-1,j} \right]$$

$$\left.\frac{\partial f}{\partial y}\right|_{i,j} = \frac{1}{a_2\, a_4\, (a_2 + a_4)} \left[a_4^2\, f_{i,j+1} + (a_2^2 - a_4^2)\, f_{i,j} - a_2^2\, f_{i,j-1} \right]$$

(2) Second irregular central differences

$$\left.\frac{\partial^2 f}{\partial x^2}\right|_{i,j} = \frac{2}{a_1 a_3 (a_1 + a_3)} \left[a_3 f_{i+1,j} - (a_1 + a_3)\, f_{i,j} + a_1 f_{i-1,j} \right]$$

$$\left.\frac{\partial^2 f}{\partial y^2}\right|_{i,j} = \frac{2}{a_2 a_4 (a_2 + a_4)} \left[a_4 f_{i,j+1} - (a_2 + a_4)\, f_{i,j} + a_2 f_{i,j-1} \right]$$

(3) Second mixed irregular central difference

$$\left.\frac{\partial^2 f}{\partial x \partial y}\right|_{i,j} = \frac{a_3^2}{a_1 a_2 a_3 a_4 (a_1 + a_3)(a_2 + a_4)} \left[a_4^2\, f_{i+1,j+1} \right.$$

$$\left. + (a_2^2 - a_4^2)\, f_{i+1,j} - a_2^2\, f_{i+1,j-1} \right]$$

$$+ \frac{(a_1^2 - a_3^2)}{a_1 a_2 a_3 a_4 (a_1 + a_3)(a_2 + a_4)} \left[a_4^2\, f_{i,j+1} \right.$$

$$\left. + (a_2^2 - a_4^2)\, f_{i,j} - a_2^2\, f_{i,j-1} \right]$$

$$- \frac{a_1^2}{a_1 a_2 a_3 a_4 (a_1 + a_3)(a_2 + a_4)} \left[a_4^2\, f_{i-1,j+1} \right.$$

$$\left. + (a_2^2 - a_4^2)\, f_{i-1,j} - a_2^2\, f_{i-1,j-1} \right]$$

(Equation continued on next page)

(4) First irregular forward differences

$$\left.\frac{\partial f}{\partial x}\right|_{i,j} = \frac{1}{a_1 a_9 (a_9 - a_1)} \left[- (a_9^2 - a_1^2) f_{i,j} + a_9^2 f_{i+1,j} - a_1^2 f_{i+2,j} \right]$$

$$\left.\frac{\partial f}{\partial y}\right|_{i,j} = \frac{1}{a_2 a_{10} (a_{10} - a_2)} \left[- (a_{10}^2 - a_2^2) f_{i,j} + a_{10}^2 f_{i,j+1} - a_2^2 f_{i,j+2} \right]$$

(5) First irregular backward differences

$$\left.\frac{\partial f}{\partial x}\right|_{i,j} = \frac{1}{a_3 a_{11} (a_{11} - a_3)} \left[(a_{11}^2 - a_3^2) f_{i,j} - a_{11}^2 f_{i-1,j} + a_3^2 f_{i-2,j} \right]$$

$$\left.\frac{\partial f}{\partial y}\right|_{i,j} = \frac{1}{a_4 a_{12} (a_{12} - a_4)} \left[(a_{12}^2 - a_4^2) f_{i,j} - a_{12}^2 f_{i,j-1} + a_4^2 f_{i,j-2} \right]$$

The terms a_1 through a_{12} represent distances measured from the point (i, j) at which the difference form is being expressed (point 0 in Figure C-2) to surrounding points (numbered 1 through 12 in Figure C-2). The subscripts on each displacement term identify the grid coordinates of that displacement in terms of the point (i, j).

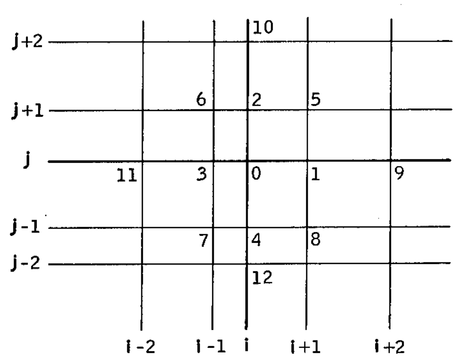


Figure C.2. Node Identification Numbering System

Central differences are used in representing the equilibrium equations, Equations (66) and (67). In representing the boundary condition equations, Equations (70) or (71), and the interface continuity equations, Equation (69), it becomes necessary to use either forward or backward differences to remain within the first quadrant of the fundamental region.

C.3 PROGRAM FORMULATION

The fundamental region is bounded by the grid lines $3 \leq i \leq M$ $3 \leq j \leq N$ (see Figure C-1). The computer storage array is bounded by the grid lines $1 \leq i \leq M + 2$ and $1 \leq j \leq N + 2$, the two additional grid lines exterior to each side of the fundamental region being used only for indexing purposes in the program.

The maximum total grid array size has been established as 17 x 17 and the minimum total grid array size must be 9 x 9. Thus, if the total grid array size is $(M + 2) \times (N + 2)$, i.e., an array with M + 2 grid lines parallel to the y-axis and N + 2 grid lines parallel to the x-axis, where $9 \leq (M + 2) \leq 17$, $9 \leq (N + 2) \leq 17$, then the usable grid node array size is $(M - 2) \times (N - 2)$ because of the unused grid lines exterior to the fundamental region.

For a maximum total grid array size of 17 x 17, the usable grid node array size is therefore 13 x 13; and for a minimum grid array size of 9 x 9, the usable grid node array size is 5 x 5.

Grid lines are located as desired in the fundamental area subject to the following restrictions. Any grid line in the y direction which intersects the matrix-inclusion interface must, at that intersection, cross a corresponding grid line in the x direction such that the intersection is a grid node lying on the interface. Also, a horizontal grid line must pass through the point at which the interface crosses the y axis. Similarly, a vertical grid line must pass through the point at which the interface crosses the x axis.

C.4 FORTRAN PROGRAM

A listing of the Fortran statements which make up the main program and its supporting subroutines is presented at the end of this appendix.

The main control program, called TRANSTRESS, generates the equations to be solved at each grid node and controls the logic flow to the

supporting, equation solving, subroutines. Initially the program clears the locations used to store the u and v displacements, the u and v residuals (REU and REV), and other storage locations which may have values from a previous problem remaining in them. The program then reads the punched input data cards. The first card read is an alphanumeric title card of 72 characters, which will be repeated on the printed output. The remaining data cards supply the program with the physical geometry, imposed stress conditions and control parameters of the problem, as detailed in Paragraph C.6.

The program then creates two grid lines outside of the fundamental region on each side, which are to be used in indexing during the relaxation process. A code, MFI, is assigned to each node, identifying it as lying in the matrix (MFI = 1), in the inclusion (MFI = 2), or on the interface (MFI = 3). Another code, KNT, is assigned to each node denoting the particular equation to be solved at that grid node (i.e., equilibrium, boundary or interface equation) and the difference representation to be employed (i.e., central, forward or backward). There are a total of 17 different equation combinations or node types and thus KNT is a number ranging from 1 through 17.

The proper stress-displacement equation coefficients, listed in Section 4, are then generated to produce a plane stress or a plane strain solution.

At every interior grid node the equilibrium equations in the x and y directions are combined into two equations, one of which eliminates the u displacement at the node and the other eliminates the v displacement at the node. The program then generates the coefficients of these equations at each interior grid node, utilizing the grid spacing surrounding each node and the proper stress-displacement equation coefficient. These coefficients are stored in the two-dimensional arrays E1 through E32, which are in common storage with the other subroutines. This eliminates the need of recalculating any coefficient at any time during the solution process.

The coefficients of the interface node equations are then generated for each node lying on the interface. These are stored in the one-dimensional arrays C1 through C38. The boundary equation coefficients are generated and stored in the one-dimensional arrays D1 through D12. The program then prints out the title, the input parameters and the problem description and begins the solution.

The remainder of the statements in the main program TRANSTRESS direct the logic flow between the subroutines and store and manipulate the interim results to produce the desired solution. This portion of the program is shown schematically in Figure 31.

C.5 SUPPORTING SUBROUTINES

C.5.1 SUBROUTINE RESDTS

Upon entry into Subroutine RESDTS, the existing displacement field is substituted into the difference equations generated for each grid node. The extent to which these equations are not satisfied is termed the residual at that grid node. The displacement field may be the initial unit displacement given to one boundary with all other displacements set equal to zero. Or it may be the displacements existing after a specified number of relaxation cycles have been executed.

Two equations have been formulated at each grid node. One equation is used to solve for the u displacement at the node and the other to solve for the v displacement. The residual errors in these equations are termed REU and REV, respectively. Using the existing displacement field, these residual quantities are computed and stored for each grid node in the array.

Special equations have been formulated for grid nodes which interact with surrounding grid nodes located across the matrix-inclusion interface. These equations involve changing coefficients, as discussed in Subroutine RELXTS. Most of the statements occurring in Subroutine RESDTS are

required for computing the correct value for these coefficients before calculating the residuals.

C.5.2 SUBROUTINE RELXTS

Subroutine RELXTS systematically adjusts the displacements at each grid node to reduce the residual at the node while calculating the corresponding effect upon surrounding residuals. This procedure (successive overrelaxation) is repeated throughout the array until the displacements satisfy the difference equations.

Special equations using varying coefficients have been formulated at grid nodes adjacent to the matrix-inclusion interface. These equations involve the displacements at grid nodes across the interface. Because the material properties of the matrix and the inclusion are different there is a discontinuity in the slope of the displacements at the interface. The coefficients of these displacements are adjusted at the beginning of each relaxation cycle to reflect an effective displacement which would exist if the material properties were constant.

After calculating these coefficients, indexing is begun in the row adjacent to the displaced boundary and progresses toward the interior of the fundamental region. This is done to transmit the boundary displacement most rapidly to the other nodes. At each node, the KNT code is tested to determine the type of equation to be satisfied at that node. The coefficients multiplying the displacements at that node in the difference equations for the node are placed in CUAT and CVAT.

The residual existing at each node represents the extent to which the difference equation is not satisfied at that node and this error is arbitrarily assumed to be entirely due to an error in displacement at that node. A change in displacement can be calculated which will cause the residual at the grid node to be reduced to zero, thus satisfying the equation at that node.

Actually, the change in displacement is further increased by multiplying it by a factor OMB, in effect "overrelaxing" the residual. In theory*, the value of OMB can vary from 0 < OMB < 2. The case of OMB < 1 is termed underrelaxation and OMB > 1 is overrelaxation. An optimum value of the relaxation factor OMB has been found to be about 1.75 for the present solution.

After computing the desired displacement changes at the node and actually changing the u and v displacement value, the program indexes to the 13 affected nodes (see Figures C-2). The residuals at each of these nodes are changed in proportion to the influence of the changed displacement on the equation at the node point. This amount is the ratio of the coefficient of the changed displacement to the coefficient stored in CUAT or CVAT. This process is repeated many times throughout the array until the residuals at each node are reduced to a value small enough such that subsequent relaxations would no longer induce a significant change in displacement at any grid node.

Two exits are possible from Subroutine RELXTS. At the beginning of each relaxation cycle, the total number of cycles already executed is compared to the input value of NRX. When these are equal, control returns to the main program. At the end of each relaxation cycle, the total number of cycles already executed is compared to the input value of NRXBT, which is the number of relaxation cycles to be executed before testing the stresses at selected test points. When the number of relaxation cycles reaches NRXBT, the stresses (σ_x in problems 1 and 3 and σ_y in problem 2) are calculated at the specified test points and compared with the stresses existing at the end of the previous relaxation cycle. If the stresses at all test points have changed by an amount less than a specified percentage, read in as PCGPRX, then control returns to the main program.

Printed output from Subroutine RELXTS consists of an I and J node index, u and v displacement and residual for each node point in the array.

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*Young, David, "Iterative Methods for Solving Partial Difference Equations of Elliptic Type," Transactions of the American Mathematical Society, Vol. 76, pp 92-111, January - June 1954.

Printout occurs for the first (NCPRLX) number of consecutive relaxation cycles following an exit from Subroutine RESDTS and every (NPRLX) multiple cycle thereafter. Printout will also occur for the last relaxation cycle executed when exit from RELXTS is a result of satisfying the condition of minimum change in stress at the test points. At the end of each printout, a record of the number of test points which have not yet satisfied the percentage change in stress condition, since testing began, is given.

C.5.3 SUBROUTINE STRSTS

Subroutine STRSTS is entered after Subroutines RESDTS and RELXTS have been executed the specified number of times, the main program, TRANSTRESS, having properly scaled, combined and stored the displacement fields from the three separate problems.

Subroutine STRSTS calculates σ_x, σ_y, σ_z and τ_{xy} at each node in the array. To conserve computer core storage, these quantities are stored in the two-dimensional arrays previously used for the equilibrium equation coefficients. Using these stresses, the principal stresses σ_1, σ_2, σ_3 are calculated. Also computed are θ, the angle between the x axis and the principal stress direction, and the von Mises sum defined in Paragraph C. 8. These are printed along with the identifying I and J indices, u and v displacements, and a heading defining the imposed load conditions.

At each interface node, where stresses can be calculated both in the inclusion and in the matrix, a zero is printed. The interface stresses are then printed on a separate page along with the effective composite elastic moduli and thermal coefficients. The stresses in the inclusion at the point where the inclusion crosses the x and y axes cannot be calculated and have been arbitrarily printed as zeros.

C.5.4 SUBROUTINE SIGMAB

This subroutine is called by the main program, TRANSTRESS, to calculate the average σ_x and σ_y stresses existing along the x = a and y = b

boundaries for each of the three intermediate solutions. The necessary arguments are transmitted through the CALL statement.

C.5.5 SUBROUTINE PART

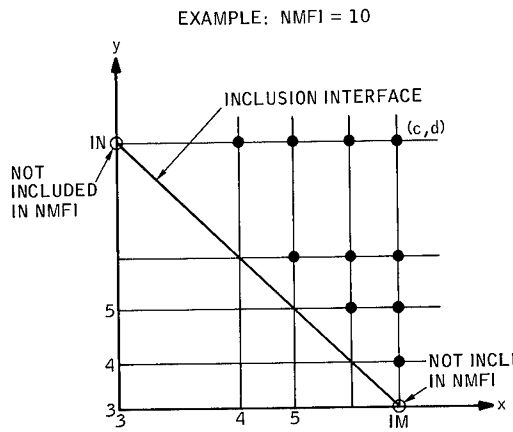
Subroutine PART is called by Subroutine STRSTS and Subroutine SIGMAB to calculate the partial derivative of u or v with respect to x or y. The CALL statement transmits the necessary arguments and indicates the difference scheme to be used, i.e., forward, central or backward.

C.6 INPUT PARAMETER DEFINITIONS

Parameter	Definition
TITLE	TITLE is an alphanumeric description of the particular problem under consideration (up to 72 characters).
M N	M and N define the grid lines bounding the fundamental region at x = a and y = b, respectively (see Figure C-1).
NRX	NRX is the maximum number of times the program will execute Subroutine RELXTS between successive returns to Subroutine RESDTS.
NRD	NRD is the number of times the program will enter Subroutine RESDTS.
IM	IM is the number of the I coordinate line at which the inclusion crosses the x-axis, grid node (IM, 3). Grid nodes are indexed in the program as (I, J).
IN	IN is the number of the J coordinate line at which the inclusion crosses the y-axis, grid node (3, IN).

Parameter	Definition
NPRLX	NPRLX is an integer such that subroutine RELXTS will be printed at every integral multiple of NPRLX.
NCPRLX	NCPRLX is an integer which indicates the number of consecutive outputs of the results of Subroutine RELXTS, beginning with the first entry to RELXTS, i. e., the first NCPRLX outputs of Subroutine RELXTS will be printed.
NL	NL is the number of grid nodes lying on the inclusion interface and includes the grid nodes referenced in the definitions of IM and IN.
NMFI	Construct a line perpendicular to the y-axis and passing through the grid node referenced in the definition of IN and another line perpendicular to the x-axis and passing through the grid node referenced in the definition of IM. These lines will intersect at some grid node (c, d). NMFI is the number of grid nodes contained in the region exterior to the inclusion and its interface node points, but lying on or within the lines constructed through point (c, d). Note: The grid nodes referenced in the definitions of IM and IN are not included in the above sum.

Parameter	Definition
	Example: NMFI = 10



NTP	NTP is the number of test points ($1 \leq$ NTP ≤ 10). Note: Choose as test points only those grid nodes which are interior to the matrix.
NRXBT	NRXBT is the number of times the program will execute Subroutine RELXTS before testing the selected test points.
KPSPS	KPSPS = 0 indicates that the program will execute the case of plane stress. KPSPS = 1 indicates that the program will execute the case of plane strain.

Parameter	Definition
KSYM	KSYM = 0 indicates an unsymmetrical inclusion or inclusion spacing. An inclusion is unsymmetrical if, when rotated 90 degrees about its longitudinal axis, the transformed inclusion does not occupy the same space as the original inclusion. KSYM = 1 indicates that both inclusion and spacing are symmetrical.
MATRIX IJTP	MATRIX IJTP contains the coordinates of the test points used in testing the percent change of stress per relax IJTP (2N-1) = I coordinate and IJTP (2N) = J coordinate of the Nth test point.
PCGPRX	PCGPRX is the maximum percent change in stress allowed at any of the test points, per relax, before exiting from Subroutine RELXTS.
MATRIX HX	HX(I) is the absolute value of the distance between grid lines I and I + 1.
MATRIX HY	HY(J) is the absolute value of the distance between grid lines J and J+1.
EM	EM is the modulus of elasticity, E_m, of the matrix (lb/in.^{2}).

Parameter	Definition
EF	EF is the modulus of elasticity, E_f, of the filament (lb/in.^{2}).
ALPHAM	ALPHAM is the coefficient of thermal expansion, α_m, of the matrix (in./in./deg F).
ALPHAF	ALPHAF is the coefficient of thermal expansion, α_f, of the filament (in./in./deg F).
PRM	PRM is the Poisson's ratio, ν_m, of the matrix.
PRF	PRF is the Poisson's ratio, ν_f, of the filament.
OMB	OMB is the relaxation factor to be used. $0 < OMB < 2$, with optimum convergence usually being obtained for OMB near 1.7.
VF	VF is the percent fiber content by volume of the composite. Note: VF is input for printout purposes only and is not used in the calculations.
T	T is the uniform temperature change (plus or minus) from that temperature corresponding to the zero thermal stress state (deg F).

Parameter	Definition
MATRICES LI, LJ	Associated with each grid node on the interface of the inclusion is an L number. The grid node referenced in the definition of IN has an L number equal to 1, i. e., L = 1. Proceeding clockwise along the interface, the next grid node has an L number equal to 2, i. e., L = 2. Continuing as described above implies that the grid node referenced in the definition of IM has an L number equal to NL, i.e., L = NL. Matrices LI and LJ contain the I and J coordinates respectively, of the grid nodes on the interface of the inclusion where LI(N) is the I coordinate and LJ(N) is the J coordinate of that grid node whose L number is equal to N, i.e., L = N.
MATRICES COST, SINT	MATRICES COST and SINT contain $\cos\theta_n$ and $\sin\theta_n$, respectively, where θ_n is defined as follows: For an arbitrary grid node (I, J) on the interface of the inclusion whose L number is some value such that $1 < L < NL$, θ_n is defined as the angle between the

Parameter	Definition
	normal to the inclusion surface at (I, J) and the positive x-axis. Thus

$$\text{COST (L)} = \text{COS}\,\theta_n$$
$$\text{SINT (L)} = \text{SIN}\,\theta_n$$

For L = 1, i.e., the grid node referenced in the definition of IN, θ_n is defined to be 90 degrees which implies

$$\text{COST (1)} = \text{COS}\,90^{\circ} = 0.0$$
$$\text{SINT (1)} = \text{SIN}\,90^{\circ} = 1.0$$

For L = NL, i.e., the grid node referenced in the definition of IM, θ_n is defined to be 0 degrees which implies

$$\text{COST (NL)} = \text{COS}\,0^{\circ} = 1.0$$
$$\text{SINT (NL)} = \text{SIN}\,0^{\circ} = 0.0$$

SIGXB	SIGXB is the desired average normal stress (lb/in.^2) at infinity in the x-direction.
SIGYB	SIGYB is the desired average normal stress (lb/in.^2) at infinity in the y-direction.
MATRICES MFII, MFIJ	MATRICES MFII and MFIJ contain the I and J coordinates respectively of those grid nodes referenced in the definition of NMFI. No particular input order is required.

INPUT DATA CARD LISTING

Card No.	Parameter	Data Field	Format
1	TITLE	1-72	12A6
2	M, N, NRX,	1-3, 4-6, 7-9,	I3
	NRD, IM, IN,	10-12, 13-15, 16-18,	I3
	NPRLX, NCPRLX ,	19-21, 22-24,	I3
	NL, NMFI, NTP,	25-27, 28-30, 31-33,	I3
	NRXBT, KPSPS,	34-36, 37-39,	I3
	KSYM	40-42	I3
3	IJTP	1-60	I3
4	PCGPRX	1-12	E12.6
5 to L	HX(I)	1-72	E12.6
	I = 3.....M-1		
	Note: Card No. L = $[\frac{M-3}{6}]$ + 5 where [ ] represents the greatest integer function. The maximum allowable value of L is 7.		
L+1 to K	HY(J)	1-72	E12.6
	J = 3...N-1		
	Note: Card No. K = $[\frac{N-3}{6}]$ + (L+1) where [ ] represents the greatest integer function. The maximum value of K is L + 3.		
K+1	EM, EF, ALPHAM	1-36	E12.6
	ALPHAF, PRM, PRF	37-72	E12.6
K+2	OMB, CHI, T	1-36	E12.6
K+3 to J	LI(L), LJ(L)	1-72	I3
	L = 1....NL		
J+1 to I	COST(L), SINT(L)	1-72	E12.6
	L = 1....NL		

Card No.	Parameter	Data Field	Format
I+1	SIGXB, SIGYB	1-24	E12.6
I+2 to LC	MFII(K), MFIJ(K) K=1...NMFI	1-72	I3

C.7 OUTPUT OF PROGRAM

(1) Repeated input data

(2) Dimensions of the first quadrant of the fundamental region, A and B, where

$$A = \sum_{I=3}^{M-1} HX(I)$$

$$B = \sum_{J=3}^{N-1} HY(J)$$

(3) Problem 1

(a) Results of the kth entry into Subroutine RESDTS

(b) Results of Subroutine RELXTS, NCPRLX consecutive times, every integral multiple of NPRLX, and the last execution.

Note: (a) and (b) are printed consecutively for each value of k where k = 1...NRD.

Output includes the I and J coordinates of each node of the grid array, the corresponding displacements in the u and v directions, and the u and v residuals at each grid node.

Problem 2

For KSYM = 0, (a) and (b) are as described for Problem 1. For KSYM = 1, the RESDTS and RELXTS Subroutines are not executed.

Problem 3

(a) and (b) are as described for Problem 1.

(4) Results of Subroutine STRSTS for Problem 1 and Problem 2 are combined to obtain the desired solution for specified values of $\bar{\sigma}_x$ and $\bar{\sigma}_y$ with T = 0, i.e., no temperature effect being included.

Note: Subroutine STRSTS will not be executed in (4) if SIGXB and SIGYB are both equal to zero.

Output will include:

(a) SIGXB, SIGYB, and Temperature (T = 0)
(b) The I and J coordinates of each grid node and the corresponding u and v displacements.
(c) The stress components at the interior and boundary nodes, i.e., SIGMA X, SIGMA Y, SIGMA Z and TAU XY.
(d) The stress components at the interface nodes for both filament and matrix.

(e) The principal stresses at the interior and boundary nodes, i.e., SIGMA 1, SIGMA 2, THETA*, and the von Mises sum.**

(f) The principal stresses at the interface nodes for both filament and matrix.

(g) EX and EY which are defined as the effective composite elastic moduli (lb/in.^{2}) in the x and y directions, respectively.

(h) ALPHAX and ALPHAY which are defined as the effective composite thermal expansion coefficients (in./in./deg F) in the x and y directions, respectively.

---

*Theta is defined as the angle (degrees) measured from the positive x-axis to the direction of the maximum principal stress axis.

**The von Mises sum represents a 2-dimensional yield criterion which is defined as follows:

(a) For a plane stress solution, i.e., if KPSPS = 1

$$\text{von Mises sum} = \sigma_1^2 - \sigma_1 \sigma_2 + \sigma_2^2$$

(b) For a plane strain solution, i.e., if KPSPS = 2

$$\text{von Mises sum} = (1 - \nu + \nu^2)\ \sigma_1^2 - (1 + 2\nu - 2\nu^2)\ \sigma_1 \sigma_2 + (1 - \nu\ \nu^2)\ \sigma_2^2$$

where ν is Poisson's ratio.

(5) Results of Subroutine STRSTS for Problems 1, 2, and 3 are combined to obtain the solution for $T \neq 0$, $\bar{\sigma}_x = \bar{\sigma}_y = 0$.

Note: Subroutine STRSTS will not be executed in (5) if temperature, T, equals zero.

Output format is the same as described in (4)

(6) Results of Subroutine STRSTS for Problems 1, 2 and 3 are combined to obtain the solution for T, $\bar{\sigma}_x$, and $\bar{\sigma}_y$ all non-zero.

Note: Subroutine STRSTS will not be executed in (6) if either temperature, T, is zero or if SIGXB and SIGYB are both equal to zero since this would be a repetition of (5) or (4), respectively.

Output format is the same as described in (4).

C.8 PROGRAM LISTING

Included at the end of this appendix is a listing of the Fortran statements which make up the transverse stress program, TRANSTRESS, and its supporting subroutines.

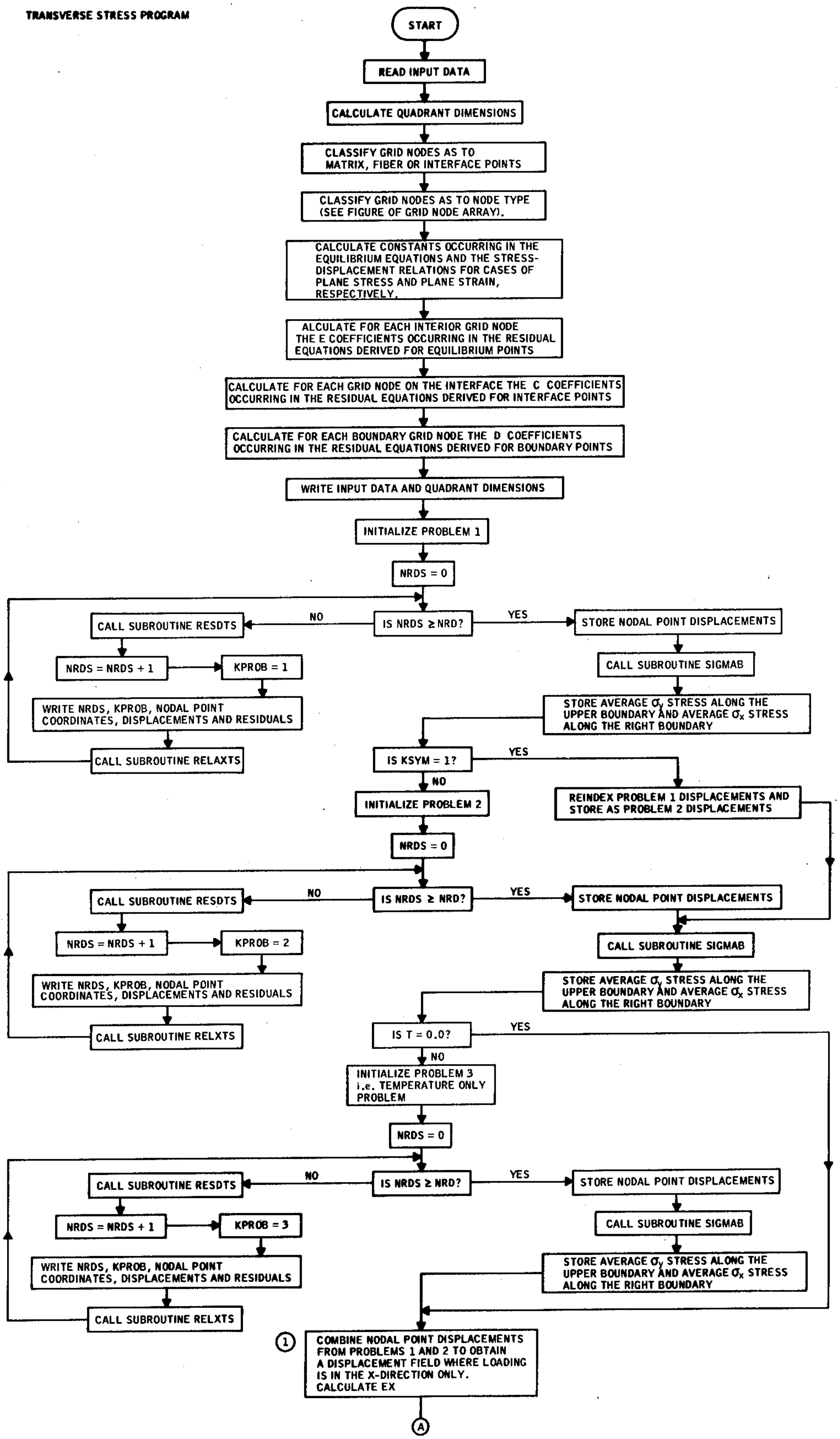
C.9 SAMPLE PROBLEM

The sample output presented at the end of this appendix is that obtained for circular elastic inclusions with a fiber to matrix modulus ratio of 21.5 to 1 and a fiber volume of 40 percent. The imposed loading consists of an average component stress $\bar{\sigma}_x$ at infinity of 1000 psi, an average component stress $\bar{\sigma}_y$ at infinity of zero psi and zero temperature change. The solution is for an assumed plane stress condition and is the result after 150 relaxation cycles.

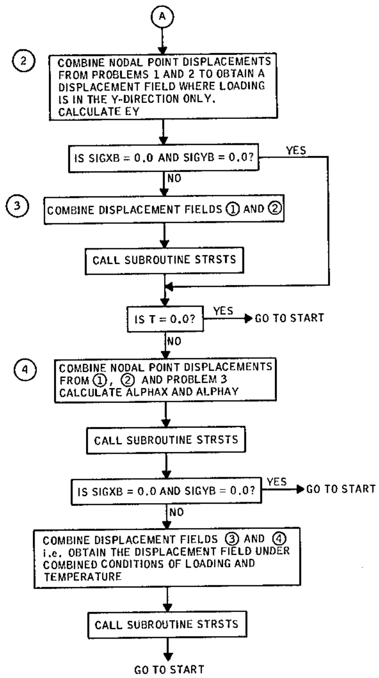
The effective composite modulii, EX and EY, are equal since the inclusion shape and spacing is symmetrical in both coordinate directions.

Program refinement is being continued in an effort to eliminate certain limitations encountered with the present solution. Particular emphasis is being directed toward improving the equations developed to allow the relaxation process to extend across the inclusion-matrix interface. This will eliminate the need for variable coefficients which in the present method must be calculated each relaxation cycle. The particular method presently used of combining the equilibrium equations into a form best suited for unequal grid spacing also has one disadvantage. In this form, certain terms are lost from the equations when equal grid spacing is used and can result in a divergent solution form.

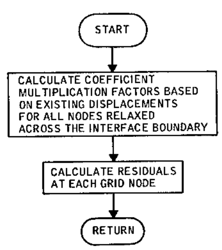
TRANSVERSE STRESS PROGRAM



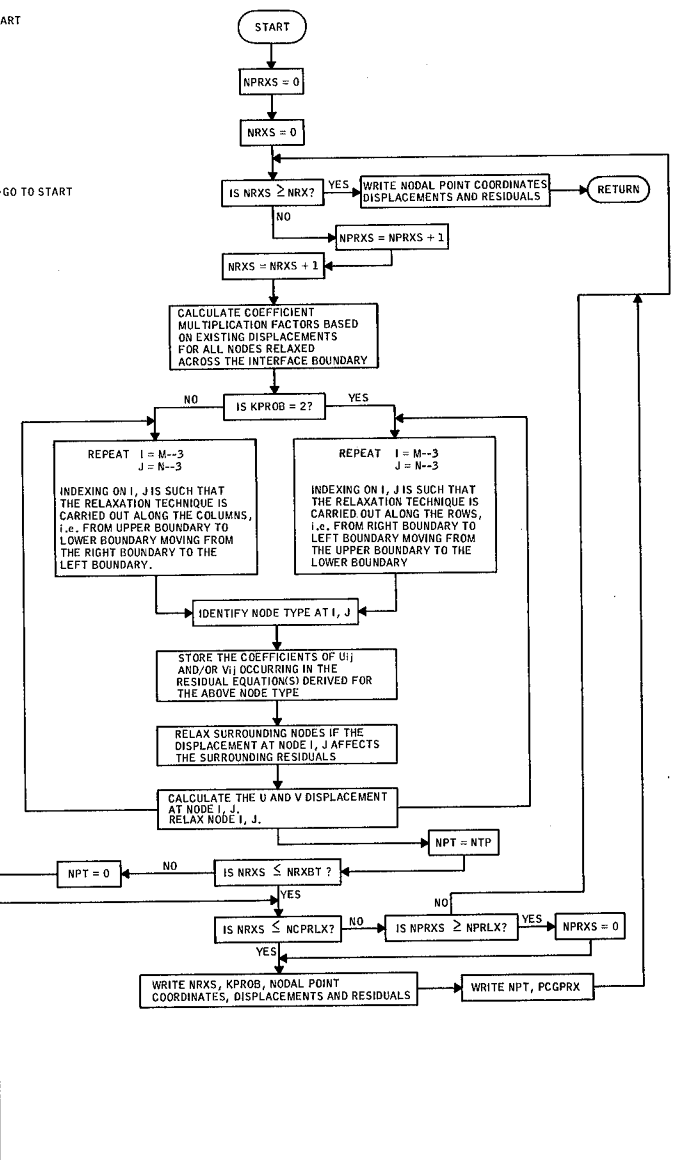
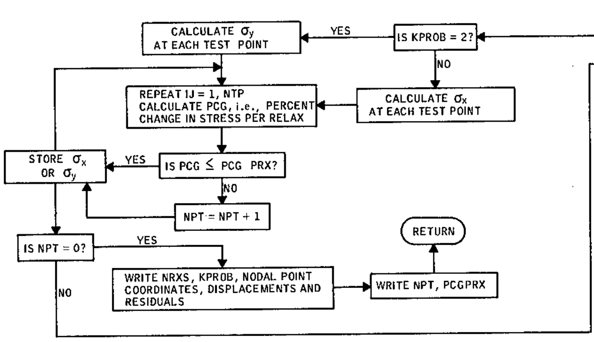
TRANSVERSE STRESS CONTINUED



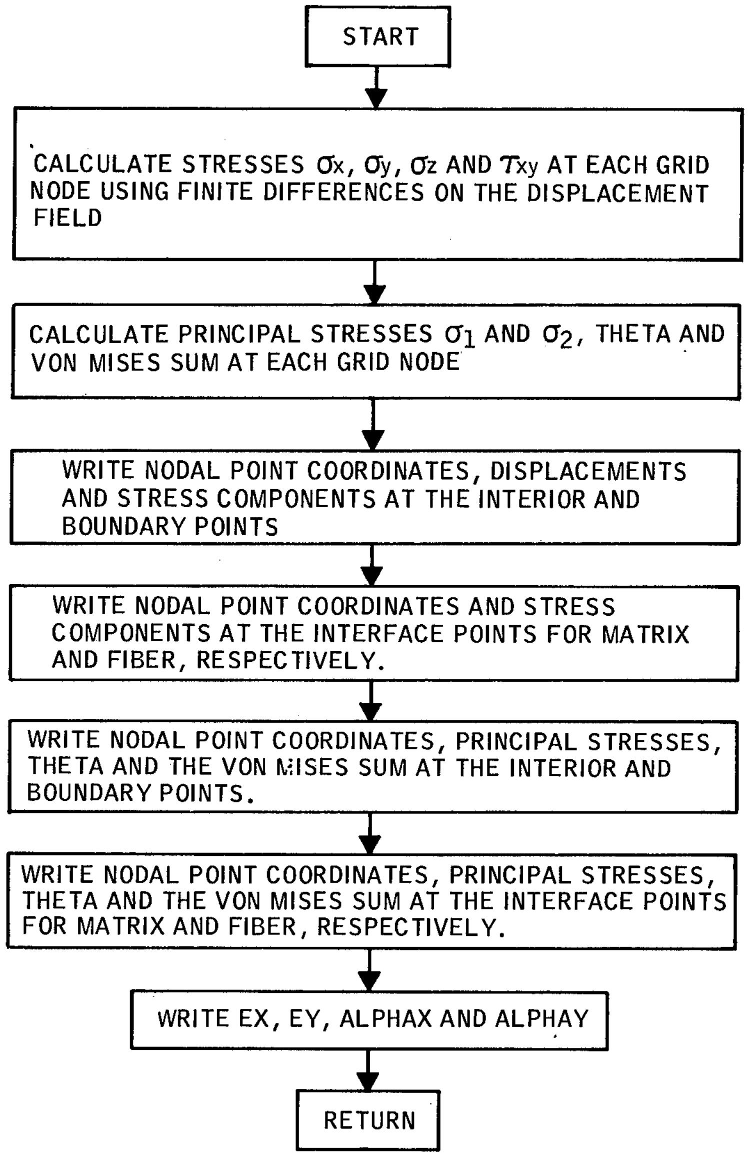
SUBROUTINE RESDTS



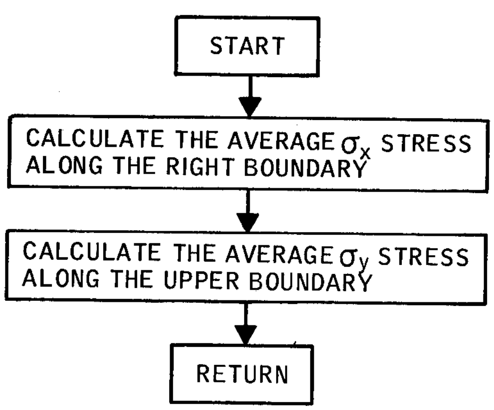
SUBROUTINE RELXTS

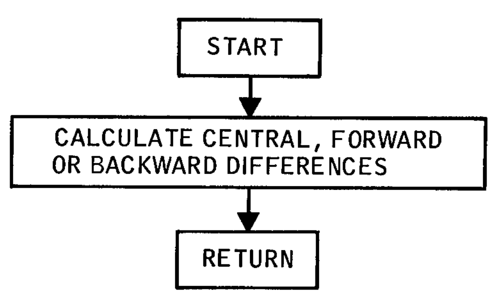
SUBROUTINE STRSTS



SUBROUTINE SIGMAB



SUBROUTINE PART



FORTRAN IV COMPUTER LISTING

FORTRAN MAP

```
CTRANSTRESS
      COMMON U,V,REU,REV,USAVE,VSAVE,U1,V2,SIGX,SIGY,SIGZ,SIGXY,CAT,
     XSIGXB,SIGXBS,SIGYB,SIGYBS,SIGXM,SIGYM,SIGZM,SIGXF,SIGYF,SIGZF,
     XHX,HY,OMB,PRM,PRF,EM,EF,ALPHAM,ALPHAF,T,EX,EY,F1,F2,COST,SINT,
     XC1,C2,C3,C4,C5,C6,C7,C8,C9,C10,C11,C12,C13,C14,C15,C16,C17,C18,
     XC19,C20,C21,C22,C23,C24,C25,C26,C27,C28,C29,C30,C31,C32,C33,C34,
     XC35,C36,C37,C38,C39,C40,C41,C42,C43,C44,C45,C46,C47,C48,
     XD1,D2,D3,D4,D5,D6,D7,D8,D9,D10,D11,D12,
     XE1,E2,E3,E4,E5,E6,E7,E8,E9,E10,E11,E12,E13,E14,E15,E16,E17,E18,E19
     X,E20,E21,E22,E23,E24,E25,E26,E27,E28,E29,E30,E31,E32
     X,AM,AF,BM,BF,CM,CF,DM,DF,          FM,FF,GM,GF,HM,HF,
     XMP2,MP1,M,MM1,MM2,MM3,NP2,NP1,N,NM1,NM2,NM3,INP3,INP2,INP1,IN,
     XINM1,INM2,INM3,IMP3,IMP2,IMP1,IM,IMM1,IMM2,IMM3,NL,NLM1,NLM2,
     XLN,LI,LJ,LAT,KNAT,NMFI,MFIJ,MFI,KNT,KPROB,IJTP,MFII,
     XNRX,NRD,NRXS,NRDS,NPRLX,NCPRLX,NTP,NPT,SIGR1,SIGR2,PCGPRX,SIGR,
     XNRXBT,UL,VL,KPSPS,A,B,KSYM,NKPROB
     X,ALPHAX,ALPHAY,IJRAIP,DA,DB,          NRAIP
      DIMENSION U(20,20),V(20,20),REU(20,20),REV(20,20),
     XE 1(17,17),E 2(17,17),E 3(17,17),E 4(17,17),E 5(17,17),E 6(17,17),
     XE 7(17,17),E 8(17,17),E 9(17,17),E10(17,17),E11(17,17),E12(17,17),
     XE13(17,17),E14(17,17),E15(17,17),E16(17,17),E17(17,17),E18(17,17),
     XE19(17,17),E20(17,17),E21(17,17),E22(17,17),E23(17,17),E24(17,17),
     XE25(17,17),E26(17,17),E27(17,17),E28(17,17),E29(17,17),E30(17,17),
     XE31(17,17),E32(17,17),
     XLN(20,20),MFI(20,20),KNT(20,20),MFII(200),MFIJ(200)
      DIMENSION SIGXM(40),SIGYM(40),SIGZM(40),SIGXF(40),SIGYF(40),
     XSIGZF(40),COST(40),SINT(40),
     XC 1(40),C 2(40),C 3(40),C 4(40),C 5(40),C 6(40),C 7(40),C 8(40),
     XC 9(40),C10(40),C11(40),C12(40),C13(40),C14(40),C15(40),C16(40),
     XC17(40),C18(40),C19(40),C20(40),C21(40),C22(40),C23(40),C24(40),
     XC25(40),C26(40),C27(40),C28(40),C29(40),C30(40),C31(40),C32(40),
     XC33(40),C34(40),C35(40),C36(40),C37(40),C38(40),LI(40),LJ(40)
      DIMENSION HX(20),HY(20),IJTP(20),D1(20),D2(20),D3(20),D4(20),
     XD5(20),D6(20),D7(20),D8(20),D9(20),D10(20),D11(20),D12(20),
     XSIGR1(10),SIGR2(10),IJRAIP(30),    DA(15),    DB(15)
      DIMENSION TITLE(12),UKP1(20,20),VKP1(20,20),UKP2(20,20),
     XVKP2(20,20)
C
C     A RELAXATION SOLUTION OF THE TRANSVERSE STRESS PROBLEM FOR A
C     DOUBLY PERIODIC RECTANGULAR ARRAY OF ELASTIC INCLUSIONS IN AN
C     INFINITE ELASTIC BODY
C
    1 DO 102 I=1,20
      DO 102 J=1,20
      U(I,J)=0.0
      V(I,J)=0.0
      REU(I,J)=0.0
      REV(I,J)=0.0
  102 MFI(I,J)=0
      ALPHAX=0.0
      ALPHAY=0.0
      FF=0.0
      FM=0.0
  101 READ (8,208) TITLE
      READ (8,201) M,N,NRX,NRD,IM,IN,NPRLX,NCPRLX,NL,NMFI,NTP
     1,NRXBT,KPSPS,KSYM,NRAIP
C KPSPS=1 FOR PLANE STRESS KPSPS=2 FOR PLANE STRAIN
C     KSYM=0 FOR UNSYMMETRICAL INCLUSION OR INCLUSION SPACING
C     KSYM=1 IF BOTH INCLUSION AND SPACING ARE SYMMETRICAL
      NTP2=NTP*2
      READ (8,201) (IJTP (IJ),IJ=1,NTP2 )



      DO 44 IJ=1,10
      SIGR1(IJ)=0.0
   44 SIGR2(IJ)=0.0
      READ(8,202) PCGPRX
      NRAIP2=2*NRAIP
      READ (8,201) (IJRAIP(IJ),IJ=1,NRAIP2)
      READ (8,202)((    DA(IJ),    DB(IJ)),IJ=1,NRAIP)
      MM1=M-1
      MM2=M-2
      MM3=M-3
      NM1=N-1
      NM2=N-2
      NM3=N-3
      MP1=M+1
      MP2=M+2
      NP1=N+1
      NP2=N+2
      NLM1=NL-1
      NLM2=NL-2
      IMP3=IM+3
      IMP2=IM+2
      IMP1=IM+1
      IMM1=IM-1
      IMM2=IM-2
      IMM3=IM-3
      INP3=IN+3
      INP2=IN+2
      INP1=IN+1
      INM1=IN-1
      INM2=IN-2
      INM3=IN-3
      READ (8,202) (HX(I),I=3,MM1 )
      READ (8,202) (HY(J),J=3,NM1)
      A=0.0
      B=0.0
      DO 42 I=3,MM1
   42 A=A+HX(I)
      DO 43 J=3,NM1
   43 B=B+HY(J)
      HX(M)=HX(MM1)
      HX(MP1)=HX(MM2)
      HY(N)=HY(NM1)
      HY(NP1)=HY(NM2)
      HX(2)=HX(3)
      HX(1)=HX(4)
      HY(2)=HY(3)
      HY(1)=HY(4)
      READ (8,202) EM,EF,ALPHAM,ALPHAF,PRM,PRF,OMB,VF ,T
      READ (8,201) ((LI(L),LJ(L)),L=1,NL)
      READ (8,202) ((COST(L),SINT(L)),L=1,NL)
      READ (8,202) SIGXB,SIGYB
      DO 33 I=3,M
      DO 33 J=INP1,N
   33 MFI(I,J)=1
      DO 34 I=IMP1,M
      DO 34 J=3,IN
   34 MFI(I,J)=1
      DO 35 I=3,IM
      DO 35 J=3,IN
   35 MFI(I,J)=2
      READ (8,201) ((MFII(K),MFIJ(K)),K=1,NMFI)
      DO 36 K=1,NMFI
```

```
      I=MFII(K)
      J=MFIJ(K)
   36 MFI(I,J)=1
      DO 37 L=1,NL
      I=LI(L)
      J=LJ(L)
   37 MFI(I,J)=3
      DO 12 L=1,NL
      I=LI(L)
      J=LJ(L)
      LN(I,J)=L
   12 CONTINUE
      DO 20 I=4,MM1
      DO 20 J=4,NM1
      KNT(I,J)=2
   20 CONTINUE
      DO 21 J=1,NP2
      KNT(1,J)=1
      KNT(2,J)=1
      KNT(MP1,J)=1
      KNT(MP2,J)=1
   21 CONTINUE
      DO 22 I=3,M
      KNT(I,1)=1
      KNT(I,2)=1
      KNT(I,NP1)=1
      KNT(I,NP2)=1
   22 CONTINUE
      DO 23 J=4,NM1
      KNT(3,J)=8
      KNT(M,J)=9
   23 CONTINUE
      DO 24 I=4,MM1
      KNT(I,3)=10
      KNT(I,N)=11
   24 CONTINUE
      KNT(3,3)=12
      KNT(3,N)=13
      KNT(M,N)=14
      KNT(M,3)=15
      DO 25 L=2,NLM1
      I=LI(L)
      J=LJ(L)
      KNT(I,J)=3
   25 CONTINUE
      GM=EM/(2.0*(1.0+PRM))
      GF=EF/(2.0*(1.0+PRF))
      GO TO (61,62),KPSPS
   61 AM=(1.0-PRM)/(1.0+PRM)
      AF=(1.0-PRF)/(1.0+PRF)
      BM=EM/((1.0+PRM)*(1.0-PRM))
      BF=EF/((1.0+PRF)*(1.0-PRF))
      CM=PRM
      CF=PRF
      DM=0.0
      DF=0.0
      HM=0.0
      HF=0.0
      GO TO 63
   62 AM=1.0-2.0*PRM
      AF=1.0-2.0*PRF
      BM=((1.0-PRM)*EM)/((1.0+PRM)*(1.0-2.0*PRM))
      BF=((1.0-PRF)*EF)/((1.0+PRF)*(1.0-2.0*PRF))
      CM=PRM/(1.0-PRM)
      CF=PRF/(1.0-PRF)
      DM=(PRM*EM)/((1.0+PRM)*(1.0-2.0*PRM))
      DF=(PRF*EF)/((1.0+PRF)*(1.0-2.0*PRF))
      HM=FM
      HF=FF
   63 CONTINUE
      DO 4 I=4,MM1
      DO 4 J=4,NM1
      A1=HX(I)
      A2=HY(J)
      A3=HX(I-1)
      A4=HY(J-1)
      IF (MFI(I,J).EQ.2) GO TO3310
      G=GM
      P=AM
      GO TO3311
 3310 G=GF
      P=AF
 3311 EE1=2.0/(A1*(A1+A3))
      EE2=-2.0/(A1*A3)
      EE3=2.0/(A3*(A1+A3))
      EE4=2.0/(A2*(A2+A4))
      EE5=-2.0/(A2*A4)
      EE6=2.0/(A4*(A2+A4))
      EE16=1.0/(A1*A2*A3*A4*(A1+A3)*(A2+A4))
      EE7=A3**2*A4**2*EE16
      EE8=EE16*(A2**2-A4**2)*A3**2
      EE9=-EE16*A2**2*A3**2
      EE10=EE16*(A1**2-A3**2)*A4**2
      EE11=EE16*(A1**2-A3**2)*(A2**2-A4**2)
      EE12=EE16*(A1**2-A3**2)*(-A2**2)
      EE13=EE16*(-A1**2)*A4**2
      EE14=EE16*(-A1**2)*(A2**2-A4**2)
      EE15=EE16*(-A1**2)*(-A2**2)
      EE21=G*(P+1.0)*EE2+G*P*EE5
      EE22=G*(P+1.0)*EE1
      EE23=G*P*EE4
      EE24=G*(P+1.0)*EE3
      EE25=G*P*EE6
      EE26=G*EE11
      EE27=G*EE8
      EE28=G*EE10
      EE29=G*EE14
      EE30=G*EE12
      EE31=G*EE7
      EE32=G*EE13
      EE33=G*EE15
      EE34=G*EE9
      EE35=G*P*EE2+G*(P+1.0)*EE5
      EE36=G*P*EE1
      EE37=G*(P+1.0)*EE4
      EE38=G*P*EE3
      EE39=G*(P+1.0)*EE6
      EE40=EE26/EE35
      EE41=EE26/EE21
      E 1(I,J)=EE21-EE40*EE26
      E 2(I,J)=EE22-EE40*EE27
      E 3(I,J)=EE23-EE40*EE28
      E 4(I,J)=EE24-EE40*EE29
      E 5(I,J)=EE25-EE40*EE30
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      E 6(I,J)=-EE40*EE31
      E 7(I,J)=EE27-EE40*EE36
      E 8(I,J)=EE28-EE40*EE37
      E 9(I,J)=EE29-EE40*EE38
      E10(I,J)=EE30-EE40*EE39
      E11(I,J)=EE31
      E12(I,J)=EE32
      E13(I,J)=EE33
      E14(I,J)=EE34
      E15(I,J)=EE35-EE41*EE26
      E16(I,J)=EE36-EE41*EE27
      E17(I,J)=EE37-EE41*EE28
      E18(I,J)=EE38-EE41*EE29
      E19(I,J)=EE39-EE41*EE30
      E20(I,J)=-EE40*EE32
      E21(I,J)=-EE40*EE33
      E22(I,J)=-EE40*EE34
      E23(I,J)=EE27-EE41*EE22
      E24(I,J)=EE28-EE41*EE23
      E25(I,J)=EE29-EE41*EE24
      E26(I,J)=EE30-EE41*EE25
      E27(I,J)=-EE41*EE31
      E28(I,J)=-EE41*EE32
      E29(I,J)=-EE41*EE33
      E30(I,J)=-EE41*EE34
      E31(I,J)=E11(I,J)
      E32(I,J)=E13(I,J)
    4 CONTINUE
      DO 7 L=3,NLM2
      I=LI(L)
      J=LJ(L)
      A1=HX(I)
      A2=HY(J)
      A3=HX(I-1)
      A4=HY(J-1)
      A9=A1+HX(I+1)
      A10=A2+HY(J+1)
      A11=A3+HX(I-2)
      A12=A4+HY(J-2)
      CC 1=COST(L)**2
      CC 2=SINT(L)**2
      CC 4=COST(L)*SINT(L)
      CC 3=2.0*CC4
      CC 5=-CC4
      CC18=CC2-CC1
      CC6=-(A9+A1)/(A1*A9)
      CC7=A9/(A1*(A9-A1))
      CC8=-A1/(A9*(A9-A1))
      CC9=-(A10+A2)/(A2*A10)
      CC10=A10/(A2*(A10-A2))
      CC11=-A2/(A10*(A10-A2))
      CC12=(A11+A3)/(A3*A11)
      CC13=-A11/(A3*(A11-A3))
      CC14=A3/(A11*(A11-A3))
      CC15=(A12+A4)/(A4*A12)
      CC16=-A12/(A4*(A12-A4))
      CC17=A4/(A12*(A12-A4))
      CC21=CC1*(BM*CC6-BF*CC12)+CC2*(BM*CM*CC6-BF*CF*CC12)
     1     +CC3*(GM*CC9-GF*CC15)
      CC22=CC1*(BM*CM*CC9-BF*CF*CC15)+CC2*(BM*CC9-BF*CC15)
     1     +CC3*(GM*CC6-GF*CC12)
      CC23=CC1*BM*CC7+CC2*BM*CM*CC7


      CC24=CC3*GM*CC10
      CC25=-CC1*BF*CC13-CC2*BF*CF*CC13
      CC26=-CC3*GF*CC16
      CC27=CC1*BM*CC8+CC2*BM*CM*CC8
      CC28=CC3*GM*CC11
      CC29=-CC1*BF*CC14-CC2*BF*CF*CC14
      CC30=-CC3*GF*CC17
      CC31=CC3*GM*CC7
      CC32=CC1*BM*CM*CC10+CC2*BM*CC10
      CC33=-CC3*GF*CC13
      CC34=-CC1*BF*CF*CC16-CC2*BF*CC16
      CC35=CC3*GM*CC8
      CC36=CC1*BM*CM*CC11+CC2*BM*CC11
      CC37=-CC3*GF*CC14
      CC38=-CC1*BF*CF*CC17-CC2*BF*CC17
      CC39=(CC1+CC2)*(FF-FM)
      CC1=CC4
      CC2=CC5
      CC3=CC18
      CC41=CC1*(BM*CC6-BF*CC12)+CC2*(BM*CM*CC6-BF*CF*CC12)
     1     +CC3*(GM*CC9-GF*CC15)
      CC42=CC1*(BM*CM*CC9-BF*CF*CC15)+CC2*(BM*CC9-BF*CC15)
     1     +CC3*(GM*CC6-GF*CC12)
      CC43=CC1*BM*CC7+CC2*BM*CM*CC7
      CC44=CC3*GM*CC10
      CC45=-CC1*BF*CC13-CC2*BF*CF*CC13
      CC46=-CC3*GF*CC16
      CC47=CC1*BM*CC8+CC2*BM*CM*CC8
      CC48=CC3*GM*CC11
      CC49=-CC1*BF*CC14-CC2*BF*CF*CC14
      CC50=-CC3*GF*CC17
      CC51=CC3*GM*CC7
      CC52=CC1*BM*CM*CC10+CC2*BM*CC10
      CC53=-CC3*GF*CC13
      CC54=-CC1*BF*CF*CC16-CC2*BF*CC16
      CC55=CC3*GM*CC8
      CC56=CC1*BM*CM*CC11+CC2*BM*CC11
      CC57=-CC3*GF*CC14
      CC58=-CC1*BF*CF*CC17-CC2*BF*CC17
      CC59=(CC1+CC2)*(FF-FM)
      CC40=CC22/CC42
      CC60=CC41/CC21
      C 1(L)=CC21-CC41*CC40
      C 2(L)=CC22-CC42*CC40
      C 3(L)=CC23-CC43*CC40
      C 4(L)=CC24-CC44*CC40
      C 5(L)=CC25-CC45*CC40
      C 6(L)=CC26-CC46*CC40
      C 7(L)=CC27-CC47*CC40
      C 8(L)=CC28-CC48*CC40
      C 9(L)=CC29-CC49*CC40
      C10(L)=CC30-CC50*CC40
      C11(L)=CC31-CC51*CC40
      C12(L)=CC32-CC52*CC40
      C13(L)=CC33-CC53*CC40
      C14(L)=CC34-CC54*CC40
      C15(L)=CC35-CC55*CC40
      C16(L)=CC36-CC56*CC40
      C17(L)=CC37-CC57*CC40
      C18(L)=CC38-CC58*CC40
      C19(L)=CC39-CC59*CC40
      C20(L)=CC41-CC21*CC60
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      C21(L)=CC42-CC22*CC60
      C22(L)=CC43-CC23*CC60
      C23(L)=CC44-CC24*CC60
      C24(L)=CC45-CC25*CC60
      C25(L)=CC46-CC26*CC60
      C26(L)=CC47-CC27*CC60
      C27(L)=CC48-CC28*CC60
      C28(L)=CC49-CC29*CC60
      C29(L)=CC50-CC30*CC60
      C30(L)=CC51-CC31*CC60
      C31(L)=CC52-CC32*CC60
      C32(L)=CC53-CC33*CC60
      C33(L)=CC54-CC34*CC60
      C34(L)=CC55-CC35*CC60
      C35(L)=CC56-CC36*CC60
      C36(L)=CC57-CC37*CC60
      C37(L)=CC58-CC38*CC60
      C38(L)=CC59-CC39*CC60
    7 CONTINUE
      A2=HY(3)
      A10=HY(4)+A2
      A4=HY(NM1)
      A12=HY(NM2)+A4
      DO 8 I=4,IMM1
      D1(I)=(-(A10**2-A2**2)/(A2*A10*(A10-A2)))*GF
      D2(I)=(A10/(A2*(A10-A2)))*GF
      D3(I)=(-A2/(A10*(A10-A2)))*GF
      D4(I)=((A12**2-A4**2)/(A4*A12*(A12-A4)))*GM
      D5(I)=(-A12/(A4*(A12-A4)))*GM
      D6(I)=(A4/(A12*(A12-A4)))*GM
    8 CONTINUE
      DO 81 I=IM   ,MM1
      D1(I)=(-(A10**2-A2**2)/(A2*A10*(A10-A2)))*GM
      D2(I)=(A10/(A2*(A10-A2)))*GM
      D3(I)=(-A2/(A10*(A10-A2)))*GM
      D4(I)=((A12**2-A4**2)/(A4*A12*(A12-A4)))*GM
      D5(I)=(-A12/(A4*(A12-A4)))*GM
      D6(I)=(A4/(A12*(A12-A4)))*GM
   81 CONTINUE
      A1=HX(3)
      A9=HX(4)+A1
      A3=HX(MM1)
      A11=HX(MM2)+A3
      DO 9 J=4,INM1
      D7(J)=(-(A9**2-A1**2)/(A1*A9*(A9-A1)))*GF
      D8(J)=(A9/(A1*(A9-A1)))*GF
      D9(J)=(-A1/(A9*(A9-A1)))*GF
      D10(J)=((A11**2-A3**2)/(A3*A11*(A11-A3)))*GM
      D11(J)=(-A11/(A3*(A11-A3)))*GM
      D12(J)=(A3/(A11*(A11-A3)))*GM
    9 CONTINUE
      DO 91 J=IN   ,NM1
      D7(J)=(-(A9**2-A1**2)/(A1*A9*(A9-A1)))*GM
      D8(J)=(A9/(A1*(A9-A1)))*GM
      D9(J)=(-A1/(A9*(A9-A1)))*GM
      D10(J)=((A11**2-A3**2)/(A3*A11*(A11-A3)))*GM
      D11(J)=(-A11/(A3*(A11-A3)))*GM
      D12(J)=(A3/(A11*(A11-A3)))*GM
   91 CONTINUE
      WRITE (5,206) TITLE,MM2,NM2,A,B,OMB,SIGXB,SIGYB,VF ,EM,EF,PRM,PRF,
     XGM,GF,  ALPHAM,ALPHAF,T,PCGPRX
  206 FORMAT (1H1,30X,53HT R A N S V E R S E    S T R E S S    A N A L Y

     X S I S ,///,12A6,///,55X,10HINPUT DATA,///,
     X45H GRID NODE ARRAY SIZE                            =,I2,4H BY ,I2,//,
     X21H QUADRANT DIMENSIONS ,6X,3HA =,1F6.3,6X,3HB =,1F6.3,        //,
     X45H RELAXATION FACTOR    (OMEGA BAR)                =,1F6.3,        //,
     X45H AVERAGE SIGMA X LOADING AT INFINITY (PSI)  =,1F9.2,//,
     X45H AVERAGE SIGMA Y LOADING AT INFINITY (PSI)  =,1F9.2,//,
     X45H PERCENT FIBER BY VOLUME                     =,1F9.2,//,
     X45H YCUNGS MODULUS E IN MATRIX  (PSI)           =,1E11.4,//,
     X45H YCUNGS MODULUS E IN FIBER   (PSI)           =,1E11.4,//,
     X45H PCISSONS RATIO   IN MATRIX                  =,1F9.4,//,
     X45H PCISSONS RATIO   IN FIBER                   =,1F9.4,//,
     X45H MATRIX SHEAR MODULUS        PSI             =,1E12.4,       //,
     X45H INCLUSION SHEAR MODULUS   PSI               =,1E12.4,       //,
     X45H THERMAL EXP. COEF. IN MATRIX (IN/IN/DEG F) =,1E11.4,//,
     X45H THERMAL EXP. COEF. IN FIBER  (IN/IN/DEG F) =,1E11.4,//,
     X45H T=AMBIENT TEMP - CURING TEMP (DEGREES F)   =,1F9.2,//,
     X45H MAX DELTA STRESS AT TEST PTS/RELAX(PERCENT)=,1F9.4,// )
      IF (KPSPS.EC.1) GO TO 212
      WRITE (5,214)
      GC TO 213
  212 WRITE (5,215)
  213 WRITE (5,207) (IJTP(IJ),IJ=1,NTP2)
  214 FORMAT (1H ,////,30H SOLUTION IS FOR PLANE STRAIN  )
  215 FORMAT (1H ,////,30H SOLUTION IS FOR PLANE STRESS  )
  207 FORMAT (1H1,//,24H TEST POINT COORDINATES ,//,6X,1HI,3X,1HJ,//,
     X(3X,2I4))
      WRITE (5,209) ((I,HX(I)),I=3,MM1)
  209 FORMAT (1H1,////,14H  GRID SPACING ,//,6X,1HI,8X,5HHX(I),//,
     1(3X,I4,3X,F12.8))
      WRITE (5,210) ((J,HY(J)),J=3,NM1)
  210 FORMAT (1H ,///,6X,1HJ,8X,5HHY(J),//,
     1(3X,I4,3X,F12.8))
      WRITE (5,211) ((LI(L),LJ(L),COST(L),SINT(L)),L=1,NL)
  211 FORMAT (1H ,///,38H COS AND SINE THETA AT INTERFACE NODES ,//,
     X6X,1HI,3X,1HJ,6X,3HCOS,8X,4HSINE,//,(3X,2I4,2F11.5))
      U1=1.0
      DO 30 J=3,N
   30 U(M,J)=U1
      NRXS=0
      NRDS=0
      FF=0.0
      FM=0.0
   10 IF (NRDS.GE.NRD) GO TO 6
      CALL RESDTS
      NRDS=NRDS+1
      KPROB=1
      WRITE(5,203) NRDS,KPROB
      WRITE(5,204)
      WRITE (5,205) (((I,J,U(I,J),V(I,J),REU(I,J),REV(I,J)),J=3,N),
     XI=3,M)
      DO 46 IJ=1,10
   46 SIGR1(IJ)=0.0
      CALL RELXTS
      GO TO 10
    6 DO 70 I=3,M
      UKP1(I,2)=U(I,4)
      VKP1(I,2)=-V(I,4)
      UKP1(I,NP1)=U(I,NM1)
   70 VKP1(I,NP1)=-V(I,NM1)
      DO 71 J=3,N
      UKP1(2,J)=-U(4,J)
      VKP1(2,J)= V(4,J)
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      UKP1(MP1,J)=-U(MM1,J)
   71 VKP1(MP1,J)= V(MM1,J)
      DO 72 I=3,M
      DO 72 J=3,N
      UKP1(I,J)=U(I,J)
   72 VKP1(I,J)=V(I,J)
      DO 105 I=1,MP2
      DO 105 J=1,NP2
      U(I,J)=0.0
      V(I,J)=0.0
      REU(I,J)=0.0
  105 REV(I,J)=0.0
      CALL SIGMAB(HX,HY,UKP1,VKP1,BM,CM,FM,M,N,A,B,SXBS,SYBS)
      SXBS1=SXBS
      SYBS1=SYBS
      V2=1.0
      IF (KSYM.EQ.1) GO TO 92
      DO 73 I=3,M
   73 V(I,N)=V2
      NRXS=0
      NRDS=0
      FF=0.0
      FM=0.0
   11 IF (NRDS.GE.NRD) GO TO 3
      CALL RESDTS
      NRDS=NRDS+1
      KPROB=2
      WRITE(5,203) NRDS,KPROB
      WRITE(5,204)
      WRITE (5,205) (((I,J,U(I,J),V(I,J),REU(I,J),REV(I,J)),J=3,N),
     XI=3,M)
      DO 47 IJ=1,10
   47 SIGR1(IJ)=0.0
      CALL RELXTS
      GO TO 11
    3 DO 74 I=3,M
      UKP2(I,2)= U(I,4)
      VKP2(I,2)=-V(I,4)
      UKP2(I,NP1)= U(I,NM1)
   74 VKP2(I,NP1)=-V(I,NM1)
      DO 75 J=3,N
      UKP2(2,J)=-U(4,J)
      VKP2(2,J)= V(4,J)
      UKP2(MP1,J)=-U(MM1,J)
   75 VKP2(MP1,J)= V(MM1,J)
      DO 76 I=3,M
      DO 76 J=3,N
      UKP2(I,J)=U(I,J)
   76 VKP2(I,J)=V(I,J)
      DO 106 I=1,MP2
      DO 106 J=1,NP2
      U(I,J)=0.0
      V(I,J)=0.0
      REU(I,J)=0.0
  106 REV(I,J)=0.0
      GO TO 94
   92 DO 93 I=2,MP1
      DO 93 J=2,NP1
      UKP2(I,J)=VKP1(J,I)
   93 VKP2(I,J)=UKP1(J,I)
   94 CALL SIGMAB(HX,HY,UKP2,VKP2,BM,CM,FM,M,N,A,B,SXBS,SYBS)
      SXBS2=SXBS
      SYBS2=SYBS
   95 IF (T.EQ.0.0) GO TO 96
      GO TO (107,108),KPSPS
  107 FM=(ALPHAM*EM*T)/(1.0-PRM)
      FF=(ALPHAF*EF*T)/(1.0-PRF)
      GO TO 109
  108 FM=(ALPHAM*EM*T)/(1.0-2.0*PRM)
      FF=(ALPHAF*EF*T)/(1.0-2.0*PRF)
      HM = FM
      HF = FF
  109 DO 110 L=1,NL
      I=LI(L)
      J=LJ(L)
      C19(L)=FF-FM
      A1=HX(I)
      A2=HY(J)
      A3=HX(I-1)
      A4=HY(J-1)
      A9=A1+HX(I+1)
      A10=A2+HY(J+1)
      A11=A3+HX(I-2)
      A12=A4+HY(J-2)
      CC 1=COST(L)**2
      CC 2=SINT(L)**2
      CC 4=COST(L)*SINT(L)
      CC 3=2.0*CC4
      CC 5=-CC4
      CC18=CC2-CC1
      CC6=-(A9+A1)/(A1*A9)
      CC7=A9/(A1*(A9-A1))
      CC8=-A1/(A9*(A9-A1))
      CC9=-(A10+A2)/(A2*A10)
      CC10=A10/(A2*(A10-A2))
      CC11=-A2/(A10*(A10-A2))
      CC12=(A11+A3)/(A3*A11)
      CC13=-A11/(A3*(A11-A3))
      CC14=A3/(A11*(A11-A3))
      CC15=(A12+A4)/(A4*A12)
      CC16=-A12/(A4*(A12-A4))
      CC17=A4/(A12*(A12-A4))
      CC21=CC1*(BM*CC6-BF*CC12)+CC2*(BM*CM*CC6-BF*CF*CC12)
     1     +CC3*(GM*CC9-GF*CC15)
      CC1=CC4
      CC2=CC5
      CC3=CC18
      CC41=CC1*(BM*CC6-BF*CC12)+CC2*(BM*CM*CC6-BF*CF*CC12)
     1     +CC3*(GM*CC9-GF*CC15)
      CC60=CC41/CC21
  110 C38(L)=(FM-FF)*CC60
      NRXS=0
      NRDS=0
   13 IF (NRDS.GE.NRD) GO TO 14
      CALL RESDTS
      NRDS=NRDS+1
      KPROB=3
      WRITE(5,203) NRDS,KPROB
      WRITE(5,204)
      WRITE (5,205) (((I,J,U(I,J),V(I,J),REU(I,J),REV(I,J)),J=3,N),
     XI=3,M)
      DO 48 IJ=1,10
   48 SIGR1(IJ)=0.0
      CALL RELXTS
```

```
      GO TO 13
   14 DO 77 I=3,M
      E 9(I,2)= U(I,4)
      E10(I,2)=-V(I,4)
      E 9(I,NP1)= U(I,NM1)
   77 E10(I,NP1)=-V(I,NM1)
      DO 78 J=3,N
      E 9(2,J)=-U(4,J)
      E10(2,J)= V(4,J)
      E 9(MP1,J)=-U(MM1,J)
   78 E10(MP1,J)= V(MM1,J)
      DO 79 I=3,M
      DO 79 J=3,N
      E 9(I,J)=U(I,J)
   79 E10(I,J)=V(I,J)
      CALL SIGMAB(HX,HY,E9,E10,BM,CM,FM,M,N,A,B,SXBS,SYBS)
      SXBS3=SXBS
      SYBS3=SYBS
   96 FF1=-SYBS1/SYBS2
      FF2=-SXBS2/SXBS1
      SXBS12=SXBS1+FF1*SXBS2
      SYBS21=SYBS2+FF2*SYBS1
      DO 82 I=2,MP1
      DO 82 J=2,NP1
      E11(I,J)=UKP1(I,J)+FF1*UKP2(I,J)
   82 E12(I,J)=VKP1(I,J)+FF1*VKP2(I,J)
      EX=SXBS12*A/U1
      DO 83 I=2,MP1
      DO 83 J=2,NP1
      E13(I,J)=UKP2(I,J)+FF2*UKP1(I,J)
   83 E14(I,J)=VKP2(I,J)+FF2*VKP1(I,J)
      EY=SYBS21*B/V2
      IF ((SIGXB.EQ.0.0).AND.(SIGYB.EQ.0.0)) GO TO 97
      FF3=SIGXB/SXBS12
      FF4=SIGYB/SYBS21
      DO 84 I=2,MP1
      DO 84 J=2,NP1
      E15(I,J)=FF3*E11(I,J)+FF4*E13(I,J)
      E16(I,J)=FF3*E12(I,J)+FF4*E14(I,J)
      U(I,J)=E15(I,J)
   84 V(I,J)=E16(I,J)
      FF=0.0
      FM=0.0
      HF = 0.0
      HM = 0.0
      KPROB=1
      CALL STRSTS
   97 IF (T.EQ.0.0) GO TO 1
      FF5=-SXBS3/SXBS12
      FF6=-SYBS3/SYBS21
      DO 85 I=2,MP1
      DO 85 J=2,NP1
      U(I,J)=FF5*E11(I,J)+FF6*E13(I,J)+E 9(I,J)
   85 V(I,J)=FF5*E12(I,J)+FF6*E14(I,J)+E10(I,J)
      ALPHAX=((FF5*U1+FF2*FF6*U1)/A)/T
      ALPHAY=((FF6*V2+FF1*FF5*V2)/B)/T
      IF (KPSPS .EQ. 2) GO TO 111
      FM=(ALPHAM*EM*T)/(1.0-PRM)
      FF=(ALPHAF*EF*T)/(1.0-PRF)
      GO TO 112
  111 FM=(ALPHAM*EM*T)/(1.0-2.0*PRM)
      FF=(ALPHAF*EF*T)/(1.0-2.0*PRF)



      HF = FF
      HM = FM
  112 KPROB=2
      CALL STRSTS
   87 IF (SIGXB.EQ.0.0) GO TO 89
      GO TO 88
   89 IF (SIGYB.EQ.0.0) GO TO 99
   88 DO 86 I=2,MP1
      DO 86 J=2,NP1
      U(I,J)=U(I,J)+E15(I,J)
   86 V(I,J)=V(I,J)+E16(I,J)
      KPROB=3
      CALL STRSTS
   99 GO TO 1
  201 FORMAT (24I3)
  202 FORMAT (6E12.6)
  203 FORMAT (1H1,49X,21HRESULTS OF RESID NO. ,I2,5X,11HPROBLEM NO.,I3/)
  204 FORMAT (1H ,//,6X,1HI,3X,1HJ,19X,1HU,18X,1HV,14X,10HU RESIDUAL,
     X10X,10HV RESIDUAL,///)
  205 FORMAT (1H ,3X,2I4,6X,4E20.8)
  208 FORMAT (12A6)
      END
                    FORTRAN MAP
CRESDTS
      SUBROUTINE RESDTS
       COMMON U,V,REU,REV,USAVE,VSAVE,U1,V2,SIGX,SIGY,SIGZ,SIGXY,CAT,
     XSIGXB,SIGXBS,SIGYB,SIGYBS,SIGXM,SIGYM,SIGZM,SIGXF,SIGYF,SIGZF,
     XHX,HY,OMB,PRM,PRF,EM,EF,ALPHAM,ALPHAF,T,EX,EY,F1,F2,COST,SINT,
     XC1,C2,C3,C4,C5,C6,C7,C8,C9,C10,C11,C12,C13,C14,C15,C16,C17,C18,
     XC19,C20,C21,C22,C23,C24,C25,C26,C27,C28,C29,C30,C31,C32,C33,C34,
     XC35,C36,C37,C38,C39,C40,C41,C42,C43,C44,C45,C46,C47,C48,
     XD1,D2,D3,D4,D5,D6,D7,D8,D9,D10,D11,D12,
     XE1,E2,E3,E4,E5,E6,E7,E8,E9,E10,E11,E12,E13,E14,E15,E16,E17,E18,E19
     X,E20,E21,E22,E23,E24,E25,E26,E27,E28,E29,E30,E31,E32
     X,AM,AF,BM,BF,CM,CF,DM,DF,         FM,FF,GM,GF,HM,HF,
     XMP2,MP1,M,MM1,MM2,MM3,NP2,NP1,N,NM1,NM2,NM3,INP3,INP2,INP1,IN,
     XINM1,INM2,INM3,IMP3,IMP2,IMP1,IM,IMM1,IMM2,IMM3,NL,NLM1,NLM2,
     XLN,LI,LJ,LAT,KNAT,NMFI,MFIJ,MFI,KNT,KPROB,IJTP,MFII,
     XNRX,NRD,NRXS,NRDS,NPRLX,NCPRLX,NTP,NPT,SIGR1,SIGR2,PCGPRX,SIGR,
     XNRXBT,UL,VL,KPSPS,A,B,KSYM,NKPROB
     X,ALPHAX,ALPHAY,IJRAIP,DA,DB,      NRAIP
       DIMENSION U(20,20),V(20,20),REU(20,20),REV(20,20),
     XE 1(17,17),E 2(17,17),E 3(17,17),E 4(17,17),E 5(17,17),E 6(17,17),
     XE 7(17,17),E 8(17,17),E 9(17,17),E10(17,17),E11(17,17),E12(17,17),
     XE13(17,17),E14(17,17),E15(17,17),E16(17,17),E17(17,17),E18(17,17),
     XE19(17,17),E20(17,17),E21(17,17),E22(17,17),E23(17,17),E24(17,17),
     XE25(17,17),E26(17,17),E27(17,17),E28(17,17),E29(17,17),E30(17,17),
     XE31(17,17),E32(17,17),
     XLN(20,20),MFI(20,20),KNT(20,20),MFII(200),MFIJ(200)
       DIMENSION SIGXM(40),SIGYM(40),SIGZM(40),SIGXF(40),SIGYF(40),
     XSIGZF(40),COST(40),SINT(40),
     XC 1(40),C 2(40),C 3(40),C 4(40),C 5(40),C 6(40),C 7(40),C 8(40),
     XC 9(40),C10(40),C11(40),C12(40),C13(40),C14(40),C15(40),C16(40),
     XC17(40),C18(40),C19(40),C20(40),C21(40),C22(40),C23(40),C24(40),
     XC25(40),C26(40),C27(40),C28(40),C29(40),C30(40),C31(40),C32(40),
     XC33(40),C34(40),C35(40),C36(40),C37(40),C38(40),LI(40),LJ(40)
       DIMENSION HX(20),HY(20),IJTP(20),D1(20),D2(20),D3(20),D4(20),
     XD5(20),D6(20),D7(20),D8(20),D9(20),D10(20),D11(20),D12(20),
     XSIGR1(10),SIGR2(10),IJRAIP(30),   DA(15),   DB(15)
     X,ABIJ(15),ABKJ(15)
       INTERIOR POINTS
       DO 5000 IJ=1,NRAIP
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      ABIJ(IJ)=((DA(IJ)+DB(IJ))/(DA(IJ)+DB(IJ)*(EF/EM)))*EF/EM
 5000 ABKJ(IJ)=((DA(IJ)+DB(IJ))/(DA(IJ)*(EM/EF)+DB(IJ)))*EM/EF
      DO 5001 IJ=1,NRAIP
      IJI=IJ*2-1
      IJJ=IJ*2
      I=IJRAIP(IJI)
      J=IJRAIP(IJJ)
      KI=I+1
      KJ=J+1
      IF (U(I,J).EQ.0.0) GO TO 5002
      EUIJ    =(U(KI,KJ)-ABIJ(IJ)*(U(KI,KJ)-U(I,J)))/U(I,J)
      A1=HX(KI)
      A2=HY(KJ)
      A3=HX(KI-1)
      A4=HY(KJ-1)
      G=GM
      P=AM
      EE1=2.0/(A1*(A1+A3))
      EE2=-2.0/(A1*A3)
      EE3=2.0/(A3*(A1+A3))
      EE4=2.0/(A2*(A2+A4))
      EE5=-2.0/(A2*A4)
      EE6=2.0/(A4*(A2+A4))
      EE16=1.0/(A1*A2*A3*A4*(A1+A3)*(A2+A4))
      EE7=A3**2*A4**2*EE16
      EE8=EE16*(A2**2-A4**2)*A3**2
      EE9=-EE16*A2**2*A3**2
      EE10=EE16*(A1**2-A3**2)*A4**2
      EE11=EE16*(A1**2-A3**2)*(A2**2-A4**2)
      EE12=EE16*(A1**2-A3**2)*(-A2**2)
      EE13=EE16*(-A1**2)*A4**2
      EE14=EE16*(-A1**2)*(A2**2-A4**2)
      EE15=EE16*(-A1**2)*(-A2**2)
      EE21=G*(P+1.0)*EE2+G*P*EE5
      EE26=G*EE11
      EE31=G*EE7
      EE33=G*EE15
      EE35=G*P*EE2+G*(P+1.0)*EE5
      EE40=EE26/EE35
      EE41=EE26/EE21
      E21(KI,KJ)=-EE40*EE33*EUIJ
      E32(KI,KJ)= EE33*EUIJ
      GO TO 5003
 5002 E21(KI,KJ)=0.0
      E32(KI,KJ)=0.0
 5003 IF (V(I,J).EQ.0.0) GO TO 5004
      EVIJ=(V(KI,KJ)-ABIJ(IJ)*(V(KI,KJ)-V(I,J)))/V(I,J)
      A1=HX(KI)
      A2=HY(KJ)
      A3=HX(KI-1)
      A4=HY(KJ-1)
      G=GM
      P=AM
      EE1=2.0/(A1*(A1+A3))
      EE2=-2.0/(A1*A3)
      EE3=2.0/(A3*(A1+A3))
      EE4=2.0/(A2*(A2+A4))
      EE5=-2.0/(A2*A4)
      EE6=2.0/(A4*(A2+A4))
      EE16=1.0/(A1*A2*A3*A4*(A1+A3)*(A2+A4))
      EE7=A3**2*A4**2*EE16
      EE8=EE16*(A2**2-A4**2)*A3**2

      EE9=-EE16*A2**2*A3**2
      EE10=EE16*(A1**2-A3**2)*A4**2
      EE11=EE16*(A1**2-A3**2)*(A2**2-A4**2)
      EE12=EE16*(A1**2-A3**2)*(-A2**2)
      EE13=EE16*(-A1**2)*A4**2
      EE14=EE16*(-A1**2)*(A2**2-A4**2)
      EE15=EE16*(-A1**2)*(-A2**2)
      EE21=G*(P+1.0)*EE2+G*P*EE5
      EE26=G*EE11
      EE31=G*EE7
      EE33=G*EE15
      EE35=G*P*EE2+G*(P+1.0)*EE5
      EE40=EE26/EE35
      EE41=EE26/EE21
      E13(KI,KJ)=EE33*EVIJ
      E29(KI,KJ)=-EE41*EE33*EVIJ
      GO TO 5005
 5004 E13(KI,KJ)=0.0
      E29(KI,KJ)=0.0
 5005 IF (U(KI,KJ).EQ.0.0) GO TO 5006
      EUKJ=(U(I,J)+ABKJ(IJ)*(U(KI,KJ)-U(I,J)))/U(KI,KJ)
      A1=HX(I)
      A2=HY(J)
      A3=HX(I-1)
      A4=HY(J-1)
      G=GF
      P=AF
      EE1=2.0/(A1*(A1+A3))
      EE2=-2.0/(A1*A3)
      EE3=2.0/(A3*(A1+A3))
      EE4=2.0/(A2*(A2+A4))
      EE5=-2.0/(A2*A4)
      EE6=2.0/(A4*(A2+A4))
      EE16=1.0/(A1*A2*A3*A4*(A1+A3)*(A2+A4))
      EE7=A3**2*A4**2*EE16
      EE8=EE16*(A2**2-A4**2)*A3**2
      EE9=-EE16*A2**2*A3**2
      EE10=EE16*(A1**2-A3**2)*A4**2
      EE11=EE16*(A1**2-A3**2)*(A2**2-A4**2)
      EE12=EE16*(A1**2-A3**2)*(-A2**2)
      EE13=EE16*(-A1**2)*A4**2
      EE14=EE16*(-A1**2)*(A2**2-A4**2)
      EE15=EE16*(-A1**2)*(-A2**2)
      EE21=G*(P+1.0)*EE2+G*P*EE5
      EE26=G*EE11
      EE31=G*EE7
      EE33=G*EE15
      EE35=G*P*EE2+G*(P+1.0)*EE5
      EE40=EE26/EE35
      EE41=EE26/EE21
      E 6(I,J)=-EE40*EE31*EUKJ
      E31(I,J)= EE31*EUKJ
      GO TO 5007
 5006 E 6(I,J)=0.0
      E31(I,J)=0.0
 5007 IF (V(KI,KJ).EQ.0.0) GO TO 5008
      EVKJ=(V(I,J)+ABKJ(IJ)*(V(KI,KJ)-V(I,J)))/V(KI,KJ)
      A1=HX(I)
      A2=HY(J)
      A3=HX(I-1)
      A4=HY(J-1)
      G=GF
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```
      P=AF
      EE1=2.0/(A1*(A1+A3))
      EE2=-2.0/(A1*A3)
      EE3=2.0/(A3*(A1+A3))
      EE4=2.0/(A2*(A2+A4))
      EE5=-2.0/(A2*A4)
      EE6=2.0/(A4*(A2+A4))
      EE16=1.0/(A1*A2*A3*A4*(A1+A3)*(A2+A4))
      EE7=A3**2*A4**2*EE16
      EE8=EE16*(A2**2-A4**2)*A3**2
      EE9=-EE16*A2**2*A3**2
      EE10=EE16*(A1**2-A3**2)*A4**2
      EE11=EE16*(A1**2-A3**2)*(A2**2-A4**2)
      EE12=EE16*(A1**2-A3**2)*(-A2**2)
      EE13=EE16*(-A1**2)*A4**2
      EE14=EE16*(-A1**2)*(A2**2-A4**2)
      EE15=EE16*(-A1**2)*(-A2**2)
      EE21=G*(P+1.0)*EE2+G*P*EE5
      EE26=G*EE11
      EE31=G*EE7
      EE33=G*EE15
      EE35=G*P*EE2+G*(P+1.0)*EE5
      EE40=EE26/EE35
      EE41=EE26/EE21
      E11(I,J)= EE31*EVKJ
      E27(I,J)=-EE41*EE31*EVKJ
      GO TO 5009
 5008 E11(I,J)=0.0
      E27(I,J)=0.0
 5009 CONTINUE
 5001 CONTINUE
      DO 10 I = 4,MM1
      DO 10 J = 4,NM1
      REU(I,J)=E1(I,J)*U(I,J)+E2(I,J)*U(I+1,J)+E3(I,J)*U(I,J+1)+E4(I,J)*
     XU(I-1,J)+E5(I,J)*U(I,J-1)+E6(I,J)*U(I+1,J+1)+E20(I,J)*U(I-1,J+1)+
     XE21(I,J)*U(I-1,J-1)+E22(I,J)*U(I+1,J-1)+E7(I,J)*V(I+1,J)+E8(I,J)*
     XV(I,J+1)+E9(I,J)*V(I-1,J)+E10(I,J)*V(I,J-1)+E11(I,J)*V(I+1,J+1)+
     XE12(I,J)*V(I-1,J+1)+E13(I,J)*V(I-1,J-1)+E14(I,J)*V(I+1,J-1)
   10 REV(I,J)=E15(I,J)*V(I,J)+E16(I,J)*V(I+1,J)+E17(I,J)*V(I,J+1)+
     XE18(I,J)*V(I-1,J)+E19(I,J)*V(I,J-1)+E27(I,J)*V(I+1,J+1)+E28(I,J)*
     XV(I-1,J+1)+E29(I,J)*V(I-1,J-1)+E30(I,J)*V(I+1,J-1)+E23(I,J)*
     XU(I+1,J)+E24(I,J)*U(I,J+1)+E25(I,J)*U(I-1,J)+E26(I,J)*U(I,J-1)+
     XE31(I,J)*U(I+1,J+1)+E12(I,J)*U(I-1,J+1)+E32(I,J)*U(I-1,J-1)+
     XE14(I,J)*U(I+1,J-1)
C     LEFT BOUNDARY  I = 3
      IF (MFI(5,INM1).NE.1) GO TO 5030
      A1 = HX(3)
      A9 = A1 + HX(4)
      D8(INM1) = (GF*(A9**2 - A1**2) + GM*A1**2)/(A1*A9*(A9 - A1))
      D9(INM1) = -GM*A1/(A9*(A9 - A1))
 5030 CONTINUE
      DO 20 J = 4,NM1
   20 REV(3,J) = D7(J)*V(3,J) + D8(J)*V(4,J) + D9(J)*V(5,J)
C     RIGHT BOUNDARY I = M
      DO 30 J = 4,NM1
   30 REV(M,J) = D10(J)*V(M,J) + D11(J)*V(MM1,J) + D12(J)*V(MM2,J)
C     LOWER BOUNDARY J = 3
      IF (MFI(IMM1,5).NE.1) GO TO 5040
      A2 = HY(3)
      A10 = A2 + HY(4)
      D2(IMM1) = (GF*(A10**2 - A2**2) + GM* A2**2)/(A2*A10*(A10 - A2))
      D3(IMM1) = -GM*A2/(A10*(A10 - A2))



 5040 CONTINUE
      DO 40 I = 4,MM1
   40 REU(I,3) = D1(I)*U(I,3) + D2(I)*U(I,4) + D3(I)*U(I,5)
C     UPPER BOUNDARY J = N
      DO 50 I = 4,MM1
   50 REU(I,N) = D4(I)*U(I,N) + D5(I)*U(I,NM1) + D6(I)*U(I,NM2)
C     INTERFACE POINTS
      L=2
      I=LI(L)
      J=LJ(L)
      A1=HX(I)
      A2=HY(J)
      A3=HX(I-1)
      A4=HY(J-1)
      A9=A1+HX(I+1)
      A10=A2+HY(J+1)
      A11=A3+HX(I-2)
      A12=A4+HY(J-2)
      CC 1=COST(L)**2
      CC 2=SINT(L)**2
      CC 4=COST(L)*SINT(L)
      CC 3=2.0*CC4
      CC 5=-CC4
      CC18=CC2-CC1
      CC6=-(A9+A1)/(A1*A9)
      CC7=A9/(A1*(A9-A1))
      CC8=-A1/(A9*(A9-A1))
      CC9=-(A10+A2)/(A2*A10)
      CC10=A10/(A2*(A10-A2))
      CC11=-A2/(A10*(A10-A2))
      CC14=A3/(A11*(A11-A3))
      CC15=(A12+A4)/(A4*A12)
      CC16=-A12/(A4*(A12-A4))
      CC17=A4/(A12*(A12-A4))
      CC12=(A1-A3)/(A1*A3)
      CC13=-A1/(A3*(A1+A3))
      CC61=A3/(A1*(A1+A3))
      CC21=CC1*(BM*CC6-BF*CC12)+CC2*(BM*CM*CC6-BF*CF*CC12)
     1    +CC3*(GM*CC9-GF*CC15)
      CC22=CC1*(BM*CM*CC9-BF*CF*CC15)+CC2*(BM*CC9-BF*CC15)
     1    +CC3*(GM*CC6-GF*CC12)
      IF (MFI(5,J).EQ.1) GO TO 5012
      CC23=CC1*BM*CC7+CC2*BM*CM*CC7-CC1*BF*CC61-CC2*BF*CF*CC61
      CC31=CC3*GM*CC7-CC3*GF*CC61
      GO TO 5015
 5012 IF (U(5,J).EQ.0.0) GO TO 5013
      CUL2=(U(4,J)+(EM/EF)*(U(5,J)-U(4,J)))/U(5,J)
      CVL2=(V(4,J)+(EM/EF)*(V(5,J)-V(4,J)))/V(5,J)
      GO TO 5014
 5013 CUL2=0.0
      CVL2=0.0
 5014 CC23=CC1*BM*CC7+CC2*BM*CM*CC7-CC1*BF*CC61*CUL2-CC2*BF*CF*CC61*CUL2
      CC31=CC3*GM*CC7-CC3*GF*CC61*CVL2
 5015 CC24=CC3*GM*CC10
      CC25=-CC1*BF*CC13-CC2*BF*CF*CC13
      CC26=-CC3*GF*CC16
      CC27=CC1*BM*CC8+CC2*BM*CM*CC8
      CC28=CC3*GM*CC11
      CC29=0.0
      CC30=-CC3*GF*CC17
      CC32=CC1*BM*CM*CC10+CC2*BM*CC10
      CC33=-CC3*GF*CC13
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      CC34=-CC1*BF*CF*CC16-CC2*BF*CC16
      CC35=CC3*GM*CC8
      CC36=CC1*BM*CM*CC11+CC2*BM*CC11
      CC37=0.0
      CC38=-CC1*BF*CF*CC17-CC2*BF*CC17
      CC39=(CC1+CC2)*(FF-FM)
      CC1=CC4
      CC2=CC5
      CC3=CC18
      IF (MFI(5,J).EQ.1) GO TO 7012
      CC43=CC1*BM*CC7+CC2*BM*CM*CC7-CC1*BF*CC61-CC2*BF*CF*CC61
      CC51=CC3*GM*CC7-CC3*GF*CC61
      GO TO 7015
 7012 CC43=CC1*BM*CC7+CC2*BM*CM*CC7-CC1*BF*CC61*CUL2-CC2*BF*CF*CC61*CUL2
      CC51=CC3*GM*CC7-CC3*GF*CC61*CVL2
 7015 CC41=CC1*(BM*CC6-BF*CC12)+CC2*(BM*CM*CC6-BF*CF*CC12)
     1     +CC3*(GM*CC9-GF*CC15)
      CC42=CC1*(BM*CM*CC9-BF*CF*CC15)+CC2*(BM*CC9-BF*CC15)
     1     +CC3*(GM*CC6-GF*CC12)
      CC44=CC3*GM*CC10
      CC45=-CC1*BF*CC13-CC2*BF*CF*CC13
      CC46=-CC3*GF*CC16
      CC47=CC1*BM*CC8+CC2*BM*CM*CC8
      CC48=CC3*GM*CC11
      CC49=0.0
      CC50=-CC3*GF*CC17
      CC52=CC1*BM*CM*CC10+CC2*BM*CC10
      CC53=-CC3*GF*CC13
      CC54=-CC1*BF*CF*CC16-CC2*BF*CC16
      CC55=CC3*GM*CC8
      CC56=CC1*BM*CM*CC11+CC2*BM*CC11
      CC57=0.0
      CC58=-CC1*BF*CF*CC17-CC2*BF*CC17
      CC59=(CC1+CC2)*(FF-FM)
      CC40=CC22/CC42
      CC60=CC41/CC21
      C 1(L)=CC21-CC41*CC40
      C 2(L)=CC22-CC42*CC40
      C 3(L)=CC23-CC43*CC40
      C 4(L)=CC24-CC44*CC40
      C 5(L)=CC25-CC45*CC40
      C 6(L)=CC26-CC46*CC40
      C 7(L)=CC27-CC47*CC40
      C 8(L)=CC28-CC48*CC40
      C 9(L)=CC29-CC49*CC40
      C10(L)=CC30-CC50*CC40
      C11(L)=CC31-CC51*CC40
      C12(L)=CC32-CC52*CC40
      C13(L)=CC33-CC53*CC40
      C14(L)=CC34-CC54*CC40
      C15(L)=CC35-CC55*CC40
      C16(L)=CC36-CC56*CC40
      C17(L)=CC37-CC57*CC40
      C18(L)=CC38-CC58*CC40
      C19(L)=CC39-CC59*CC40
      C20(L)=CC41-CC21*CC60
      C21(L)=CC42-CC22*CC60
      C22(L)=CC43-CC23*CC60
      C23(L)=CC44-CC24*CC60
      C24(L)=CC45-CC25*CC60
      C25(L)=CC46-CC26*CC60
      C26(L)=CC47-CC27*CC60



      C27(L)=CC48-CC28*CC60
      C28(L)=CC49-CC29*CC60
      C29(L)=CC50-CC30*CC60
      C30(L)=CC51-CC31*CC60
      C31(L)=CC52-CC32*CC60
      C32(L)=CC53-CC33*CC60
      C33(L)=CC54-CC34*CC60
      C34(L)=CC55-CC35*CC60
      C35(L)=CC56-CC36*CC60
      C36(L)=CC57-CC37*CC60
      C37(L)=CC58-CC38*CC60
      C38(L)=CC59-CC39*CC60
      L=NLM1
      I=LI(L)
      J=LJ(L)
      A1=HX(I)
      A2=HY(J)
      A3=HX(I-1)
      A4=HY(J-1)
      A9=A1+HX(I+1)
      A10=A2+HY(J+1)
      A11=A3+HX(I-2)
      A12=A4+HY(J-2)
      CC 1=COST(L)**2
      CC 2=SINT(L)**2
      CC 4=COST(L)*SINT(L)
      CC 3=2.0*CC4
      CC 5=-CC4
      CC18=CC2-CC1
      CC6=-(A9+A1)/(A1*A9)
      CC7=A9/(A1*(A9-A1))
      CC8=-A1/(A9*(A9-A1))
      CC9=-(A10+A2)/(A2*A10)
      CC10=A10/(A2*(A10-A2))
      CC11=-A2/(A10*(A10-A2))
      CC12=(A11+A3)/(A3*A11)
      CC13=-A11/(A3*(A11-A3))
      CC14=A3/(A11*(A11-A3))
      CC17=A4/(A12*(A12-A4))
      CC15=(A2-A4)/(A2*A4)
      CC16=-A2/(A4*(A2+A4))
      CC71=A4/(A2*(A2+A4))
      CC21=CC1*(BM*CC6-BF*CC12)+CC2*(BM*CM*CC6-BF*CF*CC12)
     1     +CC3*(GM*CC9-GF*CC15)
      CC22=CC1*(BM*CM*CC9-BF*CF*CC15)+CC2*(BM*CC9-BF*CC15)
     1     +CC3*(GM*CC6-GF*CC12)
      CC23=CC1*BM*CC7+CC2*BM*CM*CC7
      IF (MFI(I,5).EQ.1) GO TO 5022
      CC24=CC3*GM*CC10-CC3*GF*CC71
      CC32=CC1*BM*CM*CC10+CC2*BM*CC10-CC1*BF*CF*CC71-CC2*BF*CC71
      GO TO 5025
 5022 IF (U(I,5).EQ.0.0) GO TO 5023
      CUNL=(U(I,4)+(EM/EF)*(U(I,5)-U(I,4)))/U(I,5)
      CVNL=(V(I,4)+(EM/EF)*(V(I,5)-V(I,4)))/V(I,5)
      GO TO 5024
 5023 CUNL=0.0
      CVNL=0.0
 5024 CC24=CC3*GM*CC10-CC3*GF*CC71*CUNL
      CC32=CC1*BM*CM*CC10+CC2*BM*CC10-CC1*BF*CF*CC71*CVNL-CC2*BF*CC71*
     XCVNL
 5025 CC25=-CC1*BF*CC13-CC2*BF*CF*CC13
      CC26=-CC3*GF*CC16
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      CC27=CC1*BM*CC8+CC2*BM*CM*CC8
      CC28=CC3*GM*CC11
      CC29=-CC1*BF*CC14-CC2*BF*CF*CC14
      CC30=0.0
      CC31=CC3*GM*CC7
      CC33=-CC3*GF*CC13
      CC34=-CC1*BF*CF*CC16-CC2*BF*CC16
      CC35=CC3*GM*CC8
      CC36=CC1*BM*CM*CC11+CC2*BM*CC11
      CC37=-CC3*GF*CC14
      CC38=0.0
      CC39=(CC1+CC2)*(FF-FM)
      CC1=CC4
      CC2=CC5
      CC3=CC18
      IF (MFI(I,5).EQ.1) GO TO 8022
      CC44=CC3*GM*CC10-CC3*GF*CC71
      CC52=CC1*BM*CM*CC10+CC2*BM*CC10-CC1*BF*CF*CC71-CC2*BF*CC71
      GO TO 8025
 8022 CC44=CC3*GM*CC10-CC3*GF*CC71*CUNL
      CC52=CC1*BM*CM*CC10+CC2*BM*CC10-CC1*BF*CF*CC71*CVNL-CC2*BF*CC71*
     XCVNL
 8025 CC41=CC1*(BM*CC6-BF*CC12)+CC2*(BM*CM*CC6-BF*CF*CC12)
     1     +CC3*(GM*CC9-GF*CC15)
      CC42=CC1*(BM*CM*CC9-BF*CF*CC15)+CC2*(BM*CC9-BF*CC15)
     1     +CC3*(GM*CC6-GF*CC12)
      CC43=CC1*BM*CC7+CC2*BM*CM*CC7
      CC45=-CC1*BF*CC13-CC2*BF*CF*CC13
      CC46=-CC3*GF*CC16
      CC47=CC1*BM*CC8+CC2*BM*CM*CC8
      CC48=CC3*GM*CC11
      CC49=-CC1*BF*CC14-CC2*BF*CF*CC14
      CC50=0.0
      CC51=CC3*GM*CC7
      CC53=-CC3*GF*CC13
      CC54=-CC1*BF*CF*CC16-CC2*BF*CC16
      CC55=CC3*GM*CC8
      CC56=CC1*BM*CM*CC11+CC2*BM*CC11
      CC57=-CC3*GF*CC14
      CC58=0.0
      CC59=(CC1+CC2)*(FF-FM)
      CC40=CC22/CC42
      CC60=CC41/CC21
      C 1(L)=CC21-CC41*CC40
      C 2(L)=CC22-CC42*CC40
      C 3(L)=CC23-CC43*CC40
      C 4(L)=CC24-CC44*CC40
      C 5(L)=CC25-CC45*CC40
      C 6(L)=CC26-CC46*CC40
      C 7(L)=CC27-CC47*CC40
      C 8(L)=CC28-CC48*CC40
      C 9(L)=CC29-CC49*CC40
      C10(L)=CC30-CC50*CC40
      C11(L)=CC31-CC51*CC40
      C12(L)=CC32-CC52*CC40
      C13(L)=CC33-CC53*CC40
      C14(L)=CC34-CC54*CC40
      C15(L)=CC35-CC55*CC40
      C16(L)=CC36-CC56*CC40
      C17(L)=CC37-CC57*CC40
      C18(L)=CC38-CC58*CC40
      C19(L)=CC39-CC59*CC40



      C20(L)=CC41-CC21*CC60
      C21(L)=CC42-CC22*CC60
      C22(L)=CC43-CC23*CC60
      C23(L)=CC44-CC24*CC60
      C24(L)=CC45-CC25*CC60
      C25(L)=CC46-CC26*CC60
      C26(L)=CC47-CC27*CC60
      C27(L)=CC48-CC28*CC60
      C28(L)=CC49-CC29*CC60
      C29(L)=CC50-CC30*CC60
      C30(L)=CC51-CC31*CC60
      C31(L)=CC52-CC32*CC60
      C32(L)=CC53-CC33*CC60
      C33(L)=CC54-CC34*CC60
      C34(L)=CC55-CC35*CC60
      C35(L)=CC56-CC36*CC60
      C36(L)=CC57-CC37*CC60
      C37(L)=CC58-CC38*CC60
      C38(L)=CC59-CC39*CC60
      DO 60 L = 2,NLM1
      I = LI(L)
      J = LJ(L)
      REU(I,J) = C1(L)*U(I,J) + C2(L)*V(I,J) + C3(L)*U(I+1,J) + C4(L)*
     X          U(I,J+1) + C5(L)*U(I-1,J) + C6(L)*U(I,J-1) + C7(L)*
     X          U(I+2,J) + C8(L)*U(I,J+2) + C9(L)*U(I-2,J) + C10(L)*
     X          U(I,J-2) + C11(L)*V(I+1,J) + C12(L)*V(I,J+1) + C13(L)*
     X       V(I-1,J) + C14(L)*V(I,J-1) + C15(L)*V(I+2,J) + C16(L)*
     X       V(I,J+2) + C17(L)*V(I-2,J) + C18(L)*V(I,J-2) + C19(L)
   60 REV(I,J) = C20(L)*U(I,J) + C21(L)*V(I,J) + C22(L)*U(I+1,J) +
     X          C23(L)*U(I,J+1) + C24(L)*U(I-1,J) + C25(L)*U(I,J-1) +
     X          C26(L)*U(I+2,J) + C27(L)*U(I,J+2) + C28(L)*U(I-2,J) +
     X          C29(L)*U(I,J-2) + C30 (L)*V(I+1,J) + C31(L)*V(I,J+1) +
     X          C32(L)*V(I-1,J) + C33(L)*V(I,J-1) + C34(L)*V(I+2,J) +
     X          C35(L)*V(I,J+2) + C36(L)*V(I-2,J) + C37(L)*V(I,J-2) +
     X          C38(L)
      RETURN
      END
                FORTRAN MAP
CRELXTS
      SUBROUTINE RELXTS
      COMMON U,V,REU,REV,USAVE,VSAVE,U1,V2,SIGX,SIGY,SIGZ,SIGXY,CAT,
     XSIGXB,SIGXBS,SIGYB,SIGYBS,SIGXM,SIGYM,SIGZM,SIGXF,SIGYF,SIGZF,
     XHX,HY,OMB,PRM,PRF,EM,EF,ALPHAM,ALPHAF,T,EX,EY,F1,F2,COST,SINT,
     XC1,C2,C3,C4,C5,C6,C7,C8,C9,C10,C11,C12,C13,C14,C15,C16,C17,C18,
     XC19,C20,C21,C22,C23,C24,C25,C26,C27,C28,C29,C30,C31,C32,C33,C34,
     XC35,C36,C37,C38,C39,C40,C41,C42,C43,C44,C45,C46,C47,C48,
     XD1,D2,D3,D4,D5,D6,D7,D8,D9,D10,D11,D12,
     XE1,E2,E3,E4,E5,E6,E7,E8,E9,E10,E11,E12,E13,E14,E15,E16,E17,E18,E19
     X,E20,E21,E22,E23,E24,E25,E26,E27,E28,E29,E30,E31,E32
     X,AM,AF,BM,BF,CM,CF,DM,DF,         FM,FF,GM,GF,HM,HF,
     XMP2,MP1,M,MM1,MM2,MM3,NP2,NP1,N,NM1,NM2,NM3,INP3,INP2,INP1,IN,
     XINM1,INM2,INM3,IMP3,IMP2,IMP1,IM,IMM1,IMM2,IMM3,NL,NLM1,NLM2,
     XLN,LI,LJ,LAT,KNAT,NMFI,MFIJ,MFI,KNT,KPROB,IJTP,MFII,
     XNRX,NRD,NRXS,NRCS,NPRLX,NCPRLX,NTP,NPT,SIGR1,SIGR2,PCGPRX,SIGR,
     XNRXBT,UL,VL,KPSPS,A,B,KSYM,NKPROB
     X,ALPHAX,ALPHAY,IJRAIP,DA,DB,      NRAIP
      DIMENSION U(20,20),V(20,20),REU(20,20),REV(20,20),
     XE 1(17,17),E 2(17,17),E 3(17,17),E 4(17,17),E 5(17,17),E 6(17,17),
     XE 7(17,17),E 8(17,17),E 9(17,17),E10(17,17),E11(17,17),E12(17,17),
     XE13(17,17),E14(17,17),E15(17,17),E16(17,17),E17(17,17),E18(17,17),
     XE19(17,17),E20(17,17),E21(17,17),E22(17,17),E23(17,17),E24(17,17),
     XE25(17,17),E26(17,17),E27(17,17),E28(17,17),E29(17,17),E30(17,17),
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     XE31(17,17),E32(17,17),
     XLN(20,20),MFI(20,20),KNT(20,20),MFII(200),MFIJ(200)
      DIMENSION SIGXM(40),SIGYM(40),SIGZM(40),SIGXF(40),SIGYF(40),
     XSIGZF(40),COST(40),SINT(40),
     XC 1(40),C 2(40),C 3(40),C 4(40),C 5(40),C 6(40),C 7(40),C 8(40),
     XC 9(40),C10(40),C11(40),C12(40),C13(40),C14(40),C15(40),C16(40),
     XC17(40),C18(40),C19(40),C20(40),C21(40),C22(40),C23(40),C24(40),
     XC25(40),C26(40),C27(40),C28(40),C29(40),C30(40),C31(40),C32(40),
     XC33(40),C34(40),C35(40),C36(40),C37(40),C38(40),LI(40),LJ(40)
      DIMENSION HX(20),HY(20),IJTP(20),D1(20),D2(20),D3(20),D4(20),
     XD5(20),D6(20),D7(20),D8(20),D9(20),D10(20),D11(20),D12(20),
     XSIGR1(10),SIGR2(10),IJRAIP(30),    DA(15),    DB(15)
      DIMENSION PURX(10),PVRY(10),ABIJ(15),ABKJ(15)
      KNT(3,IN)=8
      KNT(IM,3)=10
      IMM4=IM-4
      INM4=IN-4
      NTCL2=0
      NTCNL=0
      REUS=0.0
      REVS=0.0
      NRXS=0
      NPRXS=0
      DO 5000 IJ=1,NRAIP
      ABIJ(IJ)=((DA(IJ)+DB(IJ))/(DA(IJ)+DB(IJ)*(EF/EM)))*EF/EM
 5000 ABKJ(IJ)=((DA(IJ)+DB(IJ))/(DA(IJ)*(EM/EF)+DB(IJ)))*EM/EF
 4001 IF (NRXS-NRX) 4002,3004,3004
 4002 NRXS=NRXS+1
      NPRXS=NPRXS+1
      DO 5001 IJ=1,NRAIP
      IJI=IJ*2-1
      IJJ=IJ*2
      I=IJRAIP(IJI)
      J=IJRAIP(IJJ)
      KI=I+1
      KJ=J+1
      IF (U(I,J).EQ.0.0) GO TO 5002
      EUIJ    =(U(KI,KJ)-ABIJ(IJ)*(U(KI,KJ)-U(I,J)))/U(I,J)
      A1=HX(KI)
      A2=HY(KJ)
      A3=HX(KI-1)
      A4=HY(KJ-1)
      G=GM
      P=AM
      EE1=2.0/(A1*(A1+A3))
      EE2=-2.0/(A1*A3)
      EE3=2.0/(A3*(A1+A3))
      EE4=2.0/(A2*(A2+A4))
      EE5=-2.0/(A2*A4)
      EE6=2.0/(A4*(A2+A4))
      EE16=1.0/(A1*A2*A3*A4*(A1+A3)*(A2+A4))
      EE7=A3**2*A4**2*EE16
      EE8=EE16*(A2**2-A4**2)*A3**2
      EE9=-EE16*A2**2*A3**2
      EE10=EE16*(A1**2-A3**2)*A4**2
      EE11=EE16*(A1**2-A3**2)*(A2**2-A4**2)
      EE12=EE16*(A1**2-A3**2)*(-A2**2)
      EE13=EE16*(-A1**2)*A4**2
      EE14=EE16*(-A1**2)*(A2**2-A4**2)
      EE15=EE16*(-A1**2)*(-A2**2)
      EE21=G*(P+1.0)*EE2+G*P*EE5
      EE26=G*EE11



      EE31=G*EE7
      EE33=G*EE15
      EE35=G*P*EE2+G*(P+1.0)*EE5
      EE40=EE26/EE35
      EE41=EE26/EE21
      E21(KI,KJ)=-EE40*EE33*EUIJ
      E32(KI,KJ)= EE33*EUIJ
      GO TO 5003
 5002 E21(KI,KJ)=0.0
      E32(KI,KJ)=0.0
 5003 IF (V(I,J).EQ.0.0) GO TO 5004
      EVIJ=(V(KI,KJ)-ABIJ(IJ)*(V(KI,KJ)-V(I,J)))/V(I,J)
      A1=HX(KI)
      A2=HY(KJ)
      A3=HX(KI-1)
      A4=HY(KJ-1)
      G=GM
      P=AM
      EE1=2.0/(A1*(A1+A3))
      EE2=-2.0/(A1*A3)
      EE3=2.0/(A3*(A1+A3))
      EE4=2.0/(A2*(A2+A4))
      EE5=-2.0/(A2*A4)
      EE6=2.0/(A4*(A2+A4))
      EE16=1.0/(A1*A2*A3*A4*(A1+A3)*(A2+A4))
      EE7=A3**2*A4**2*EE16
      EE8=EE16*(A2**2-A4**2)*A3**2
      EE9=-EE16*A2**2*A3**2
      EE10=EE16*(A1**2-A3**2)*A4**2
      EE11=EE16*(A1**2-A3**2)*(A2**2-A4**2)
      EE12=EE16*(A1**2-A3**2)*(-A2**2)
      EE13=EE16*(-A1**2)*A4**2
      EE14=EE16*(-A1**2)*(A2**2-A4**2)
      EE15=EE16*(-A1**2)*(-A2**2)
      EE21=G*(P+1.0)*EE2+G*P*EE5
      EE26=G*EE11
      EE31=G*EE7
      EE33=G*EE15
      EE35=G*P*EE2+G*(P+1.0)*EE5
      EE40=EE26/EE35
      EE41=EE26/EE21
      E13(KI,KJ)=EE33*EVIJ
      E29(KI,KJ)=-EE41*EE33*EVIJ
      GO TO 5005
 5004 E13(KI,KJ)=0.0
      E29(KI,KJ)=0.0
 5005 IF (U(KI,KJ).EQ.0.0) GO TO 5006
      EUKJ=(U(I,J)+ABKJ(IJ)*(U(KI,KJ)-U(I,J)))/U(KI,KJ)
      A1=HX(I)
      A2=HY(J)
      A3=HX(I-1)
      A4=HY(J-1)
      G=GF
      P=AF
      EE1=2.0/(A1*(A1+A3))
      EE2=-2.0/(A1*A3)
      EE3=2.0/(A3*(A1+A3))
      EE4=2.0/(A2*(A2+A4))
      EE5=-2.0/(A2*A4)
      EE6=2.0/(A4*(A2+A4))
      EE16=1.0/(A1*A2*A3*A4*(A1+A3)*(A2+A4))
      EE7=A3**2*A4**2*EE16
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      EE8=EE16*(A2**2-A4**2)*A3**2
      EE9=-EE16*A2**2*A3**2
      EE10=EE16*(A1**2-A3**2)*A4**2
      EE11=EE16*(A1**2-A3**2)*(A2**2-A4**2)
      EE12=EE16*(A1**2-A3**2)*(-A2**2)
      EE13=EE16*(-A1**2)*A4**2
      EE14=EE16*(-A1**2)*(A2**2-A4**2)
      EE15=EE16*(-A1**2)*(-A2**2)
      EE21=G*(P+1.0)*EE2+G*P*EE5
      EE26=G*EE11
      EE31=G*EE7
      EE33=G*EE15
      EE35=G*P*EE2+G*(P+1.0)*EE5
      EE40=EE26/EE35
      EE41=EE26/EE21
      E 6(I,J)=-EE40*EE31*EUKJ
      E31(I,J)= EE31*EUKJ
      GO TO 5007
 5006 E 6(I,J)=0.0
      E31(I,J)=0.0
 5007 IF (V(KI,KJ).EQ.0.0) GO TO 5008
      EVKJ=(V(I,J)+ABKJ(IJ)*(V(KI,KJ)-V(I,J)))/V(KI,KJ)
      A1=HX(I)
      A2=HY(J)
      A3=HX(I-1)
      A4=HY(J-1)
      G=GF
      P=AF
      EE1=2.0/(A1*(A1+A3))
      EE2=-2.0/(A1*A3)
      EE3=2.0/(A3*(A1+A3))
      EE4=2.0/(A2*(A2+A4))
      EE5=-2.0/(A2*A4)
      EE6=2.0/(A4*(A2+A4))
      EE16=1.0/(A1*A2*A3*A4*(A1+A3)*(A2+A4))
      EE7=A3**2*A4**2*EE16
      EE8=EE16*(A2**2-A4**2)*A3**2
      EE9=-EE16*A2**2*A3**2
      EE10=EE16*(A1**2-A3**2)*A4**2
      EE11=EE16*(A1**2-A3**2)*(A2**2-A4**2)
      EE12=EE16*(A1**2-A3**2)*(-A2**2)
      EE13=EE16*(-A1**2)*A4**2
      EE14=EE16*(-A1**2)*(A2**2-A4**2)
      EE15=EE16*(-A1**2)*(-A2**2)
      EE21=G*(P+1.0)*EE2+G*P*EE5
      EE26=G*EE11
      EE31=G*EE7
      EE33=G*EE15
      EE35=G*P*EE2+G*(P+1.0)*EE5
      EE40=EE26/EE35
      EE41=EE26/EE21
      E11(I,J)= EE31*EVKJ
      E27(I,J)=-EE41*EE31*EVKJ
      GO TO 5009
 5008 E11(I,J)=0.0
      E27(I,J)=0.0
 5009 CONTINUE
 5001 CONTINUE
      IF (NTCL2.NE.0) GO TO 5010
      L=2
      I=LI(L)
      J=LJ(L)



      A1=HX(I)
      A2=HY(J)
      A3=HX(I-1)
      A4=HY(J-1)
      A9=A1+HX(I+1)
      A10=A2+HY(J+1)
      A11=A3+HX(I-2)
      A12=A4+HY(J-2)
      CC 1=COST(L)**2
      CC 2=SINT(L)**2
      CC 4=COST(L)*SINT(L)
      CC 3=2.0*CC4
      CC 5=-CC4
      CC18=CC2-CC1
      CC6=-(A9+A1)/(A1*A9)
      CC7=A9/(A1*(A9-A1))
      CC8=-A1/(A9*(A9-A1))
      CC9=-(A10+A2)/(A2*A10)
      CC10=A10/(A2*(A10-A2))
      CC11=-A2/(A10*(A10-A2))
      CC14=A3/(A11*(A11-A3))
      CC15=(A12+A4)/(A4*A12)
      CC16=-A12/(A4*(A12-A4))
      CC17=A4/(A12*(A12-A4))
      CC12=(A1-A3)/(A1*A3)
      CC13=-A1/(A3*(A1+A3))
      CC61=A3/(A1*(A1+A3))
      CC21=CC1*(BM*CC6-BF*CC12)+CC2*(BM*CM*CC6-BF*CF*CC12)
     1     +CC3*(GM*CC9-GF*CC15)
      CC22=CC1*(BM*CM*CC9-BF*CF*CC15)+CC2*(BM*CC9-BF*CC15)
     1     +CC3*(GM*CC6-GF*CC12)
      IF (MFI(5,J).EQ.1) GO TO 5012
      NTCL2=1
      CC23=CC1*BM*CC7+CC2*BM*CM*CC7-CC1*BF*CC61-CC2*BF*CF*CC61
      CC31=CC3*GM*CC7-CC3*GF*CC61
      GO TO 5015
 5012 IF (U(5,J).EQ.0.0) GO TO 5013
      CUL2=(U(4,J)+(EM/EF)*(U(5,J)-U(4,J)))/U(5,J)
      CVL2=(V(4,J)+(EM/EF)*(V(5,J)-V(4,J)))/V(5,J)
      GO TO 5014
 5013 CUL2=0.0
      CVL2=0.0
 5014 CC23=CC1*BM*CC7+CC2*BM*CM*CC7-CC1*BF*CC61*CUL2-CC2*BF*CF*CC61*CUL2
      CC31=CC3*GM*CC7-CC3*GF*CC61*CVL2
 5015 CC24=CC3*GM*CC10
      CC25=-CC1*BF*CC13-CC2*BF*CF*CC13
      CC26=-CC3*GF*CC16
      CC27=CC1*BM*CC8+CC2*BM*CM*CC8
      CC28=CC3*GM*CC11
      CC29=0.0
      CC30=-CC3*GF*CC17
      CC32=CC1*BM*CM*CC10+CC2*BM*CC10
      CC33=-CC3*GF*CC13
      CC34=-CC1*BF*CF*CC16-CC2*BF*CC16
      CC35=CC3*GM*CC8
      CC36=CC1*BM*CM*CC11+CC2*BM*CC11
      CC37=0.0
      CC38=-CC1*BF*CF*CC17-CC2*BF*CC17
      CC39=(CC1+CC2)*(FF-FM)
      CC1=CC4
      CC2=CC5
      CC3=CC18
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      IF (MFI(5,J).EQ.1) GO TO 7012
      CC43=CC1*BM*CC7+CC2*BM*CM*CC7-CC1*BF*CC61-CC2*BF*CF*CC61
      CC51=CC3*GM*CC7-CC3*GF*CC61
      GO TO 7015
 7012 CC43=CC1*BM*CC7+CC2*BM*CM*CC7-CC1*BF*CC61*CUL2-CC2*BF*CF*CC61*CUL2
      CC51=CC3*GM*CC7-CC3*GF*CC61*CVL2
 7015 CC41=CC1*(BM*CC6-BF*CC12)+CC2*(BM*CM*CC6-BF*CF*CC12)
     1     +CC3*(GM*CC9-GF*CC15)
      CC42=CC1*(BM*CM*CC9-BF*CF*CC15)+CC2*(BM*CC9-BF*CC15)
     1     +CC3*(GM*CC6-GF*CC12)
      CC44=CC3*GM*CC10
      CC45=-CC1*BF*CC13-CC2*BF*CF*CC13
      CC46=-CC3*GF*CC16
      CC47=CC1*BM*CC8+CC2*BM*CM*CC8
      CC48=CC3*GM*CC11
      CC49=0.0
      CC50=-CC3*GF*CC17
      CC52=CC1*BM*CM*CC10+CC2*BM*CC10
      CC53=-CC3*GF*CC13
      CC54=-CC1*BF*CF*CC16-CC2*BF*CC16
      CC55=CC3*GM*CC8
      CC56=CC1*BM*CM*CC11+CC2*BM*CC11
      CC57=0.0
      CC58=-CC1*BF*CF*CC17-CC2*BF*CC17
      CC59=(CC1+CC2)*(FF-FM)
      CC40=CC22/CC42
      CC60=CC41/CC21
      C 1(L)=CC21-CC41*CC40
      C 2(L)=CC22-CC42*CC40
      C 3(L)=CC23-CC43*CC40
      C 4(L)=CC24-CC44*CC40
      C 5(L)=CC25-CC45*CC40
      C 6(L)=CC26-CC46*CC40
      C 7(L)=CC27-CC47*CC40
      C 8(L)=CC28-CC48*CC40
      C 9(L)=CC29-CC49*CC40
      C10(L)=CC30-CC50*CC40
      C11(L)=CC31-CC51*CC40
      C12(L)=CC32-CC52*CC40
      C13(L)=CC33-CC53*CC40
      C14(L)=CC34-CC54*CC40
      C15(L)=CC35-CC55*CC40
      C16(L)=CC36-CC56*CC40
      C17(L)=CC37-CC57*CC40
      C18(L)=CC38-CC58*CC40
      C19(L)=CC39-CC59*CC40
      C20(L)=CC41-CC21*CC60
      C21(L)=CC42-CC22*CC60
      C22(L)=CC43-CC23*CC60
      C23(L)=CC44-CC24*CC60
      C24(L)=CC45-CC25*CC60
      C25(L)=CC46-CC26*CC60
      C26(L)=CC47-CC27*CC60
      C27(L)=CC48-CC28*CC60
      C28(L)=CC49-CC29*CC60
      C29(L)=CC50-CC30*CC60
      C30(L)=CC51-CC31*CC60
      C31(L)=CC52-CC32*CC60
      C32(L)=CC53-CC33*CC60
      C33(L)=CC54-CC34*CC60
      C34(L)=CC55-CC35*CC60



      C35(L)=CC56-CC36*CC60
      C36(L)=CC57-CC37*CC60
      C37(L)=CC58-CC38*CC60
      C38(L)=CC59-CC39*CC60
 5010 CONTINUE
      IF (NTCNL.NE.0) GO TO 5020
      L=NLM1
      I=LI(L)
      J=LJ(L)
      A1=HX(I)
      A2=HY(J)
      A3=HX(I-1)
      A4=HY(J-1)
      A9=A1+HX(I+1)
      A10=A2+HY(J+1)
      A11=A3+HX(I-2)
      A12=A4+HY(J-2)
      CC 1=COST(L)**2
      CC 2=SINT(L)**2
      CC 4=COST(L)*SINT(L)
      CC 3=2.0*CC4
      CC 5=-CC4
      CC18=CC2-CC1
      CC6=-(A9+A1)/(A1*A9)
      CC7=A9/(A1*(A9-A1))
      CC8=-A1/(A9*(A9-A1))
      CC9=-(A10+A2)/(A2*A10)
      CC10=A10/(A2*(A10-A2))
      CC11=-A2/(A10*(A10-A2))
      CC12=(A11+A3)/(A3*A11)
      CC13=-A11/(A3*(A11-A3))
      CC14=A3/(A11*(A11-A3))
      CC17=A4/(A12*(A12-A4))
      CC15=(A2-A4)/(A2*A4)
      CC16=-A2/(A4*(A2+A4))
      CC71=A4/(A2*(A2+A4))
      CC21=CC1*(BM*CC6-BF*CC12)+CC2*(BM*CM*CC6-BF*CF*CC12)
     1     +CC3*(GM*CC9-GF*CC15)
      CC22=CC1*(BM*CM*CC9-BF*CF*CC15)+CC2*(BM*CC9-BF*CC15)
     1     +CC3*(GM*CC6-GF*CC12)
      CC23=CC1*BM*CC7+CC2*BM*CM*CC7
      IF (MFI(I,5).EQ.1) GO TO 5022
      NTCNL=1
      CC24=CC3*GM*CC10-CC3*GF*CC71
      CC32=CC1*BM*CM*CC10+CC2*BM*CC10-CC1*BF*CF*CC71-CC2*BF*CC71
      GO TO 5025
 5022 IF (U(I,5).EQ.0.0) GO TO 5023
      CUNL=(U(I,4)+(EM/EF)*(U(I,5)-U(I,4)))/U(I,5)
      CVNL=(V(I,4)+(EM/EF)*(V(I,5)-V(I,4)))/V(I,5)
      GO TO 5024
 5023 CUNL=0.0
      CVNL=0.0
 5024 CC24=CC3*GM*CC10-CC3*GF*CC71*CUNL
      CC32=CC1*BM*CM*CC10+CC2*BM*CC10-CC1*BF*CF*CC71*CVNL-CC2*BF*CC71*
     XCVNL
 5025 CC25=-CC1*BF*CC13-CC2*BF*CF*CC13
      CC26=-CC3*GF*CC16
      CC27=CC1*BM*CC8+CC2*BM*CM*CC8
      CC28=CC3*GM*CC11
      CC29=-CC1*BF*CC14-CC2*BF*CF*CC14
      CC30=0.0
      CC31=CC3*GM*CC7
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      CC33=-CC3*GF*CC13
      CC34=-CC1*BF*CF*CC16-CC2*BF*CC16
      CC35=CC3*GM*CC8
      CC36=CC1*BM*CM*CC11+CC2*BM*CC11
      CC37=-CC3*GF*CC14
      CC38=0.0
      CC39=(CC1+CC2)*(FF-FM)
      CC1=CC4
      CC2=CC5
      CC3=CC18
      IF (MFI(I,5).EQ.1) GO TO 8022
      CC44=CC3*GM*CC10-CC3*GF*CC71
      CC52=CC1*BM*CM*CC10+CC2*BM*CC10-CC1*BF*CF*CC71-CC2*BF*CC71
      GO TO 8025
 8022 CC44=CC3*GM*CC10-CC3*GF*CC71*CUNL
      CC52=CC1*BM*CM*CC10+CC2*BM*CC10-CC1*BF*CF*CC71*CVNL-CC2*BF*CC71*
     XCVNL
 8025 CC41=CC1*(BM*CC6-BF*CC12)+CC2*(BM*CM*CC6-BF*CF*CC12)
     1     +CC3*(GM*CC9-GF*CC15)
      CC42=CC1*(BM*CM*CC9-BF*CF*CC15)+CC2*(BM*CC9-BF*CC15)
     1     +CC3*(GM*CC6-GF*CC12)
      CC43=CC1*BM*CC7+CC2*BM*CM*CC7
      CC45=-CC1*BF*CC13-CC2*BF*CF*CC13
      CC46=-CC3*GF*CC16
      CC47=CC1*BM*CC8+CC2*BM*CM*CC8
      CC48=CC3*GM*CC11
      CC49=-CC1*BF*CC14-CC2*BF*CF*CC14
      CC50=0.0
      CC51=CC3*GM*CC7
      CC53=-CC3*GF*CC13
      CC54=-CC1*BF*CF*CC16-CC2*BF*CC16
      CC55=CC3*GM*CC8
      CC56=CC1*BM*CM*CC11+CC2*BM*CC11
      CC57=-CC3*GF*CC14
      CC58=0.0
      CC59=(CC1+CC2)*(FF-FM)
      CC40=CC22/CC42
      CC60=CC41/CC21
      C 1(L)=CC21-CC41*CC40
      C 2(L)=CC22-CC42*CC40
      C 3(L)=CC23-CC43*CC40
      C 4(L)=CC24-CC44*CC40
      C 5(L)=CC25-CC45*CC40
      C 6(L)=CC26-CC46*CC40
      C 7(L)=CC27-CC47*CC40
      C 8(L)=CC28-CC48*CC40
      C 9(L)=CC29-CC49*CC40
      C10(L)=CC30-CC50*CC40
      C11(L)=CC31-CC51*CC40
      C12(L)=CC32-CC52*CC40
      C13(L)=CC33-CC53*CC40
      C14(L)=CC34-CC54*CC40
      C15(L)=CC35-CC55*CC40
      C16(L)=CC36-CC56*CC40
      C17(L)=CC37-CC57*CC40
      C18(L)=CC38-CC58*CC40
      C19(L)=CC39-CC59*CC40
      C20(L)=CC41-CC21*CC60
      C21(L)=CC42-CC22*CC60
      C22(L)=CC43-CC23*CC60
      C23(L)=CC44-CC24*CC60
      C24(L)=CC45-CC25*CC60
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      C25(L)=CC46-CC26*CC60
      C26(L)=CC47-CC27*CC60
      C27(L)=CC48-CC28*CC60
      C28(L)=CC49-CC29*CC60
      C29(L)=CC50-CC30*CC60
      C30(L)=CC51-CC31*CC60
      C31(L)=CC52-CC32*CC60
      C32(L)=CC53-CC33*CC60
      C33(L)=CC54-CC34*CC60
      C34(L)=CC55-CC35*CC60
      C35(L)=CC56-CC36*CC60
      C36(L)=CC57-CC37*CC60
      C37(L)=CC58-CC38*CC60
      C38(L)=CC59-CC39*CC60
 5020 CONTINUE
      IF (MFI(5,INM1).NE.1) GO TO 5030
      A1 = HX(3)
      A9 = A1 + HX(4)
      D8(INM1) = (GF*(A9**2 - A1**2) + GM*A1**2)/(A1*A9*(A9 - A1))
      D9(INM1) = -GM*A1/(A9*(A9 - A1))
 5030 CONTINUE
      IF (MFI(IMM1,5).NE.1) GO TO 5040
      A2 = HY(3)
      A10 = A2 + HY(4)
      D2(IMM1) = (GF*(A10**2 - A2**2) + GM* A2**2)/(A2*A10*(A10 - A2))
      D3(IMM1) = -GM*A2/(A10*(A10 - A2))
 5040 CONTINUE
      GO TO (4201,4202,4201),KPROB
 4201 KMM1=MM1
      KMP2=MP2
      KNM1=NM1
      KNP2=NP2
      GO TO 4203
 4202 KMM1=NM1
      KMP2=NP2
      KNM1=MM1
      KNP2=MP2
 4203 DO 50 III=2,KMM1
      II=KMP2-III
      DO 50 JJJ=2,KNM1
      JJ=KNP2-JJJ
      GO TO (4204,4205,4204),KPROB
 4204 I=II
      J=JJ
      GO TO 4113
 4205 I=JJ
      J=II
 4113 KNAT=KNT(I,J)
      MFIAT=MFI(I,J)
      GO TO (50,2002,2003,50,50,2006,2007,2008,2009,2010,2011,50,50,
     X50,50,50,50),KNAT
 2002 CUAT=E1(I,J)
      CVAT=E15(I,J)
      GO TO 1
 2003 LAT=LN(I,J)
      CUAT=C1(LAT)
      CVAT=C21(LAT)
      GO TO 1
 2006 LAT=LN(I,J)
      GO TO 1
 2007 LAT=LN(I,J)
      GO TO 1
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2008 CVAT=D7(J)
     GO TO 1
2009 CVAT=D10(J)
     GO TO 1
2010 CUAT=D1(I)
     GO TO 1
2011 CUAT=D4(I)
     GO TO 1
   1 DO 51 KIJ=1,13
     GO TO (9013,9002,9003,9004,9005,9006,9007,9008,9009,9010,9011,
    X9012,9001),KIJ
9001 KI=I+1
     KJ=J
     GO TO 30
9002 KI=I
     KJ=J+1
     GO TO 30
9003 KI=I-1
     KJ=J
     GO TO 30
9004 KI=I
     KJ=J-1
     GO TO 30
9005 KI=I+1
     KJ=J+1
     GO TO 30
9006 KI=I-1
     KJ=J+1
     GO TO 30
9007 KI=I-1
     KJ=J-1
     GO TO 30
9008 KI=I+1
     KJ=J-1
     GO TO 30
9009 KI=I+2
     KJ=J
     GO TO 30
9010 KI=I
     KJ=J+2
     GO TO 30
9011 KI=I-2
     KJ=J
     GO TO 30
9012 KI=I
     KJ=J-2
     GO TO 30
9013 KI=I
     KJ=J
  30 KN=KNT(KI,KJ)
     GO TO (51,2,3,3,3,6,7,8,9,10,11,51,51,51,51,51,51),KN
   2 GO TO (213,202,203,204,205,206,207,208,51,51,51,51,201),KIJ
 201 REU(KI,KJ)=REU(KI,KJ)-REUS    *OMB*(E 4(KI,KJ)/CUAT)
     REU(KI,KJ)=REU(KI,KJ)-REVS    *OMB*(E 9(KI,KJ)/CVAT)
     REV(KI,KJ)=REV(KI,KJ)-REUS    *OMB*(E25(KI,KJ)/CUAT)
     REV(KI,KJ)=REV(KI,KJ)-REVS    *OMB*(E18(KI,KJ)/CVAT)
     GO TO 51
 202 REU(KI,KJ)=REU(KI,KJ)-REUS    *OMB*(E 5(KI,KJ)/CUAT)
     REU(KI,KJ)=REU(KI,KJ)-REVS    *OMB*(E10(KI,KJ)/CVAT)
     REV(KI,KJ)=REV(KI,KJ)-REUS    *OMB*(E26(KI,KJ)/CUAT)
     REV(KI,KJ)=REV(KI,KJ)-REVS    *OMB*(E19(KI,KJ)/CVAT)
     GO TO 51


 203 REU(KI,KJ)=REU(KI,KJ)-REUS    *OMB*(E 2(KI,KJ)/CUAT)
     REU(KI,KJ)=REU(KI,KJ)-REVS    *OMB*(E 7(KI,KJ)/CVAT)
     REV(KI,KJ)=REV(KI,KJ)-REUS    *OMB*(E23(KI,KJ)/CUAT)
     REV(KI,KJ)=REV(KI,KJ)-REVS    *OMB*(E16(KI,KJ)/CVAT)
     GO TO 51
 204 REU(KI,KJ)=REU(KI,KJ)-REUS    *OMB*(E 3(KI,KJ)/CUAT)
     REU(KI,KJ)=REU(KI,KJ)-REVS    *OMB*(E 8(KI,KJ)/CVAT)
     REV(KI,KJ)=REV(KI,KJ)-REUS    *OMB*(E24(KI,KJ)/CUAT)
     REV(KI,KJ)=REV(KI,KJ)-REVS    *OMB*(E17(KI,KJ)/CVAT)
     GO TO 51
 205 REU(KI,KJ)=REU(KI,KJ)-REVS    *OMB*(E13(KI,KJ)/CVAT)
     REU(KI,KJ)=REU(KI,KJ)-REUS    *OMB*(E21(KI,KJ)/CUAT)
     REV(KI,KJ)=REV(KI,KJ)-REUS    *OMB*(E32(KI,KJ)/CUAT)
     REV(KI,KJ)=REV(KI,KJ)-REVS    *OMB*(E29(KI,KJ)/CVAT)
     GO TO 51
 206 REU(KI,KJ)=REU(KI,KJ)-REVS    *OMB*(E14(KI,KJ)/CVAT)
     REU(KI,KJ)=REU(KI,KJ)-REUS    *OMB*(E22(KI,KJ)/CUAT)
     REV(KI,KJ)=REV(KI,KJ)-REUS    *OMB*(E14(KI,KJ)/CUAT)
     REV(KI,KJ)=REV(KI,KJ)-REVS    *OMB*(E30(KI,KJ)/CVAT)
     GO TO 51
 207 REU(KI,KJ)=REU(KI,KJ)-REVS    *OMB*(E11(KI,KJ)/CVAT)
     REU(KI,KJ)=REU(KI,KJ)-REUS    *OMB*(E 6(KI,KJ)/CUAT)
     REV(KI,KJ)=REV(KI,KJ)-REUS    *OMB*(E31(KI,KJ)/CUAT)
     REV(KI,KJ)=REV(KI,KJ)-REVS    *OMB*(E27(KI,KJ)/CVAT)
     GO TO 51
 208 REU(KI,KJ)=REU(KI,KJ)-REVS    *OMB*(E12(KI,KJ)/CVAT)
     REU(KI,KJ)=REU(KI,KJ)-REUS    *OMB*(E20(KI,KJ)/CUAT)
     REV(KI,KJ)=REV(KI,KJ)-REUS    *OMB*(E12(KI,KJ)/CUAT)
     REV(KI,KJ)=REV(KI,KJ)-REVS    *OMB*(E28(KI,KJ)/CVAT)
     GO TO 51
 213 REUS=REU(I,J)
     REU(KI,KJ)=REU(KI,KJ)-REUS    *OMB*(E 1(KI,KJ)/CUAT)
     REVS=REV(I,J)
     REV(KI,KJ)=REV(KI,KJ)-REVS    *OMB*(E15(KI,KJ)/CVAT)
     U(I,J)=U(I,J)-REUS*OMB/CUAT
     V(I,J)=V(I,J)-REVS*OMB/CVAT
     GO TO 51
   3 L=LN(KI,KJ)
     GO TO (313,302,303,304,51,51,51,51,309,310,311,312,301),KIJ
 301 REU(KI,KJ)=REU(KI,KJ)-REUS    *OMB*(C 5(L)/CUAT)
     REU(KI,KJ)=REU(KI,KJ)-REVS    *OMB*(C13(L)/CVAT)
     REV(KI,KJ)=REV(KI,KJ)-REUS    *OMB*(C24(L)/CUAT)
     REV(KI,KJ)=REV(KI,KJ)-REVS    *OMB*(C32(L)/CVAT)
     GO TO 51
 302 REU(KI,KJ)=REU(KI,KJ)-REUS    *OMB*(C 6(L)/CUAT)
     REU(KI,KJ)=REU(KI,KJ)-REVS    *OMB*(C14(L)/CVAT)
     REV(KI,KJ)=REV(KI,KJ)-REUS    *OMB*(C25(L)/CUAT)
     REV(KI,KJ)=REV(KI,KJ)-REVS    *OMB*(C33(L)/CVAT)
     GO TO 51
 303 REU(KI,KJ)=REU(KI,KJ)-REUS    *OMB*(C 3(L)/CUAT)
     REU(KI,KJ)=REU(KI,KJ)-REVS    *OMB*(C11(L)/CVAT)
     REV(KI,KJ)=REV(KI,KJ)-REUS    *OMB*(C22(L)/CUAT)
     REV(KI,KJ)=REV(KI,KJ)-REVS    *OMB*(C30(L)/CVAT)
     GO TO 51
 304 REU(KI,KJ)=REU(KI,KJ)-REUS    *OMB*(C 4(L)/CUAT)
     REU(KI,KJ)=REU(KI,KJ)-REVS    *OMB*(C12(L)/CVAT)
     REV(KI,KJ)=REV(KI,KJ)-REUS    *OMB*(C23(L)/CUAT)
     REV(KI,KJ)=REV(KI,KJ)-REVS    *OMB*(C31(L)/CVAT)
     GO TO 51
 309 REU(KI,KJ)=REU(KI,KJ)-REUS    *OMB*(C 9(L)/CUAT)
     REU(KI,KJ)=REU(KI,KJ)-REVS    *OMB*(C17(L)/CVAT)
     REV(KI,KJ)=REV(KI,KJ)-REUS    *OMB*(C28(L)/CUAT)
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      REV(KI,KJ)=REV(KI,KJ)-REVS     *OMB*(C36(L)/CVAT)
      GO TO 51
  310 REU(KI,KJ)=REU(KI,KJ)-REUS     *OMB*(C10(L)/CUAT)
      REU(KI,KJ)=REU(KI,KJ)-REVS     *OMB*(C18(L)/CVAT)
      REV(KI,KJ)=REV(KI,KJ)-REUS     *OMB*(C29(L)/CUAT)
      REV(KI,KJ)=REV(KI,KJ)-REVS     *OMB*(C37(L)/CVAT)
      GO TO 51
  311 REU(KI,KJ)=REU(KI,KJ)-REUS     *OMB*(C 7(L)/CUAT)
      REU(KI,KJ)=REU(KI,KJ)-REVS     *OMB*(C15(L)/CVAT)
      REV(KI,KJ)=REV(KI,KJ)-REUS     *OMB*(C26(L)/CUAT)
      REV(KI,KJ)=REV(KI,KJ)-REVS     *OMB*(C34(L)/CVAT)
      GO TO 51
  312 REU(KI,KJ)=REU(KI,KJ)-REUS     *OMB*(C 8(L)/CUAT)
      REU(KI,KJ)=REU(KI,KJ)-REVS     *OMB*(C16(L)/CVAT)
      REV(KI,KJ)=REV(KI,KJ)-REUS     *OMB*(C27(L)/CUAT)
      REV(KI,KJ)=REV(KI,KJ)-REVS     *OMB*(C35(L)/CVAT)
      GO TO 51
  313 REUS=REU(I,J)
  314 REU(KI,KJ)=REU(KI,KJ)-REUS     *OMB*(C 1(L)/CUAT)
      REVS=REV(I,J)
      REV(KI,KJ)=REV(KI,KJ)-REVS     *OMB*(C21(L)/CVAT)
      U(I,J)=U(I,J)-REUS*OMB/CUAT
      V(I,J)=V(I,J)-REVS*OMB/CVAT
      GO TO 51
    6 L=LN(KI,KJ)
      GO TO 51
    7 L=LN(KI,KJ)
      GO TO 51
    8 GO TO(813,51,803,51,51,51,51,51,51,51,811,51,51 ),KIJ
  803 REV(KI,KJ)=REV(KI,KJ)-REVS     *OMB*(D 8(J)/CVAT)
      GO TO 51
  811 REV(KI,KJ)=REV(KI,KJ)-REVS     *OMB*(D 9(J)/CVAT)
      GO TO 51
  813 REVS=REV(I,J)
      REV(KI,KJ)=REV(KI,KJ)-REVS     *OMB*(D 7(J)/CVAT)
      V(I,J)=V(I,J)-REVS*OMB/CVAT
      REUS=0.0
      GO TO 51
    9 GO TO (913,51,51,51,51,51,51,51,909,51,51,51,901),KIJ
  901 REV(KI,KJ)=REV(KI,KJ)-REVS     *OMB*(D11(J)/CVAT)
      GO TO 51
  909 REV(KI,KJ)=REV(KI,KJ)-REVS     *OMB*(D12(J)/CVAT)
      GO TO 51
  913 REVS=REV(I,J)
      REV(KI,KJ)=REV(KI,KJ)-REVS     *OMB*(D10(J)/CVAT)
      V(I,J)=V(I,J)-REVS*OMB/CVAT
      REUS=0.0
      GO TO 51
   10 GO TO (1013,51,51,1004,51,51,51,51,51,51,51,1012,51),KIJ
 1004 REU(KI,KJ)=REU(KI,KJ)-REUS     *OMB*(D 2(I)/CUAT)
      GO TO 51
 1012 REU(KI,KJ)=REU(KI,KJ)-REUS     *OMB*(D 3(I)/CUAT)
      GO TO 51
 1013 REUS=REU(I,J)
      REU(KI,KJ)=REU(KI,KJ)-REUS     *OMB*(D 1(I)/CUAT)
      U(I,J)=U(I,J)-REUS*OMB/CUAT
      REVS=0.0
      GO TO 51
   11 GO TO (1113,1102,51,51,51,51,51,51,51,1110,51,51,51),KIJ
 1102 REU(KI,KJ)=REU(KI,KJ)-REUS     *OMB*(D 5(I)/CUAT)
      GO TO 51
 1110 REU(KI,KJ)=REU(KI,KJ)-REUS     *OMB*(D 6(I)/CUAT)



      GO TO 51
 1113 REUS=REU(I,J)
      REU(KI,KJ)=REU(KI,KJ)-REUS     *OMB*(D 4(I)/CUAT)
      U(I,J)=U(I,J)-REUS*OMB/CUAT
      REVS=0.0
      GO TO 51
   51 CONTINUE
   50 CONTINUE
      NPT=NTP
  167 IF (NRXS.LE.NRXBT) GO TO 3005
      NPT=0
      DO 3001 IJ=1,NTP
      I=IJTP(2*IJ-1)
      J=IJTP(2*IJ)
      A1=HX(I)
      A2=HY(J)
      A3=HX(I-1)
      A4=HY(J-1)
      PURX(IJ)=(1.0/(A1*A3*(A1+A3)))*(A3**2*U(I+1,J)+(A1**2-A3**2)*U(I,
     1J)-A1**2*U(I-1,J))
      PVRY(IJ)=(1.0/(A2*A4*(A2+A4)))*(A4**2*V(I,J+1)+(A2**2-A4**2)*V(I,
     1J)-A2**2*V(I,J-1))
      GO TO (3100,3200,3100),KPROB
 3100 SIGR2(IJ)=BM*PURX(IJ)+BM*CM*PVRY(IJ)-FM
      GO TO 3001
 3200 SIGR2(IJ)=BM*CM*PURX(IJ)+BM*PVRY(IJ)-FM
 3001 CONTINUE
      DO 3002 IJ=1,NTP
      I=IJTP(2*IJ-1)
      J=IJTP(2*IJ)
      PCG=ABS(((SIGR2(IJ)-SIGR1(IJ))/SIGR2(IJ))*100.0)
      IF (PCG.LE .PCGPRX) GO TO 3002
      NPT=NPT+1
 3002 SIGR1(IJ)=SIGR2(IJ)
      IF (NPT.EQ.0) GO TO 3004
 3005 CONTINUE
      IF(NRXS-NCPRLX) 4005,4005,4004
 4004 CONTINUE
      IF (NPRXS-NPRLX) 4001,4006,4006
 4006 NPRXS=0
 4005 CONTINUE
      WRITE (5,4041) NRXS,KPROB
 4041 FORMAT(1H1,49X,21H RESULTS OF RELAX NO. ,I4,5X,11HPROBLEM NO.,I3/)
      WRITE (5,4042) (((I,J,U(I,J),V(I,J),REU(I,J),REV(I,J)),J=3,N),I=3,
     1M)
 4042 FORMAT(1H ,//,6X,1HI,3X,1HJ,18X,1HU,19X,1HV,14X,10HU RESIDUAL,10X,
     110HV RESIDUAL,///,(3X,2I4,6X,4E20.8))
      WRITE (5,4043) NPT,PCGPRX
      LPRX=NRXS
      GO TO 4001
 3004 IF (NRXS.EQ.LPRX) GO TO 4044
      WRITE (5,4041) NRXS,KPROB
      WRITE (5,4042) (((I,J,U(I,J),V(I,J),REU(I,J),REV(I,J)),J=3,N),I=3,
     1M)
      WRITE (5,4043) NPT,PCGPRX
 4043 FORMAT(1H ,///,I10,92H  TEST POINTS HAVE NOT YET CONVERGED TO THE
     1SPECIFIED MINIMUM CHANGE IN STRESS PER RELAX OF ,F8.3,7HPERCENT)
 4044 RETURN
      END
                FORTRAN MAP
CSTRSTS
      SUBROUTINE STRSTS
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      COMMON U,V,REU,REV,USAVE,VSAVE,UI,V2,SIGX,SIGY,SIGZ,SIGXY,CAT,
     XSIGXB,SIGXBS,SIGYB,SIGYBS,SIGXM,SIGYM,SIGZM,SIGXF,SIGYF,SIGZF,
     XHX,HY,OMB,PRM,PRF,EM,EF,ALPHAM,ALPHAF,T,EX,EY,F1,F2,COST,SINT,
     XC1,C2,C3,C4,C5,C6,C7,C8,C9,C10,C11,C12,C13,C14,C15,C16,C17,C18,
     XC19,C20,C21,C22,C23,C24,C25,C26,C27,C28,C29,C30,C31,C32,C33,C34,
     XC35,C36,C37,C38,C39,C40,C41,C42,C43,C44,C45,C46,C47,C48,
     XD1,D2,D3,D4,D5,D6,D7,D8,D9,D10,D11,D12,
     XE1,E2,E3,E4,E5,E6,E7,E8,E9,E10,E11,E12,E13,E14,E15,E16,E17,E18,E19
     X,E20,E21,E22,E23,E24,E25,E26,E27,E28,E29,E30,E31,E32
     X,AM,AF,BM,BF,CM,CF,DM,DF,         FM,FF,GM,GF,HM,HF,
     XMP2,MP1,M,MM1,MM2,MM3,NP2,NP1,N,NM1,NM2,NM3,INP3,INP2,INP1,IN,
     XINM1,INM2,INM3,IMP3,IMP2,IMP1,IM,IMM1,IMM2,IMM3,NL,NLM1,NLM2,
     XLN,LI,LJ,LAT,KNAT,NMFI,MFIJ,MFI,KNT,KPROB,IJTP,MFII,
     XNRX,NRD,NRXS,NRDS,NPRLX,NCPRLX,NTP,NPT,SIGR1,SIGR2,PCGPRX,SIGR,
     XNRXBT,UL,VL,KPSPS,A,B,KSYM,NKPROB
     X,ALPHAX,ALPHAY
      DIMENSICN U(20,20),V(20,20),REU(20,20),REV(20,20),
     XE 1(17,17),E 2(17,17),E 3(17,17),E 4(17,17),E 5(17,17),E 6(17,17),
     XE 7(17,17),E 8(17,17),E 9(17,17),E10(17,17),E11(17,17),E12(17,17),
     XE13(17,17),E14(17,17),E15(17,17),E16(17,17),E17(17,17),E18(17,17),
     XE19(17,17),E20(17,17),E21(17,17),E22(17,17),E23(17,17),E24(17,17),
     XE25(17,17),E26(17,17),E27(17,17),E28(17,17),E29(17,17),E30(17,17),
     XE31(17,17),E32(17,17),
     XLN(20,20),MFI(20,20),KNT(20,20),MFII(200),MFIJ(200)
      DIMENSION SIGXM(40),SIGYM(40),SIGZM(40),SIGXF(40),SIGYF(40),
     XSIGZF(40),COST(40),SINT(40),
     XC 1(40),C 2(40),C 3(40),C 4(40),C 5(40),C 6(40),C 7(40),C 8(40),
     XC 9(40),C10(40),C11(40),C12(40),C13(40),C14(40),C15(40),C16(40),
     XC17(40),C18(40),C19(40),C20(40),C21(40),C22(40),C23(40),C24(40),
     XC25(40),C26(40),C27(40),C28(40),C29(40),C30(40),C31(40),C32(40),
     XC33(40),C34(40),C35(40),C36(40),C37(40),C38(40),LI(40),LJ(40)
      DIMENSION HX(20),HY(20),IJTP(20),D1(20),D2(20),D3(20),D4(20),
     XD5(20),D6(20),D7(20),D8(20),D9(20),D10(20),D11(20),D12(20),
     XSIGR1(10),SIGR2(10),TXYF(40),TXYM(40)
C     MATRIX SIGX IS STORED IN E5 MATRIX
C     MATRIX SIGY IS STORED IN E6 MATRIX
C     MATRIX SIGZ IS STORED IN E7 MATRIX
C     MATRIX TXY  IS STORED IN E8 MATRIX
      KNT(3,IN)=6
      KNT(IM,3)=7
  232 DO 1OC I=3,M
      DO 100 J = 3,N
      E1(I,J)=0.0
      E2(I,J)=0.0
      E3(I,J)=0.0
      E4(I,J)=0.0
      E5(I,J) = 0.0
      E6(I,J) = 0.C
      E7(I,J) = 0.0
      E8(I,J) = 0.0
      KN = KNT(I,J)
      GO TO (100,2,3,3,3,6,7,8,9,10,11,12,13,14,15,16,17),KN
    2 KMFI = MFI(I,J)
      GO TO (21,22),KMFI
   21 BB = BM
      CC = CM
      FMF= FM
      DD = CM
      HH = HM
      GG = GM
      IF (KN .LT. 3) GO TO 23
      IF (KN .LT. 9) GO TO 30



      GO TO 35
   22 BB = BF
      CC = CF
      FMF= FF
      DD = CF
      HH = HF
      GG = GF
      IF (KN .LT. 3) CO TO 23
      IF (KN .LT. 9) GO TO 30
      GO TO 35
   23 CALL PART (1,HX(I),HX(I-1),U(I+1,J),U(I,J),U(I-1,J),PUX)
      CALL PART (1,HY(J),HY(J-1),V(I,J+1),V(I,J),V(I,J-1),PVY)
      CALL PART (1,HY(J),HY(J-1),U(I,J+1),U(I,J),U(I,J-1),PUY)
      CALL PART (1,HX(I),HX(I-1),V(I+1,J),V(I,J),V(I-1,J),PVX)
      GO TO 4C
    3 A11 = HX(I-1) + HX(I-2)
      A12 = HY(J-1) + HY(J-2)
      CALL PART (4,HX(I-1),A11,U(I,J),U(I-1,J),U(I-2,J),PUX)
      CALL PART (4,HY(J-1),A12,V(I,J),V(I,J-1),V(I,J-2),PVY)
      CALL PART (4,HY(J-1),A12,U(I,J),U(I,J-1),U(I,J-2),PUY)
      CALL PART (4,HX(I-1),A11,V(I,J),V(I-1,J),V(I-2,J),PVX)
      L = LN(I,J)
      SIGXF(L) = BF*(PUX + CF*PVY) - FF
      SIGYF(L) = BF*(CF*PUX + PVY) - FF
      SIGZF(L) = DF*(PUX + PVY) - HF
      TXYF(L)  = GF*(PUY + PVX)
   24 A9 = HX(I) + HX(I+1)
      A10= HY(J) + HY(J+1)
      CALL PART (2,HX(I),A9,U(I,J),U(I+1,J),U(I+2,J),PUX)
      CALL PART (2,HY(J),A10,V(I,J),V(I,J+1),V(I,J+2),PVY)
      CALL PART (2,HY(J),A10,U(I,J),U(I,J+1),U(I,J+2),PUY)
      CALL PART (2,HX(I),A9,V(I,J),V(I+1,J),V(I+2,J),PVX)
      IF (KN .LT. 6) GO TO 25
      IF (KN .EC. 6) GO TO 26
      IF (KN .EQ. 7 ) GO TO 27
      GO  TC  28
   25 SIGXM(L) = BM*(PUX + CM*PVY) - FM
      SIGYM(L) = BM*(CM*PUX + PVY) - FM
      SIGZM(L) = DM*(PUX + PVY) - HM
      TXYM(L)  = GM*(PUY + PVX)
      GO TO 1C0
    6 GO TO 24
C     PUX = PARTIAL DERIVATIVE OF U W.R. TO X
C     PVX = PARTIAL DERIVATIVE OF V W.R. TO X
C     WE CANNOT COMPUTE PUX OR PVX
C     L = 1
   26 SIGXM(1) = BM*(PUX + CM*PVY) - FM
      SIGYM(1) = BM*(CM*PUX + PVY) - FM
      SIGZM(1) = DM*(PUX + PVY) - HM
      TXYM(1)  = GM*(PUY + PVX)
      GO TO 1CC
    7 GO TO 24
C     PUY = PARTIAL DERIVATIVE OF U W.R. TO Y
C     PVY = PARTIAL DERIVATIVE OF V W.R. TO Y
C     L = NL
C     WE CANNOT COMPUTE PUY OR PVY
   27 SIGXM(NL) = BM*(PUX + CM*PVY) - FM
      SIGYM(NL) = BM*(CM*PUX + PVY) - FM
      SIGZM(NL) = DM*(PUX + PVY) - HM
      TXYM(NL)  = GM*(PUY + PVX)
      GO TO 100
    8 GO TO 2
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   30 A9 = HX(I) + HX(I+1)
      CALL PART (2,HX(I),A9,U(I,J),U(I+1,J),U(I+2,J),PUX)
      CALL PART (2,HX(I),A9,V(I,J),V(I+1,J),V(I+2,J),PVX)
      CALL PART (1,HY(J),HY(J-1),V(I,J+1),V(I,J),V(I,J-1),PVY)
      CALL PART (1,HY(J),HY(J-1),U(I,J+1),U(I,J),U(I,J-1),PUY)
      GO TO 40
    9 A11 = HX(I-1) + HX(I-2)
      CALL PART (3,HX(I-1),A11,U(I,J),U(I-1,J),U(I-2,J),PUX)
      CALL PART (3,HX(I-1),A11,V(I,J),V(I-1,J),V(I-2,J),PVX)
      CALL PART (1,HY(J),HY(J-1),V(I,J+1),V(I,J),V(I,J-1),PVY)
      CALL PART (1,HY(J),HY(J-1),U(I,J+1),U(I,J),U(I,J-1),PUY)
      GO TO 45
   10 GO TO 2
   35 A10 = HY(J) + HY(J+1)
      CALL PART (2,HY(J),A10,V(I,J),V(I,J+1),V(I,J+2),PVY)
      CALL PART (2,HY(J),A10,U(I,J),U(I,J+1),U(I,J+2),PUY)
      CALL PART (1,HX(I),HX(I-1),U(I+1,J),U(I,J),U(I-1,J),PUX)
      CALL PART (1,HX(I),HX(I-1),V(I+1,J),V(I,J),V(I-1,J),PVX)
      GO TO 40
   11 A12 = HY(J-1) + HY(J-2)
      CALL PART (3,HY(J-1),A12,V(I,J),V(I,J-1),V(I,J-2),PVY)
      CALL PART (3,HY(J-1),A12,U(I,J),U(I,J-1),U(I,J-2),PUY)
      CALL PART (1,HX(I),HX(I-1),U(I+1,J),U(I,J),U(I-1,J),PUX)
      CALL PART (1,HX(I),HX(I-1),V(I+1,J),V(I,J),V(I-1,J),PVX)
      GO TO 45
   12 GO  TC 24
   28 E5(I,J) = BF*(PUX + CF*PVY) - FF
      E6(I,J) = BF*(CF*PUX + PVY) - FF
      E7(I,J) = DF*(PUX + PVY) - HF
      E8(I,J) = GF*(PUY + PVX)
      GO TO 1C0
   13 A9 = HX(I) + HX(I+1)
      A12= HY(J-1) + HY(J-2)
      CALL PART (2,HX(I),A9,U(I,J),U(I+1,J),U(I+2,J),PUX)
      CALL PART (2,HX(I),A9,V(I,J),V(I+1,J),V(I+2,J),PVX)
      CALL PART (3,HY(J-1),A12,V(I,J),V(I,J-1),V(I,J-2),PVY)
      CALL PART (3,HY(J-1),A12,U(I,J),U(I,J-1),U(I,J-2),PUY)
      GO TO 45
   14 A11 = HX(I-1) + HX(I-2)
      A12 = HY(J-1) + HY(J-2)
      CALL PART (3,HX(I-1),A11,U(I,J),U(I-1,J),U(I-2,J),PUX)
      CALL PART (3,HX(I-1),A11,V(I,J),V(I-1,J),V(I-2,J),PVX)
      CALL PART (3,HY(J-1),A12,V(I,J),V(I,J-1),V(I,J-2),PVY)
      CALL PART (3,HY(J-1),A12,U(I,J),U(I,J-1),U(I,J-2),PUY)
      GO TO 45
   15 A11 = HX(I-1) + HX(I-2)
      A10 = HY(J) + HY(J+1)
      CALL PART (3,HX(I-1),A11,U(I,J),U(I-1,J),U(I-2,J),PUX)
      CALL PART (3,HX(I-1),A11,V(I,J),V(I-1,J),V(I-2,J),PVX)
      CALL PART (2,HY(J),A1C,V(I,J),V(I,J+1),V(I,J+2),PVY)
      CALL PART (2,HY(J),A10,U(I,J),U(I,J+1),U(I,J+2),PUY)
      GO TO 45
   16 GO TO 1C0
C     WE CANNOT CCMPUTE PUY OR PVY
   17 GO TO 1C0
C     WE CANNOT COMPUTE PUX OR PVX
   40 E5(I,J) = BB*(PUX + CC*PVY) - FMF
      E6(I,J) = BB*(CC*PUX + PVY) - FMF
      E7(I,J) = DD*(PUX + PVY) - HH
      E8(I,J) = GG*(PUY + PVX)
      GO TO 1C0
   45 E5(I,J) = BM*(PUX + CM*PVY) - FM



      E6(I,J) = BM*(CM*PUX + PVY) - FM
      E7(I,J) = DM*(PUX + PUY) - HM
      E8(I,J) = GM*(PUY + PVX)
  100 CONTINUE
C     FOR INTERIOR POINTS
C       THE VALUES OF SIGMA 1 ARE STORED IN E1 MATRIX
C       THE VALUES OF SIGMA 2 ARE STORED IN E2 MATRIX
C       THE VALUES OF THETA ARE STORED IN E3 MATRIX
C       THE VALUES OF THE VON MISES SUM ARE STORED IN E4 MATRIX
      DO 60 I=3,M
      DC 60 J=3,N
      IF (MFI(I,J) .EC. 3) GO TO 65
      VTZS = .5*(E5(I,J) + E6(I,J))
      VTZM = .5*(E5(I,J) - E6(I,J))
      RADIUS = VTZM**2 + E8(I,J)**2
      RADIUS = SQRT(RADIUS)
      E1(I,J) = VTZS + RADIUS
      E2(I,J) = VTZS - RADIUS
      E3(I,J) = -E8(I,J)/VTZM
      E3(I,J) = .5*ATAN(E3(I,J))
      E3(I,J) = 57.29578*E3(I,J)
      IF (KPSPS .EQ. 2) GO TO 62
      E4(I,J) = E1(I,J)**2 - E1(I,J)*E2(I,J) + E2(I,J)**2
      GO TO 6C
   62 IF (MFI(I,J) .EC. 2) GO TO 64
   61 SMIT1 = 1. - PRM + PRM**2
      SMIT2 = 1. + 2.*PRM - 2.*(PRM**2)
      IF (MFI(I,J) .EC. 3) GO TO 69
   63 E4(I,J) = SMIT1*(E1(I,J)**2) - SMIT2*E1(I,J)*E2(I,J) +
     X    SMIT1*(E2(I,J)**2)
      GO TO 6C
   64 SMIT1 = 1. - PRF + PRF**2
      SMIT2 = 1. + 2.*PRF - 2.*(PRF**2)
      IF (MFI(I,J) .EC. 3) GO TO 71
      GO TO 63
   65 L  = LN(I,J)
C     FOR INTERFACE PCINTS
C      MATRIX
C        THE VALUES OF SIGMA 1 ARE STORED IN C1 MATRIX
C        THE VALUES OF SIGMA 2 ARE STORED IN C2 MATRIX
C        THE VALUES OF THETA ARE STORED IN C3 MATRIX
C        THE VALUES OF THE VON MISES SUM ARE STORED IN C4 MATRIX
C      FIBER
C        THE VALUES OF SIGMA 1 ARE STORED IN C11 MATRIX
C        THE VALUES OF SIGMA 2 ARE STORED IN C12 MATRIX
C        THE VALUES OF THETA ARE STORED IN C13 MATRIX
C        THE VALUES OF VON MISES SUM ARE STORED IN C14 MATRIX
      VTZMS = .5*(SIGXM(L) + SIGYM(L))
      VTZMM = .5*(SIGXM(L) - SIGYM(L))
      RADIUS = VTZMM**2 + TXYM(L)**2
      RADIUS = SQRT(RADIUS)
      C1(L) = VTZMS + RADIUS
      C2(L) = VTZMS - RADIUS
      C3(L) = -TXYM(L)/VTZMM
      C3(L) = .5*ATAN(C3(L))
      C3(L) = 57.29578*C3(L)
      VTZFS = .5*(SIGXF(L) + SIGYF(L))
      VTZFM = .5*(SIGXF(L) - SIGYF(L))
      RADIUS = VTZFM**2 + TXYF(L)**2
      RADIUS = SQRT(RADIUS)
      C11(L) = VTZFS + RADIUS
      C12(L) = VTZFS - RADIUS
```

```
      C13(L) = -TXYF(L)/VTZFM
      C13(L) = .5*ATAN(C13(L))
      C13(L) = 57.29578*C13(L)
      IF (KPSPS .EQ. 2) GO TO 67
      C4(L) = C1(L)**2 - C1(L)*C2(L) + C2(L)**2
      C14(L) = C11(L)**2 - C11(L)*C12(L) + C12(L)**2
      GO TO 60
   67 GO TO 61
   69 C4(L) = SMIT1*(C1(L)**2) - SMIT2*C1(L)*C2(L) + SMIT1*(C2(L)**2)
      GO TO 64
   71 C14(L) = SMIT1*(C11(L)**2) - SMIT2*C11(L)*C12(L) +
     X      SMIT1*(C12(L)**2)
   60 CONTINUE
      E1(IMM1,3)=0.0
      E2(IMM1,3)=0.0
      E3(IMM1,3)=0.0
      E4(IMM1,3)=0.0
      E5(IMM1,3)=0.0
      E6(IMM1,3)=0.0
      E7(IMM1,3)=0.0
      E8(IMM1,3)=0.0
      E1(3,INM1)=0.0
      E2(3,INM1)=0.0
      E3(3,INM1)=0.0
      E4(3,INM1)=0.0
      E5(3,INM1)=0.0
      E6(3,INM1)=0.0
      E7(3,INM1)=0.0
      E8(3,INM1)=0.0
   80 GO TO (81,82,83),KPROB
   81 SX=SIGXB
      SY=SIGYB
      TT=0.0
      GO TO 500
   82 SX=0.0
      SY=0.0
      TT=T
      GO TO 500
   83 SX=SIGXB
      SY=SIGYB
      TT=T
      GO TO 500
  500 CONTINUE
      SIGXF(NL) = 0.0
      SIGYF(NL) = 0.0
      SIGZF(NL) = 0.0
      TXYF(NL) = 0.0
      SIGXF(1) = 0.0
      SIGYF(1) = 0.0
      SIGZF(1) = 0.0
      TXYF(1) = 0.0
      WRITE (5,405) SX,SY,TT
      WRITE (5,400) (((I,J,U(I,J),V(I,J),E5(I,J),E6(I,J),E7(I,J),
     1E8(I,J)),J=3,N),I=3,M)
  400 FORMAT (1H ,35X,50H  STRESS COMPONENTS - INTERIOR AND BOUNDARY POI
     1NTS,/////,
     26X,1HI,3X,1HJ,15X,1HU,19X,1HV,15X,7HSIGMA X,8X,7HSIGMA Y,8X,
     37HSIGMA Z,8X,8H TAU XY ,////,(3X,2I4,4X,2E20.8,4F15.3))
      WRITE (5,405) SX,SY,TT
      WRITE (5,401) ((LI(L),LJ(L),SIGXM(L),SIGYM(L),SIGZM(L),TXYM(L),
     1SIGXF(L),SIGYF(L),SIGZF(L),TXYF(L)),L=1,NL)
  401 FORMAT (1H ,35X,38H  STRESS COMPONENTS - INTERFACE POINTS,/////,


     138X,9HIN MATRIX,43X,8HIN FIBER,//,
     26X,1HI,3X,1HJ,9X,7HSIGMA X,6X,7HSIGMA Y,6X,7HSIGMA Z,5X,8H TAU XY
     3,6X,7HSIGMA X,6X,7HSIGMA Y,6X,7HSIGMA Z,5X,8H TAU XY ,////,
     4(3X,2I4,4X,8F13.3))
      WRITE (5,405) SX,SY,TT
      WRITE (5,402)(((I,J,E1(I,J),E2(I,J),E3(I,J),E4(I,J)),J=3,N),I=3,M)
  402 FORMAT (1H ,36X,49HPRINCIPAL STRESSES - INTERIOR AND BOUNDARY POIN
     1TS,/////,
     26X,1HI,3X,1HJ,18X,7HSIGMA 1,13X,7HSIGMA 2,12X,9HTHETA DEG,12X,
     39HVON MISES,////,(3X,2I4,6X,4F20.3))
      WRITE (5,405) SX,SY,TT
      WRITE (5,403)((LI(L),LJ(L),C1(L),C2(L),C3(L),C4(L),C11(L),C12(L),
     1C13(L),C14(L)),L=1,NL)
  403 FORMAT (1H ,36X,37HPRINCIPAL STRESSES - INTERFACE POINTS,/////,
     138X,9HIN MATRIX,43X,8HIN FIBER,//,
     26X,1HI,3X,1HJ,9X,7HSIGMA 1,6X, 7HSIGMA 2,7X,5HTHETA,6X,9HVON MISES
     3,5X,7HSIGMA 1,6X,7HSIGMA 2,7X,5HTHETA,6X,9HVON MISES,////,
     4(3X,2I4,4X,8F13.3))
      WRITE (5,404) EX,EY,ALPHAX,ALPHAY
  404 FORMAT (1H ,//,36H  EFFECTIVE COMPOSITE ELASTIC MODULI,//,6H  EX =
     X,1E12.5,//,6H  EY =,1E12.5,///,
     X57H  EFFECTIVE COMPOSITE THERMAL EXP. COEF.  (IN/IN/DEG. F) ,//,
     X11H  ALPHA X =,1E12.5,//,11H  ALPHA Y =,1E12.5)
  405 FORMAT (1H1,44X,35H * * * * STRESS CONDITION * * * *  ,//,
     X50X,41H AVERAGE COMPOSITE SIGMA X   (PSI)      =,1F11.2,/,
     X50X,41H AVERAGE COMPOSITE SIGMA Y   (PSI)      =,1F11.2,/,
     X50X,41H TEMP.  (AMBIENT - CURING)   (DEG. F)   =,1F11.2,////)
      RETURN
      END
                FORTRAN MAP
CSIGMAB
      SUBROUTINE SIGMAB (HX,HY,U,V,BM,CM,FM,M,N,A,B,SXBS,SYBS)
      DIMENSION HX(20),HY(20),U(20,20),V(20,20),SIGX(20),SIGY(20)
      MM1=M-1
      MM2=M-2
      NM1=N-1
      NM2=N-2
      A3=HX(MM1)
      A11=A3+HX(MM2)
      A2=HY(3)
      A10=A2+HY(4)
      CALL PART (3,A3,A11,U(M,3),U(MM1,3),U(MM2,3),PUX)
      CALL PART (2,A2,A10,V(M,3),V(M,4),V(M,5),PVY)
      SIGX(  3)=(BM*(PUX+CM*PVY)-FM)*A2/2.0
      DO 50 J=4,NM1
      A2=HY(J)
      A4=HY(J-1)
      CALL PART (3,A3,A11,U(M,J),U(MM1,J),U(MM2,J),PUX)
      CALL PART (1,A2,A4,V(M,J+1),V(M,J),V(M,J-1),PVY)
   50 SIGX(  J)=(BM*(PUX+CM*PVY)-FM)*((A4/2.0)+(A2/2.0))
      A4=HY(NM1)
      A12=A4+HY(NM2)
      CALL PART (3,A3,A11,U(M,N),U(MM1,N),U(MM2,N),PUX)
      CALL PART (3,A4,A12,V(M,N),V(M,NM1),V(M,NM2),PVY)
      SIGX(  N)=(BM*(PUX+CM*PVY)-FM)*A4/2.0
      SIGXBS=0.0
      DO 4 J = 3,N
    4 SIGXBS=SIGXBS+SIGX(  J)
      SXBS=SIGXBS/B
      A4=HY(NM1)
      A12=A4+HY(NM2)
      A1=HX(3)
```

```
      A9=A1+HX(4)
      CALL PART (2,A1,A9,U(3,N),U(4,N),U(5,N),PUX)
      CALL PART (3,A4,A12,V(3,N),V(3,NM1),V(3,NM2),PVY)
      SIGY(3  )=(BM*(CM*PUX+PVY)-FM)*A1/2.0
      DO 20 I=4,MM1
      A1=HX(I)
      A3=HX(I-1)
      CALL PART (1,A1,A3,U(I+1,N),U(I,N),U(I-1,N),PUX)
      CALL PART (3,A4,A12,V(I,N),V(I,NM1),V(I,NM2),PVY)
   20 SIGY(I  )=(BM*(CM*PUX+PVY)-FM)*((A1/2.0)+(A3/2.0))
      A3=HX(MM1)
      A11=A3+HX(MM2)
      CALL PART (3,A3,A11,U(M,N),U(MM1,N),U(MM2,N),PUX)
      CALL PART (3,A4,A12,V(M,N),V(M,NM1),V(M,NM2),PVY)
      SIGY(M  )=(BM*(CM*PUX+PVY)-FM)*(A3/2.0)
      SIGYBS=0.0
      DO56    I = 3,M
   56 SIGYBS=SIGYBS+SIGY(I   )
      SYBS=SIGYBS/A
      RETURN
      END
                 FORTRAN MAP
CPART
      SUBROUTINE PART (KP,AA,AB,F1,F2,F3,P)
      GO TO (1,2,3,3),KP
    1 P=(1.0/(AA*AB*(AA+AB)))*(AB**2*F1+(AA**2-AB**2)*F2-AA**2*F3)
      RETURN
    2 P=(1.0/(AA*AB*(AB-AA)))*((AA**2-AB**2)*F1+AB**2*F2-AA**2*F3)
      RETURN
    3 P=(1.0/(AA*AB*(AB-AA)))*((AB**2-AA**2)*F1-AB**2*F2+AA**2*F3)
      RETURN
      END
```

COMPUTER OUTPUT SAMPLE PROBLEM

T R A N S V E R S E   S T R E S S   A N A L Y S I S

SAMPLE PROBLEM   CIRCULAR INCLUSION

INPUT DATA

GRID NODE ARRAY SIZE =13 BY 13

QUADRANT DIMENSIONS   A = 1.400   B = 1.400

RELAXATION FACTOR (OMEGA BAR) = 1.700

AVERAGE SIGMA X LOADING AT INFINITY (PSI) = 1000.00

AVERAGE SIGMA Y LOADING AT INFINITY (PSI) = 0.

PERCENT FIBER BY VOLUME = 40.00

YOUNGS MODULUS E IN MATRIX (PSI) = 0.1000+007

YOUNGS MODULUS E IN FIBER (PSI) = 0.2151+008

POISSONS RATIO IN MATRIX = 0.3000

POISSONS RATIO IN FIBER = 0.3000

MATRIX SHEAR MODULUS PSI = 0.3846+006

INCLUSION SHEAR MODULUS PSI = 0.8271+007

THERMAL EXP. COEF. IN MATRIX (IN/IN/DEG F) = 0.

THERMAL EXP. COEF. IN FIBER (IN/IN/DEG F) = 0.

T=AMBIENT TEMP - CURING TEMP (DEGREES F) = 0.

MAX DELTA STRESS AT TEST PTS/RELAX(PERCENT)= 0.

SOLUTION IS FOR PLANE STRESS

GRID SPACING

I	HX(I)
3	0.30600000
4	0.16500000
5	0.16800000
6	0.13000000
7	0.11300000
8	0.07000000
9	0.04800000
10	0.05000000
11	0.06000000
12	0.08000000
13	0.10000000
14	0.11000000

J	HY(J)
3	0.30600000
4	0.16500000
5	0.16800000
6	0.13000000
7	0.11300000
8	0.07000000
9	0.04800000
10	0.05000000
11	0.06000000
12	0.08000000
13	0.10000000
14	0.11000000

COS AND SINE THETA AT INTERFACE NODES

I	J	COS	SINE
3	10	0.	1.00000
4	9	0.30597	0.95204
5	8	0.47101	0.88213
6	7	0.63899	0.76921
7	6	0.76921	0.63899
8	5	0.88213	0.47101
9	4	0.95204	0.30597
10	3	1.00000	0.

RESULTS OF RESID NO. 1 PROBLEM NO. 1

I	J	U	V	U RESIDUAL	V RESIDUAL
3	3	0.	0.	0.	0.
3	4	0.	0.	0.	0.
3	5	0.	0.	0.	0.
3	6	0.	0.	0.	0.
3	7	0.	0.	0.	0.
3	8	0.	0.	0.	0.
3	9	0.	0.	0.	0.
3	10	0.	0.	0.	0.
3	11	0.	0.	0.	0.
3	12	0.	0.	0.	0.
3	13	0.	0.	0.	0.
3	14	0.	0.	0.	0.
3	15	0.	0.	0.	0.
4	3	0.	0.	0.	0.
4	4	0.	0.	0.	0.
4	5	0.	0.	0.	0.
4	6	0.	0.	0.	0.
4	7	0.	0.	0.	0.
4	8	0.	0.	0.	0.
4	9	0.	0.	0.	0.
4	10	0.	0.	0.	0.
4	11	0.	0.	0.	0.
4	12	0.	0.	0.	0.
4	13	0.	0.	0.	0.
4	14	0.	0.	0.	0.
4	15	0.	0.	0.	0.
5	3	0.	0.	0.	0.
5	4	0.	0.	0.	0.
5	5	0.	0.	0.	0.
5	6	0.	0.	0.	0.
5	7	0.	0.	0.	0.
5	8	0.	0.	0.	0.
5	9	0.	0.	0.	0.
5	10	0.	0.	0.	0.
5	11	0.	0.	0.	0.
5	12	0.	0.	0.	0.
5	13	0.	0.	0.	0.
5	14	0.	0.	0.	0.
5	15	0.	0.	0.	0.
6	3	0.	0.	0.	0.
6	4	0.	0.	0.	0.
6	5	0.	0.	0.	0.
6	6	0.	0.	0.	0.
6	7	0.	0.	0.	0.
6	8	0.	0.	0.	0.
6	9	0.	0.	0.	0.
6	10	0.	0.	0.	0.
6	11	0.	0.	0.	0.
6	12	0.	0.	0.	0.
6	13	0.	0.	0.	0.
6	14	0.	0.	0.	0.
6	15	0.	0.	0.	0.
7	3	0.	0.	0.	0.
7	4	0.	0.	0.	0.
7	5	0.	0.	0.	0.
7	6	0.	0.	0.	0.
7	7	0.	0.	0.	0.
7	8	0.	0.	0.	0.
7	9	0.	0.	0.	0.
7	10	0.	0.	0.	0.
7	11	0.	0.	0.	0.
7	12	0.	0.	0.	0.
7	13	0.	0.	0.	0.
7	14	0.	0.	0.	0.
7	15	0.	0.	0.	0.
8	3	0.	0.	0.	0.
8	4	0.	0.	0.	0.
8	5	0.	0.	0.	0.
8	6	0.	0.	0.	0.
8	7	0.	0.	0.	0.
8	8	0.	0.	0.	0.
8	9	0.	0.	0.	0.
8	10	0.	0.	0.	0.
8	11	0.	0.	0.	0.
8	12	0.	0.	0.	0.
8	13	0.	0.	0.	0.
8	14	0.	0.	0.	0.
8	15	0.	0.	0.	0.
9	3	0.	0.	0.	0.
9	4	0.	0.	0.	0.
9	5	0.	0.	0.	0.
9	6	0.	0.	0.	0.
9	7	0.	0.	0.	0.
9	8	0.	0.	0.	0.
9	9	0.	0.	0.	0.
9	10	0.	0.	0.	0.
9	11	0.	0.	0.	0.
9	12	0.	0.	0.	0.
9	13	0.	0.	0.	0.
9	14	0.	0.	0.	0.
9	15	0.	0.	0.	0.
10	3	0.	0.	0.	0.
10	4	0.	0.	0.	0.
10	5	0.	0.	0.	0.
10	6	0.	0.	0.	0.
10	7	0.	0.	0.	0.
10	8	0.	0.	0.	0.
10	9	0.	0.	0.	0.
10	10	0.	0.	0.	0.
10	11	0.	0.	0.	0.
10	12	0.	0.	0.	0.
10	13	0.	0.	0.	0.
10	14	0.	0.	0.	0.
10	15	0.	0.	0.	0.
11	3	0.	0.	0.	0.
11	4	0.	0.	0.	0.
11	5	0.	0.	0.	0.
11	6	0.	0.	0.	0.
11	7	0.	0.	0.	0.
11	8	0.	0.	0.	0.
11	9	0.	0.	0.	0.
11	10	0.	0.	0.	0.
11	11	0.	0.	0.	0.
11	12	0.	0.	0.	0.
11	13	0.	0.	0.	0.
11	14	0.	0.	0.	0.
11	15	0.	0.	0.	0.
12	3	0.	0.	0.	0.

12	4	0.	0.	0.	0.
12	5	0.	0.	0.	0.
12	6	0.	0.	0.	0.
12	7	0.	0.	0.	0.
12	8	0.	0.	0.	0.
12	9	0.	0.	0.	0.
12	10	0.	0.	0.	0.
12	11	0.	0.	0.	0.
12	12	0.	0.	0.	0.
12	13	0.	0.	0.	0.
12	14	0.	0.	0.	0.
12	15	0.	0.	0.	0.
13	3	0.	0.	0.	0.
13	4	0.	0.	0.	0.
13	5	0.	0.	0.	0.
13	6	0.	0.	0.	0.
13	7	0.	0.	0.	0.
13	8	0.	0.	0.	0.
13	9	0.	0.	0.	0.
13	10	0.	0.	0.	0.
13	11	0.	0.	0.	0.
13	12	0.	0.	0.	0.
13	13	0.	0.	0.	0.
13	14	0.	0.	0.	0.
13	15	0.	0.	0.	0.
14	3	0.	0.	0.	0.
14	4	0.	0.	0.51230820+008	-0.43203024+006
14	5	0.	0.	0.51230820+008	0.15821999+005
14	6	0.	0.	0.51230820+008	-0.24628285+006
14	7	0.	0.	0.51230820+008	-0.15267006+006
14	8	0.	0.	0.51230820+008	-0.60880163+006
14	9	0.	0.	0.51230820+008	-0.50804489+006
14	10	0.	0.	0.51230820+008	0.53280056+005
14	11	0.	0.	0.51230820+008	0.24306594+006
14	12	0.	0.	0.51230820+008	0.38497148+006
14	13	0.	0.	0.51230820+008	0.28101294+006
14	14	0.	0.	0.51230820+008	0.11212132+006
14	15	0.	0.	0.	0.
15	3	0.10000000+001	0.	0.	0.
15	4	0.10000000+001	0.	0.	0.
15	5	0.10000000+001	0.	0.	0.
15	6	0.10000000+001	0.	0.	0.
15	7	0.10000000+001	0.	0.	0.
15	8	0.10000000+001	0.	0.	0.
15	9	0.10000000+001	0.	0.	0.
15	10	0.10000000+001	0.	0.	0.
15	11	0.10000000+001	0.	0.	0.
15	12	0.10000000+001	0.	0.	0.
15	13	0.10000000+001	0.	0.	0.
15	14	0.10000000+001	0.	0.	0.
15	15	0.10000000+001	0.	0.	0.

RESULTS OF RELAX NO. 150 PROBLEM NO. 1

I	J	U	V	U RESIDUAL	V RESIDUAL
3	3	0.	0.	0.	0.
3	4	0.	-0.10950214-001	0.	-0.12522907+000
3	5	0.	-0.19794400-001	0.	-0.20896980+000
3	6	0.	-0.32161600-001	0.	-0.30269726+000
3	7	0.	-0.45185495-001	0.	-0.39069290+000
3	8	0.	-0.61963622-001	0.	-0.48143399+000
3	9	0.	-0.71116235-001	0.	-0.50912167+000
3	10	0.	-0.53244888-001	0.	-0.25169414-001
3	11	0.	-0.41258342-001	0.	-0.23226759-001
3	12	0.	-0.30167784-001	0.	-0.21098207-001
3	13	0.	-0.19130836-001	0.	-0.16066789-001
3	14	0.	-0.89748405-002	0.	-0.92321837-002
3	15	0.	0.	0.	0.
4	3	-0.12944586-002	0.	0.79486471-001	0.
4	4	0.72623753-002	-0.18193618-001	0.93313756+000	-0.19046771+001
4	5	0.18978261-001	-0.30584595-001	0.14165246+001	-0.18018782+001
4	6	0.34630416-001	-0.45730972-001	0.20693014+001	-0.29264841+001
4	7	0.47672323-001	-0.59093717-001	0.27366390+001	-0.25478235+001
4	8	0.55645754-001	-0.70441756-001	0.43862784+001	-0.84015276+001
4	9	0.59996453-001	-0.71315589-001	0.62592640-001	-0.11721251+002
4	10	0.10966563+000	-0.59643464-001	0.76033251-001	0.15096288+001
4	11	0.12987161+000	-0.48568331-001	0.11243477+000	0.16916290+001
4	12	0.15002395+000	-0.37495860-001	0.12442705+000	0.12989794+001
4	13	0.17000032+000	-0.25282540-001	0.10803862+000	0.81119836+000
4	14	0.18512345+000	-0.12517859-001	0.74313004-001	0.57897524+000
4	15	0.19084188+000	0.	-0.14968656-001	0.
5	3	-0.90927518-003	0.	0.13338207+000	0.
5	4	0.12367040-001	-0.28111174-001	0.25412779+000	-0.16218576+001
5	5	0.30544767-001	-0.45358360-001	-0.18262968-001	-0.94963616+000
5	6	0.54084530-001	-0.64309941-001	0.63535509-001	-0.17502580+001
5	7	0.72665523-001	-0.78136633-001	-0.37682891+000	-0.50105352+000
5	8	0.78244830-001	-0.82049875-001	-0.40373664+001	-0.34294742+001
5	9	0.14958609+000	-0.77185188-001	-0.19206666+000	0.13609361+000
5	10	0.18774576+000	-0.68404281-001	-0.26569517+000	0.14320838+001
5	11	0.21844019+000	-0.58577042-001	-0.18812990+000	0.15625534+001
5	12	0.24664865+000	-0.47529336-001	-0.60211118-001	0.11966589+001
5	13	0.27328515+000	-0.33705350-001	0.11757092-001	0.74095560+000
5	14	0.29301199+000	-0.17368901-001	0.58932235-001	0.51031507+000
5	15	0.30047120+000	0.	-0.21189501-001	0.
6	3	0.64211642-003	0.	0.19327213+000	0.
6	4	0.19098476-001	-0.43998913-001	0.11278455+001	-0.25192482+001
6	5	0.44368640-001	-0.69016185-001	0.12581752+001	-0.19102063+001
6	6	0.73293057-001	-0.94826070-001	0.15326570+001	-0.28406253+001
6	7	0.90263645-001	-0.99920825-001	-0.35020939+001	-0.22725301+001
6	8	0.21841518+000	-0.99872628-001	-0.29847610-001	-0.27615548-001
6	9	0.27447318+000	-0.90969976-001	-0.54902068-001	0.94240072-001
6	10	0.30663651+000	-0.83087331-001	-0.10697626+000	0.12003117+001
6	11	0.33496148+000	-0.73843077-001	-0.57695492-001	0.13212012+001
6	12	0.36248940+000	-0.61925302-001	0.13292295-001	0.99954706+000
6	13	0.38943464+000	-0.45259099-001	0.56912825-001	0.62288783+000
6	14	0.40979546+000	-0.23843185-001	0.67407054-001	0.43511275+000
6	15	0.41749439+000	0.	-0.26998054-001	0.
7	3	0.41330909-002	0.	0.24820073+000	0.
7	4	0.25560614-001	-0.60644302-001	0.44509344+000	-0.28636141+001
7	5	0.54898848-001	-0.91664341-001	0.93276205-001	-0.15256565+001
7	6	0.82867358-001	-0.10609357+000	-0.35936187+001	-0.16799880+001

7	7	0.23729224+000	-0.12154731+000	-0.13475655+000	0.19442301+000
7	8	0.33718950+000	-0.11489516+000	-0.95267428-001	0.29891588-001
7	9	0.38505578+000	-0.10507332+000	-0.12133113+000	0.14499655+000
7	10	0.41272527+000	-0.96619418-001	-0.15427059+000	0.10681989+001
7	11	0.43738930+000	-0.86614173-001	-0.95723941-001	0.11680595+001
7	12	0.46174787+000	-0.73361480-001	-0.61565097-004	0.86545861+000
7	13	0.48604732+000	-0.54194665-001	0.53353517-001	0.53647139+000
7	14	0.50469475+000	-0.28791387-001	0.92195981-001	0.36543407+000
7	15	0.51174581+000	0.	-0.29669002-001	0.
8	3	0.48246310-002	0.	0.30351280+000	0.
8	4	0.30004951-001	-0.81541373-001	0.44840628+001	-0.64761763+001
8	5	0.64481458-001	-0.10446399+000	-0.36688487+001	-0.13871753+001
8	6	0.24740465+000	-0.13732897+000	0.88599024-001	-0.26815469-002
8	7	0.36777295+000	-0.14228362+000	0.62234961-001	0.66656267-001
8	8	0.44859241+000	-0.13113360+000	0.10472418+000	-0.63342676-001
8	9	0.48835836+000	-0.11932677+000	0.11836858+000	0.49486223-001
8	10	0.51151970+000	-0.10957110+000	0.11817720+000	0.80558844+000
8	11	0.53229567+000	-0.98210924-001	0.12756374+000	0.91036470+000
8	12	0.55297187+000	-0.83263429-001	0.12541940+000	0.69323866+000
8	13	0.57380349+000	-0.61635262-001	0.11368947+000	0.44932084+000
8	14	0.58994191+000	-0.32815175-001	0.73512153-001	0.34119760+000
8	15	0.59604425+000	0.	-0.31885887-001	0.
9	3	0.32522066-001	0.	0.34118881+000	0.
9	4	0.37792431-001	-0.67616061-001	-0.46764407+001	-0.11825910+001
9	5	0.19297724+000	-0.12290835+000	-0.28328000-001	0.22874989+000
9	6	0.34883121+000	-0.15438361+000	-0.56259289-002	0.30820375-001
9	7	0.45058475+000	-0.15454120+000	0.39256489-001	0.64521092-001
9	8	0.52000796+000	-0.14083273+000	0.10111106+000	-0.50972884-001
9	9	0.55455133+000	-0.12772810+000	0.14960059+000	0.55414363-001
9	10	0.57476016+000	-0.11712565+000	0.15023522+000	0.68929321+000
9	11	0.59294727+000	-0.10489635+000	0.17527149+000	0.80759353+000
9	12	0.61111037+000	-0.88895871-001	0.18670383+000	0.61006211+000
9	13	0.62948819+000	-0.65808246-001	0.14894533+000	0.41408319+000
9	14	0.64378512+000	-0.35048383-001	0.13505866+000	0.31948921+000
9	15	0.64919114+000	0.	-0.31824072-001	0.
10	3	0.45760919-001	0.	0.17738570-001	0.
10	4	0.14516965+000	-0.89412866-001	-0.11229241+001	0.36641229+000
10	5	0.28127856+000	-0.14034080+000	-0.11241578+001	0.62341759+000
10	6	0.41839460+000	-0.16499088+000	-0.81914391+000	0.44623128+000
10	7	0.50802413+000	-0.16224553+000	-0.60613425+000	0.42006999+000
10	8	0.56978622+000	-0.14695931+000	-0.35544251+000	0.25567194+000
10	9	0.60070434+000	-0.13303073+000	-0.28265216+000	0.30985311+000
10	10	0.61884113+000	-0.12188690+000	-0.19275441+000	0.86405604+000
10	11	0.63519541+000	-0.10910015+000	-0.24760653-001	0.87945334+000
10	12	0.65156051+000	-0.92425543-001	0.19621136+000	0.65252087+000
10	13	0.66815753+000	-0.68411623-001	0.42673677+000	0.39321782+000
10	14	0.68109824+000	-0.36436165-001	0.58814317+000	0.24060043+000
10	15	0.68599143+000	0.	-0.32237354-001	0.
11	3	0.16999740+000	0.	0.16608637-001	0.
11	4	0.25538098+000	-0.10858343+000	-0.13225183+001	0.37209132+000
11	5	0.37228690+000	-0.15474008+000	-0.12371873+001	0.54858788+000
11	6	0.49071549+000	-0.17454369+000	-0.91272026+000	0.44698657+000
11	7	0.56832692+000	-0.16942160+000	-0.66714862+000	0.43957650+000
11	8	0.62222907+000	-0.15273374+000	-0.44747946+000	0.28759032+000
11	9	0.64934682+000	-0.13804269+000	-0.35813693+000	0.34969205+000
11	10	0.66529250+000	-0.12639070+000	-0.30266117+000	0.76981640+000
11	11	0.67969619+000	-0.11307660+000	-0.92476015-001	0.79275489+000
11	12	0.69413453+000	-0.95761568-001	0.16484115+000	0.55115788+000
11	13	0.70880647+000	-0.70867912-001	0.39921579+000	0.32840425+000
11	14	0.72026795+000	-0.37743104-001	0.63445087+000	0.16368173+000
11	15	0.72460182+000	0.	-0.30639466-001	0.
12	3	0.31647705+000	0.	0.15820472-001	0.
12	4	0.38573819+000	-0.12357420+000	-0.10300211+001	0.23993632+000
12	5	0.48056949+000	-0.16749890+000	-0.95012126+000	0.36506228+000
12	6	0.57751444+000	-0.18384863+000	-0.69460000+000	0.31464856+000
12	7	0.64125825+000	-0.17672198+000	-0.52788910+000	0.32184464+000
12	8	0.68584131+000	-0.15870806+000	-0.34710302+000	0.21718815+000
12	9	0.70837186+000	-0.14325642+000	-0.32279963+000	0.26191751+000
12	10	0.72165101+000	-0.13108671+000	-0.25329260+000	0.60101914+000
12	11	0.73366664+000	-0.11722862+000	-0.10087261+000	0.58514384+000
12	12	0.74573160+000	-0.99247194-001	0.11285045+000	0.39426671+000
12	13	0.75801487+000	-0.73433803-001	0.33030849+000	0.20843232+000
12	14	0.76762747+000	-0.39107329-001	0.49437433+000	0.93374371-001
12	15	0.77126224+000	0.	-0.28585711-001	0.
13	3	0.50820606+000	0.	0.12082613-001	0.
13	4	0.55715736+000	-0.13507940+000	-0.64253253+000	0.88161518-001
13	5	0.62418073+000	-0.17850902+000	-0.58045814+000	0.16856171+000
13	6	0.69353987+000	-0.19278921+000	-0.42012211+000	0.15028438+000
13	7	0.73936127+000	-0.18412357+000	-0.30340142+000	0.17561812+000
13	8	0.77161942+000	-0.16491205+000	-0.20574723+000	0.11135586+000
13	9	0.78798904+000	-0.14871797+000	-0.17297964+000	0.16910289+000
13	10	0.79765867+000	-0.13602631+000	-0.15852915+000	0.36828564+000
13	11	0.80642294+000	-0.12160918+000	-0.48378219-001	0.37247903+000
13	12	0.81523781+000	-0.10293299+000	0.82307148-001	0.23315322+000
13	13	0.82422846+000	-0.76150802-001	0.20063778+000	0.12040490+000
13	14	0.83127621+000	-0.40552513-001	0.31539956+000	0.45993579-001
13	15	0.83394113+000	0.	-0.22099911-001	0.
14	3	0.74371593+000	0.	0.73794477-002	0.
14	4	0.76883736+000	-0.14194359+000	-0.43791732+000	0.23282046-001
14	5	0.80323323+000	-0.18575783+000	-0.38248476+000	0.63037201-001
14	6	0.83921528+000	-0.19929689+000	-0.26718938+000	0.63145171-001
14	7	0.86309839+000	-0.18980311+000	-0.19310702+000	0.82750532-001
14	8	0.87999440+000	-0.16979765+000	-0.10562094+000	0.62215078-001
14	9	0.88859434+000	-0.15306269+000	-0.94306610-001	0.95991293-001
14	10	0.89368289+000	-0.13997579+000	-0.48480782-001	0.21248485+000
14	11	0.89830084+000	-0.12512552+000	-0.54555887-002	0.19125866+000
14	12	0.90295127+000	-0.10590142+000	0.83553305-001	0.11391309+000
14	13	0.90770087+000	-0.78344661-001	0.16155771+000	0.50975202-001
14	14	0.91142870+000	-0.41721132-001	0.22376864+000	0.14716334-001
14	15	0.91283828+000	0.	-0.12859822-001	0.
15	3	0.10000000+001	0.	0.	0.
15	4	0.10000000+001	-0.14453911+000	0.	-0.90713407-002
15	5	0.10000000+001	-0.18849878+000	0.	-0.11685569-001
15	6	0.10000000+001	-0.20175761+000	0.	-0.10575163-001
15	7	0.10000000+001	-0.19195068+000	0.	-0.90835593-002
15	8	0.10000000+001	-0.17164502+000	0.	-0.64877770-002
15	9	0.10000000+001	-0.15470554+000	0.	-0.52828388-002
15	10	0.10000000+001	-0.14146918+000	0.	-0.38950645-002
15	11	0.10000000+001	-0.12645514+000	0.	-0.29234606-002
15	12	0.10000000+001	-0.10702385+000	0.	-0.16974082-002
15	13	0.10000000+001	-0.79174214-001	0.	-0.41551751-003
15	14	0.10000000+001	-0.42163016-001	0.	0.33242084-003
15	15	0.10000000+001	0.	0.	0.

1 TEST POINTS HAVE NOT YET CONVERGED TO THE SPECIFIED MINIMUM CHANGE IN STRESS PER RELAX OF 0. PERCENT

* * * * STRESS CONDITION * * * *

AVERAGE COMPOSITE SIGMA X (PSI) = 1000.00
AVERAGE COMPOSITE SIGMA Y (PSI) = 0.
TEMP. (AMBIENT - CURING) (DEG. F) = 0.

STRESS COMPONENTS - INTERIOR AND BOUNDARY POINTS

I	J	U	V	SIGMA X	SIGMA Y	SIGMA Z	TAU XY
3	3	0.	0.	-122.223	-357.256	0.	0.
3	4	0.	-0.72864189-005	327.677	-605.272	0.	-0.000
3	5	0.	-0.13476831-004	1028.775	-661.177	0.	-0.000
3	6	0.	-0.22349317-004	2033.772	-793.688	0.	-0.000
3	7	0.	-0.32089341-004	2980.693	-1049.875	0.	-0.000
3	8	0.	-0.43824683-004	4159.456	-1927.130	0.	-0.000
3	9	0.	-0.56057771-004	0.	0.	0.	0.
3	10	0.	-0.46540775-004	0.	0.	0.	0.
3	11	0.	-0.64779973-004	236.835	-306.754	0.	-0.000
3	12	0.	-0.88385677-004	307.856	-310.858	0.	-0.000
3	13	0.	-0.12168665-003	378.007	-310.096	0.	-0.000
3	14	0.	-0.16494086-003	430.151	-307.499	0.	-0.000
3	15	0.	-0.21344497-003	448.592	-310.771	0.	0.
4	3	0.14432562-005	0.	-30.744	-828.837	0.	0.000
4	4	0.88990623-005	-0.14115456-004	431.042	-1035.115	0.	33.884
4	5	0.19107437-004	-0.23762792-004	1144.920	-1045.516	0.	48.549
4	6	0.33308684-004	-0.35660358-004	2103.088	-1022.790	0.	35.934
4	7	0.45868891-004	-0.46268588-004	2899.346	-840.793	0.	-4.615
4	8	0.55836079-004	-0.55054680-004	3065.701	-149.707	0.	-325.659
4	9	0.55868581-004	-0.57320387-004	0.	0.	0.	0.
4	10	0.94824791-004	-0.72178214-004	296.328	-224.540	0.	162.807
4	11	0.11287181-003	-0.88053281-004	341.815	-223.494	0.	67.094
4	12	0.12998964-003	-0.10823022-003	379.777	-229.050	0.	46.304
4	13	0.14624195-003	-0.13638371-003	415.003	-232.637	0.	26.822
4	14	0.15815180-003	-0.17274987-003	440.960	-234.371	0.	11.862
4	15	0.16265521-003	-0.21344497-003	449.788	-238.316	0.	-0.000
5	3	0.35970285-005	0.	-81.153	-1527.215	0.	0.000
5	4	0.15069367-004	-0.23465667-004	403.335	-1674.418	0.	34.549
5	5	0.30777116-004	-0.37846169-004	1143.515	-1619.877	0.	39.707
5	6	0.52084389-004	-0.53885627-004	2007.364	-1393.662	0.	-34.343
5	7	0.69751401-004	-0.65682588-004	2328.975	-692.219	0.	-223.057
5	8	0.76336744-004	-0.70430611-004	0.	0.	0.	0.
5	9	0.12954511-003	-0.94497616-004	450.691	-162.631	0.	169.628
5	10	0.15962078-003	-0.10728060-003	415.289	-135.089	0.	128.421
5	11	0.18389320-003	-0.11991873-003	429.038	-126.449	0.	88.434
5	12	0.20609855-003	-0.13540106-003	440.454	-128.380	0.	57.381
5	13	0.22684498-003	-0.15650668-003	452.914	-130.404	0.	31.836
5	14	0.24201646-003	-0.18344185-003	463.232	-132.003	0.	13.759
5	15	0.24775317-003	-0.21344497-003	466.543	-134.576	0.	-0.000
6	3	0.73082066-005	0.	-86.015	-2591.206	0.	0.000
6	4	0.22951320-004	-0.37779334-004	328.500	-2646.237	0.	26.045
6	5	0.44369630-004	-0.59209781-004	958.870	-2512.171	0.	25.739
6	6	0.70859659-004	-0.81135222-004	1366.615	-1333.124	0.	81.305
6	7	0.84985308-004	-0.86697477-004	0.	0.	0.	0.
6	8	0.18015962-003	-0.12178385-003	607.984	-73.108	0.	156.174
6	9	0.22251604-003	-0.13728426-003	561.400	-37.875	0.	115.816
6	10	0.24699357-003	-0.14668811-003	533.506	-28.577	0.	94.601
6	11	0.26859507-003	-0.15574013-003	515.335	-22.148	0.	73.701
6	12	0.28959320-003	-0.16603598-003	501.954	-18.476	0.	52.729
6	13	0.31011112-003	-0.17929059-003	492.531	-16.744	0.	31.853
6	14	0.32556226-003	-0.19559347-003	487.450	-16.440	0.	14.518
6	15	0.33140472-003	-0.21344497-003	485.558	-16.230	0.	-0.000
7	3	0.12499117-004	0.	-349.870	-3874.769	0.	0.000
7	4	0.30266577-004	-0.52059125-004	63.302	-3528.528	0.	-93.782
7	5	0.54593507-004	-0.78817645-004	831.206	-2380.475	0.	226.952
7	6	0.78559556-004	-0.92539353-004	0.	0.	0.	0.
7	7	0.18982845-003	-0.13459492-003	787.678	-24.235	0.	155.972
7	8	0.26324809-003	-0.15785110-003	656.883	22.257	0.	115.172
7	9	0.29892304-003	-0.16874345-003	609.271	40.123	0.	90.199
7	10	0.31967729-003	-0.17516495-003	581.962	47.937	0.	75.326
7	11	0.33824309-003	-0.18112620-003	558.798	54.017	0.	60.997
7	12	0.35662445-003	-0.18754012-003	537.525	58.911	0.	45.799
7	13	0.37498659-003	-0.19524224-003	518.377	62.620	0.	29.290
7	14	0.38907762-003	-0.20410865-003	505.123	64.677	0.	13.918
7	15	0.39440579-003	-0.21344497-003	500.668	67.309	0.	-0.000
8	3	0.16557933-004	0.	2689.863	-5350.254	0.	0.000
8	4	0.35758220-004	-0.68193480-004	469.322	-3287.130	0.	1220.199
8	5	0.62046954-004	-0.88848538-004	0.	0.	0.	0.
8	6	0.19218619-003	-0.14146524-003	997.116	37.726	0.	129.721
8	7	0.27852449-003	-0.17023893-003	805.005	62.459	0.	116.941
8	8	0.33780809-003	-0.18631661-003	692.485	90.788	0.	88.803
8	9	0.36734249-003	-0.19340557-003	641.133	103.104	0.	71.453
8	10	0.38464645-003	-0.19729278-003	611.812	109.431	0.	60.546
8	11	0.40022779-003	-0.20064059-003	586.066	114.644	0.	49.990
8	12	0.41578291-003	-0.20389488-003	561.150	119.290	0.	38.544
8	13	0.43149438-003	-0.20726639-003	537.050	123.357	0.	25.512
8	14	0.44368311-003	-0.21049403-003	519.188	126.075	0.	12.456
8	15	0.44829197-003	-0.21344497-003	513.310	130.021	0.	-0.000
9	3	0.37640618-004	0.	0.	0.	0.	0.
9	4	0.41323122-004	-0.59504662-004	0.	0.	0.	0.
9	5	0.14975363-003	-0.11681448-003	1258.044	59.448	0.	94.506
9	6	0.26033575-003	-0.16520925-003	989.499	68.354	0.	112.152
9	7	0.33362165-003	-0.18892138-003	818.501	101.164	0.	98.551
9	8	0.38461080-003	-0.20150311-003	708.693	125.224	0.	75.857
9	9	0.41026124-003	-0.20658102-003	656.210	136.104	0.	61.694
9	10	0.42535019-003	-0.20910961-003	625.977	141.947	0.	52.599
9	11	0.43898081-003	-0.21104599-003	599.139	146.891	0.	43.749
9	12	0.45263792-003	-0.21259393-003	572.771	151.472	0.	34.077
9	13	0.46649621-003	-0.21364246-003	546.696	155.707	0.	22.871
9	14	0.47729769-003	-0.21387200-003	526.920	158.722	0.	11.293
9	15	0.48138199-003	-0.21344497-003	520.461	163.256	0.	-0.000
10	3	0.42969421-004	0.	0.	0.	0.	0.
10	4	0.11299134-003	-0.85160162-004	1531.622	157.878	0.	5.005
10	5	0.20886429-003	-0.13699912-003	1250.993	91.738	0.	79.055
10	6	0.30669688-003	-0.17940021-003	991.999	97.522	0.	100.122
10	7	0.37148745-003	-0.20014826-003	826.729	123.950	0.	87.439
10	8	0.41690762-003	-0.21067808-003	718.003	145.542	0.	67.811
10	9	0.43987356-003	-0.21455673-003	665.019	155.803	0.	55.428
10	10	0.45341592-003	-0.21626915-003	634.290	161.469	0.	47.400
10	11	0.46567224-003	-0.21735285-003	606.845	166.343	0.	39.564
10	12	0.47797706-003	-0.21786605-003	579.681	170.954	0.	30.966
10	13	0.49049405-003	-0.21750440-003	552.545	175.332	0.	20.923
10	14	0.50027448-003	-0.21591659-003	531.741	178.539	0.	10.389
10	15	0.50397270-003	-0.21344497-003	524.973	183.413	0.	-0.000
11	3	0.12621430-003	0.	1703.805	157.357	0.	0.000
11	4	0.18674436-003	-0.10271309-003	1509.298	135.246	0.	35.813
11	5	0.26962120-003	-0.15349548-003	1242.638	103.826	0.	81.508

11	6	0.35467182-003	-0.19204364-003	993.811	118.577	0.	89.998
11	7	0.41099972-003	-0.21036880-003	834.214	143.374	0.	77.067
11	8	0.45070227-003	-0.21910068-003	726.431	163.676	0.	59.854
11	9	0.47085798-003	-0.22190016-003	673.066	173.660	0.	49.062
11	10	0.48276808-003	-0.22287042-003	641.906	179.289	0.	42.043
11	11	0.49356467-003	-0.22317357-003	613.923	184.194	0.	35.180
11	12	0.50442267-003	-0.22273474-003	586.058	188.907	0.	27.632
11	13	0.51549079-003	-0.22107152-003	557.999	193.468	0.	18.764
11	14	0.52415715-003	-0.21780490-003	536.309	196.878	0.	9.355
11	15	0.52743412-003	-0.21344497-003	529.278	202.070	0.	-0.000
12	3	0.22501258-003	0.	1657.214	65.076	0.	0.000
12	4	0.27441161-003	-0.11736781-003	1488.666	111.578	0.	45.343
12	5	0.34204800-003	-0.16832826-003	1238.136	109.395	0.	77.263
12	6	0.41207533-003	-0.20434575-003	997.480	135.187	0.	77.803
12	7	0.45854069-003	-0.22060998-003	842.183	161.194	0.	65.110
12	8	0.49144530-003	-0.22764006-003	735.087	181.400	0.	50.443
12	9	0.50820813-003	-0.22937782-003	681.332	191.472	0.	41.396
12	10	0.51813271-003	-0.22960675-003	649.735	197.227	0.	35.519
12	11	0.52714330-003	-0.22912293-003	621.209	202.289	0.	29.775
12	12	0.53621990-003	-0.22771728-003	592.642	207.209	0.	23.450
12	13	0.54549000-003	-0.22472513-003	563.677	212.039	0.	15.988
12	14	0.55276250-003	-0.21973971-003	541.126	215.702	0.	7.998
12	15	0.55551241-003	-0.21344497-003	533.844	221.239	0.	-0.000
13	3	0.35507355-003	0.	1613.598	-9.583	0.	0.000
13	4	0.39019459-003	-0.12957768-003	1467.828	87.098	0.	42.055
13	5	0.43828178-003	-0.18161777-003	1235.168	109.224	0.	62.437
13	6	0.48865044-003	-0.21627185-003	1002.828	147.390	0.	59.303
13	7	0.52220403-003	-0.23090845-003	850.942	177.064	0.	48.607
13	8	0.54607113-003	-0.23637121-003	744.246	198.596	0.	37.477
13	9	0.55826743-003	-0.23707247-003	690.044	209.262	0.	30.747
13	10	0.56550140-003	-0.23656081-003	657.979	215.379	0.	26.396
13	11	0.57207868-003	-0.23528000-003	628.885	220.785	0.	22.153
13	12	0.57871431-003	-0.23288466-003	599.597	226.071	0.	17.482
13	13	0.58550359-003	-0.22852062-003	569.715	231.305	0.	11.956
13	14	0.59083935-003	-0.22175159-003	546.303	235.307	0.	5.996
13	15	0.59285693-003	-0.21344497-003	538.778	241.247	0.	-0.000
14	3	0.51555961-003	0.	1581.402	-57.287	0.	0.000
14	4	0.53366585-003	-0.13754635-003	1451.533	68.171	0.	24.445
14	5	0.55845663-003	-0.19083472-003	1233.542	106.031	0.	34.540
14	6	0.58468937-003	-0.22511226-003	1007.935	153.966	0.	32.097
14	7	0.60224030-003	-0.23881120-003	858.268	187.747	0.	26.062
14	8	0.61476830-003	-0.24319010-003	751.693	211.262	0.	20.049
14	9	0.62118449-003	-0.24312474-003	697.097	222.784	0.	16.450
14	10	0.62499508-003	-0.24205062-003	664.649	229.382	0.	14.130
14	11	0.62846340-003	-0.24015494-003	635.098	235.213	0.	11.869
14	12	0.63196639-003	-0.23698655-003	605.238	240.924	0.	9.380
14	13	0.63555515-003	-0.23154011-003	574.639	246.595	0.	6.429
14	14	0.63837919-003	-0.22335438-003	550.562	250.944	0.	3.229
14	15	0.63944703-003	-0.21344497-003	542.857	257.253	0.	-0.000
15	3	0.69064539-003	0.	1561.496	-77.644	0.	-0.000
15	4	0.69064539-003	-0.14055951-003	1441.004	59.705	0.	-0.000
15	5	0.69064539-003	-0.19431988-003	1233.167	104.895	0.	-0.000
15	6	0.69064539-003	-0.22845504-003	1011.842	157.045	0.	-0.000
15	7	0.69064539-003	-0.24179942-003	863.147	192.419	0.	0.000
15	8	0.69064539-003	-0.24576849-003	756.240	216.570	0.	0.000
15	9	0.69064539-003	-0.24541325-003	701.158	228.315	0.	0.000
15	10	0.69064539-003	-0.24412646-003	668.314	235.019	0.	0.000
15	11	0.69064539-003	-0.24199827-003	638.326	240.932	0.	0.000
15	12	0.69064539-003	-0.23853758-003	607.943	246.712	0.	0.000
15	13	0.69064539-003	-0.23268185-003	576.712	252.440	0.	0.000
15	14	0.69064539-003	-0.22396043-003	552.063	256.824	0.	0.000
15	15	0.69064539-003	-0.21344497-003	544.169	263.236	0.	0.000

* * * * STRESS CONDITION * * * *

AVERAGE COMPOSITE SIGMA X	(PSI)	=	1000.00
AVERAGE COMPOSITE SIGMA Y	(PSI)	=	0.
TEMP. (AMBIENT - CURING)	(DEG. F)	=	0.

STRESS COMPONENTS - INTERFACE POINTS

		IN MATRIX				IN FIBER			
I	J	SIGMA X	SIGMA Y	SIGMA Z	TAU XY	SIGMA X	SIGMA Y	SIGMA Z	TAU XY
3	10	165.440	-302.132	0.	-0.000	0.	0.	0.	0.
4	9	331.734	-206.118	0.	315.980	4208.321	939.775	0.	-342.019
5	8	537.509	-228.538	0.	199.134	2316.953	278.776	0.	-750.993
6	7	778.433	-131.965	0.	227.651	1466.305	342.717	0.	-343.770
7	6	999.288	-86.684	0.	182.824	567.066	-713.025	0.	703.130
8	5	1270.550	16.184	0.	173.029	774.453	-1723.917	0.	1102.149
9	4	1526.272	81.182	0.	12.419	779.401	-3948.108	0.	2070.228
10	3	1754.614	271.390	0.	0.000	0.	0.	0.	0.

* * * * STRESS CONDITION * * * *

AVERAGE COMPOSITE SIGMA X (PSI) = 1000.00
AVERAGE COMPOSITE SIGMA Y (PSI) = 0.
TEMP. (AMBIENT - CURING) (DEG. F) = 0.

PRINCIPAL STRESSES - INTERIOR AND BOUNDARY POINTS

I	J	SIGMA 1	SIGMA 2	THETA DEG	VON MISES
3	3	-122.223	-357.256	0.	98905.568
3	4	327.677	-605.272	0.000	672060.232
3	5	1028.775	-661.177	0.000	2175735.973
3	6	2033.772	-793.688	0.000	6380346.151
3	7	2980.693	-1049.875	0.000	13116125.448
3	8	4159.456	-1927.130	0.000	29030714.446
3	9	0.	0.	0.	0.
3	10	0.	0.	0.	0.
3	11	236.835	-306.754	0.000	222838.529
3	12	307.856	-310.858	0.000	287106.989
3	13	378.007	-310.096	0.000	356267.011
3	14	430.151	-307.499	0.000	411856.420
3	15	448.592	-310.771	0.	437222.682
4	3	-30.744	-828.837	-0.000	662433.618
4	4	431.824	-1035.897	-1.323	1706880.601
4	5	1145.996	-1046.592	-1.269	3608049.820
4	6	2103.501	-1023.203	-0.659	7623967.353
4	7	2899.352	-840.799	0.071	11550957.365
4	8	3098.353	-182.359	5.725	10198057.591
4	9	0.	0.	0.	0.
4	10	343.029	-271.241	-16.005	284284.330
4	11	349.669	-231.348	-6.677	256685.334
4	12	383.279	-232.552	-4.324	290115.086
4	13	416.112	-233.746	-2.367	325050.467
4	14	441.168	-234.579	-1.006	353145.572
4	15	449.788	-238.316	0.000	366295.476
5	3	-81.153	-1527.215	-0.000	2215034.306
5	4	403.909	-1674.992	-0.952	3645285.505
5	5	1144.085	-1620.447	-0.823	5788708.084
5	6	2007.711	-1394.009	0.578	8772930.501
5	7	2345.354	-708.599	4.200	7664715.766
5	8	0.	0.	0.	0.
5	9	494.479	-206.419	-14.474	389187.951
5	10	443.779	-163.579	-12.508	296291.198
5	11	442.777	-140.189	-8.831	277776.855
5	12	446.184	-134.111	-5.703	276904.497
5	13	454.646	-132.137	-3.115	284239.154
5	14	463.550	-132.321	-1.323	293724.671
5	15	466.543	-134.576	0.000	298558.516
6	3	-86.015	-2591.206	-0.000	6498864.856
6	4	328.728	-2646.465	-0.502	7981804.196
6	5	959.061	-2512.362	-0.425	9641271.130
6	6	1369.062	-1335.571	-1.723	5486557.378
6	7	0.	0.	0.	0.
6	8	642.087	-107.211	-12.318	492607.857
6	9	583.004	-59.479	-10.566	378107.236
6	10	549.001	-44.072	-9.302	327539.627
6	11	525.258	-32.071	-7.668	293769.704
6	12	507.242	-23.765	-5.728	269914.164
6	13	494.516	-18.728	-3.565	254158.119
6	14	487.868	-16.858	-1.649	246524.092
6	15	485.558	-16.230	0.000	243910.282
7	3	-349.870	-3874.769	-0.000	13780580.120
7	4	65.749	-3530.975	1.495	12704267.737
7	5	847.165	-2396.434	-4.022	8490755.323
7	6	0.	0.	0.	0.
7	7	816.610	-53.167	-10.509	713095.767
7	8	677.138	2.002	-9.974	457164.328
7	9	623.224	26.170	-8.793	372783.340
7	10	592.384	37.516	-7.877	330102.315
7	11	566.064	46.751	-6.793	296150.280
7	12	541.869	54.568	-5.417	267030.489
7	13	520.252	60.745	-3.662	242749.380
7	14	505.562	64.238	-1.808	227243.807
7	15	500.668	67.309	0.000	221499.188
8	3	2689.863	-5350.254	-0.000	50252037.471
8	4	830.877	-3648.685	-16.505	17034868.828
8	5	0.	0.	0.	0.
8	6	1014.346	20.496	-7.566	1008528.785
8	7	822.987	44.477	-8.742	642681.023
8	8	705.318	77.956	-8.223	448566.925
8	9	650.460	93.776	-7.437	370894.592
8	10	619.006	102.237	-6.776	330335.272
8	11	591.308	109.402	-5.987	296924.205
8	12	564.487	115.953	-4.948	266636.267
8	13	538.617	121.789	-3.516	239343.084
8	14	519.583	125.681	-1.813	220460.125
8	15	513.310	130.021	0.000	213651.441
9	3	0.	0.	0.	0.
9	4	0.	0.	0.	0.
9	5	1265.450	52.042	-4.481	1538215.253
9	6	1002.957	54.896	-6.843	953877.849
9	7	831.794	87.871	-7.682	626511.549
9	8	718.394	115.523	-7.288	446444.065
9	9	663.427	128.886	-6.673	371240.989
9	10	631.627	136.298	-6.131	331440.087
9	11	603.333	142.697	-5.475	298278.734
9	12	575.510	148.733	-4.595	267735.800
9	13	548.029	154.374	-3.336	239566.019
9	14	527.266	158.376	-1.755	219586.325
9	15	520.461	163.256	0.000	212563.453
10	3	0.	0.	0.	0.
10	4	1531.640	157.860	-0.209	2129055.656
10	5	1256.360	86.372	-3.883	1477385.164
10	6	1003.069	86.452	-6.309	926904.691
10	7	837.445	113.234	-6.987	619308.578
10	8	725.926	137.619	-6.664	446006.468
10	9	670.982	149.839	-6.141	372129.279
10	10	638.995	156.764	-5.669	332717.937
10	11	610.370	162.818	-5.092	299682.208
10	12	582.014	168.621	-4.308	269033.611
10	13	553.702	174.175	-3.165	240481.426
10	14	532.046	178.234	-1.683	220011.524
10	15	524.973	183.413	0.000	212949.640
11	3	1703.805	157.357	-0.000	2659606.586
11	4	1510.231	134.314	-1.492	2095992.981
11	5	1248.443	98.022	-4.073	1445842.384

11	6	1002.970	109.419	-5.811	908176.855
11	7	842.707	134.881	-6.289	614682.359
11	8	732.727	157.381	-6.004	446339.820
11	9	677.840	168.886	-5.558	373511.797
11	10	645.696	175.499	-5.151	334403.885
11	11	616.784	181.333	-4.649	301460.763
11	12	587.971	186.993	-3.961	270729.726
11	13	558.962	192.505	-2.939	241893.948
11	14	536.566	196.620	-1.578	221063.181
11	15	529.278	202.070	0.000	214016.255
12	3	1657.214	65.076	-0.000	2642748.375
12	4	1490.157	110.087	-1.884	2068639.954
12	5	1243.400	104.130	-3.898	1427410.078
12	6	1004.444	128.223	-5.115	896555.433
12	7	848.352	155.024	-5.413	612218.727
12	8	739.645	176.841	-5.163	447547.888
12	9	684.806	187.998	-4.796	375559.849
12	10	652.506	194.456	-4.461	336693.414
12	11	623.314	200.184	-4.045	303816.797
12	12	594.064	205.788	-3.469	273009.174
12	13	564.403	211.314	-2.598	243937.671
12	14	541.322	215.505	-1.407	222814.230
12	15	533.844	221.239	0.000	215828.917
13	3	1613.598	-9.583	-0.000	2619252.329
13	4	1469.108	85.818	-1.743	2039565.881
13	5	1238.620	105.773	-3.164	1414354.192
13	6	1006.920	143.298	-3.947	890132.182
13	7	854.430	173.576	-4.104	611871.339
13	8	746.808	196.034	-3.911	449751.225
13	9	692.002	207.303	-3.644	378387.470
13	10	659.548	213.810	-3.401	339700.264
13	11	630.084	219.586	-3.098	306866.753
13	12	600.414	225.254	-2.674	275990.208
13	13	570.137	230.883	-2.021	246728.057
13	14	546.418	235.192	-1.104	225375.168
13	15	538.778	241.247	0.000	218503.364
14	3	1581.402	-57.287	-0.000	2594708.405
14	4	1451.964	67.740	-1.012	2014434.025
14	5	1234.599	104.973	-1.753	1405654.694
14	6	1009.140	152.761	-2.149	887541.226
14	7	859.279	186.735	-2.223	612772.921
14	8	752.436	210.519	-2.122	452075.904
14	9	697.667	222.214	-1.984	381086.960
14	10	665.107	228.923	-1.857	342514.359
14	11	635.450	234.861	-1.699	309714.417
14	12	605.480	240.683	-1.474	278805.281
14	13	574.765	246.469	-1.122	249440.207
14	14	550.597	250.909	-0.617	227962.472
14	15	542.857	257.253	0.000	221221.304
15	3	1561.496	-77.644	0.000	2565540.093
15	4	1441.004	59.705	0.000	1994021.504
15	5	1233.167	104.895	0.000	1402351.787
15	6	1011.842	157.045	0.000	889581.615
15	7	863.147	192.419	-0.000	615962.483
15	8	756.240	216.570	-0.000	455022.246
15	9	701.158	228.315	-0.000	383665.271
15	10	668.314	235.019	-0.000	344810.661
15	11	638.326	240.932	-0.000	311715.638
15	12	607.943	246.712	-0.000	280474.948
15	13	576.712	252.440	-0.000	250737.516
15	14	552.063	256.824	-0.000	228948.980
15	15	544.169	263.236	-0.000	222167.956

* * * * STRESS CONDITION * * * *

AVERAGE COMPOSITE SIGMA X	(PSI)	=	1000.00
AVERAGE COMPOSITE SIGMA Y	(PSI)	=	0.
TEMP. (AMBIENT - CURING)	(DEG. F)	=	0.

PRINCIPAL STRESSES - INTERFACE POINTS

		IN MATRIX				IN FIBER			
I	J	SIGMA 1	SIGMA 2	THETA	VON MISES	SIGMA 1	SIGMA 2	THETA	VON MISES
3	10	165.440	-302.132	0.000	168638.702	0.	0.	0.	0.
4	9	477.735	-352.119	-24.800	520437.115	4243.726	904.369	5.910	14989199.108
5	8	586.182	-277.210	-13.735	582949.831	2563.776	31.953	18.194	6492047.258
6	7	832.185	-185.717	-13.285	881573.587	1563.138	245.884	15.732	2119509.419
7	6	1029.240	-116.636	-9.304	1192985.945	877.836	-1023.795	-23.845	2717477.675
8	5	1293.981	-7.246	-7.712	1683814.637	1191.160	-2140.624	-20.711	8550957.136
9	4	1526.379	81.076	-0.492	2212654.467	1557.808	-4726.515	-20.606	32129712.611
10	3	1754.614	271.390	-0.000	2676139.654	0.	0.	0.	0.

EFFECTIVE COMPOSITE ELASTIC MODULI

EX = 0.20271+007

EY = 0.20271+007

EFFECTIVE COMPOSITE THERMAL EXP. COEF. (IN/IN/DEG. F)

ALPHA X = 0.

ALPHA Y = 0.

NASA-Langley, 1966 CR-620

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—NATIONAL AERONAUTICS AND SPACE ACT OF 1958

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